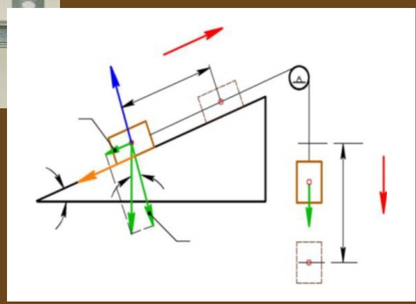
Claudiu SCHONSTEIN

Gabriel FODOR Aurora Felicia CRISTEA

MECHANICS
Laboratory Work

Statics-Kinematics-Dynamics



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Foreword

The current guide appeared as a necessity regarding the modernization of classical mechanics laboratory papers for the 1st and 2nd university years, but also as the need to add new data to them, compared to previous editions. In addition, there was a need to edit them in an international language as well, due to the mechanical specializations taught in another language of international circulation, as well as the flow of foreign students who attend the faculty with Erasmus scholarships.

Theoretical Mechanics is a basic discipline that lays the foundations of the future mechanical engineer, for this reason it is taught in all specializations of the 1st and 2nd years of study.

The presented laboratory papers are clearly designed, having the following parts: - the theme and requirement of the paper, - the theoretical concept part and - the experimental part containing either measurements or the graphic part.

The editing of the book was realized by the authors, both in terms of the text and the graphical part.

The authors thank to all those who supported them, both within the department and outside.

2022 Authors,

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STATICS

ANALITICAL AND GRAPHICAL REDUCTION OF A COPLANAR SYSTEM OF FORCES

1. Purpose of the work

There is considered a rectangular plate, on which is acting a system of five forces, located in the plate`s plane as can be seen in Figure 1. Determine:

a) Analytically the wrench (force-couple system) $\bar{\bar{\tau}}_O = \left(\bar{R}^{an}, \bar{M}_O^{an}\right)$, the equations of "equipollent screwdriver" line of action and the distance d^{an} from point O to its line.

b) Graphically the wrench (force-couple system) $\bar{\bar{\tau}}_O = \left(\bar{R}^{gr}, \bar{M}_O^{gr}\right)$, the resultant vector R^{gr} and the distance d^{gr} from point O to the "equipollent screwdriver" line of action;

c) Compare the analytical and graphical obtained results, by computing the relative errors ε_r^d and ε_r^R.

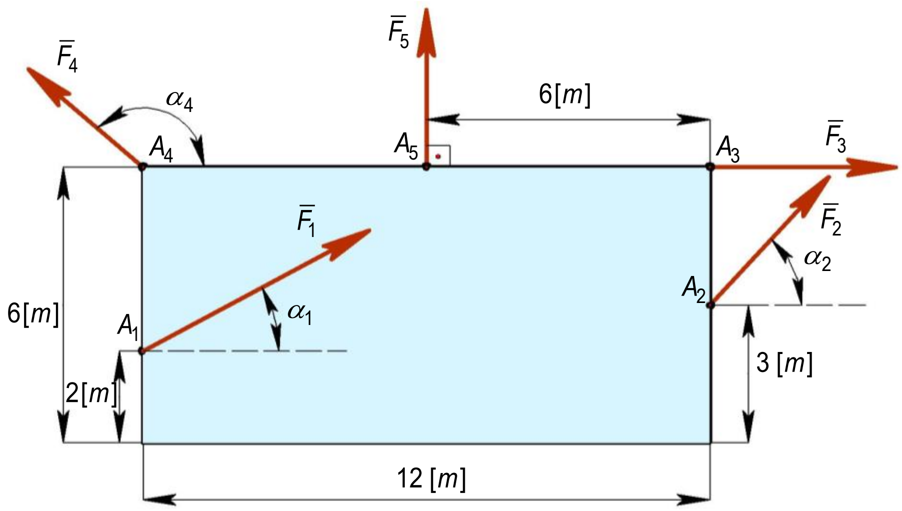


Figure 1

The input data are:

$$F_1 = 20 + n\ [N]; \qquad \alpha_1 = 48 - n\ [^0]\,;$$

$$F_2 = 80 - n\ [N]; \qquad \alpha_2 = 50 + n\ [^0];$$

$$F_3 = 35 + 2n\ [N]\,; \qquad \alpha_3 = 0\ [^0]$$

$$F_4 = 8 + n + 2m\ [N]\ ; \quad \alpha_4 = 92 + 2n\ [^0]$$

$$F_5 = 10 + n + 3m\ [N]\,; \ \alpha_5 = 90\ [^0] \tag{1}$$

where, n is the student order number in the group, and m the semigroup number.

2. Theoretical considerations

There is considered in Figure 2, a rigid body (S) on which a system of $i = 1 \rightarrow n$ forces is acting, denoted F_i in points A_i, each force being linked to origin O with the position vector $\bar{r}_i$.

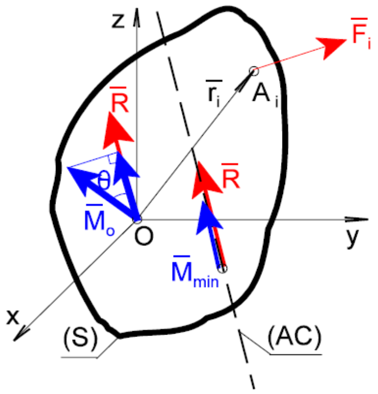


Figure 2

The wrench (force-couple system) $\bar{\bar{\tau}}_O$, is having the following components:

$$\bar{\bar{\tau}}_O = \begin{bmatrix} \bar{R} \\ -- \\ \bar{M}_O \end{bmatrix} \tag{2}$$

where $\bar{R}$ is the resultant vector of the forces, and $\bar{M}_O$ is the resultant moment (couple) with respect to point O, known by components as:

$$\bar{R} = R_x \cdot \bar{i} + R_y \cdot \bar{j} + R_z \cdot \bar{k} \tag{3}$$

$$\bar{M}_O = M_x \cdot \bar{i} + M_y \cdot \bar{j} + M_z \cdot \bar{k} \tag{4}$$

Hence, the scalar components of the force-couple system (wrench) (2), which have to be determined are:

$$\tau_O = \begin{bmatrix} R_x & R_y & R_z \\ -- & -- & -- \\ M_{Ox} & M_{Oy} & M_{Oz} \end{bmatrix} \tag{5}$$

The components of the wrench (5) are expressed as:

$$R_x = \sum_{i=1}^{n} F_{ix} \quad R_y = \sum_{i=1}^{n} F_{iy} \quad R_z = \sum_{i=1}^{n} F_{iz} \tag{6}$$

$$\begin{aligned} M_{Ox} &= \sum_{i=1}^{n} y_i \cdot F_{iz} - z_i \cdot F_{iy} \\ M_{Oy} &= \sum_{i=1}^{n} z_i \cdot F_{ix} - x_i \cdot F_{iz} \\ M_{Oz} &= \sum_{i=1}^{n} x_i \cdot F_{iy} - y_i \cdot F_{ix} \end{aligned} \tag{7}$$

In expressions (6)-(7) is well known the fact that, F_{ix}, F_{iy}, F_{iz} are the projections of the forces on the reference system axis, and x_i, y_i, z_i, are the coordinates of each point A_i.

The modules of the two vectors are:

$$R^{an} = \sqrt{R_x^2 + R_y^2 + R_z^2} \tag{8}$$

$$M_O = \sqrt{M_{Ox}^2 + M_{Oy}^2 + M_{Oz}^2} \tag{9}$$

The expressions of the equations of "equipollent screwdriver" line of action are obtained from:

$$\frac{M_{Ox} - y \cdot R_z + z \cdot R_y}{R_x} = \frac{M_{Oy} - z \cdot R_x + x \cdot R_z}{R_y} = \frac{M_{Oz} - x \cdot R_y + y \cdot R_x}{R_z} \tag{10}$$

In the case of the rectangular plate considered in Figure 1, on which is acting a system of coplanar forces in the plate`s plane, i.e. $xOy \equiv (z_i = 0)$, in concordance with Figure 3, there can be written that:

$$\begin{gathered} R_x \neq 0,\ R_y \neq 0,\ R_z = 0 \\ M_{Ox} = 0,\ M_{Oy} = 0,\ M_{Oz} \neq 0 \end{gathered} \tag{11}$$

According to (11), the expressions of equipollent screwdriver line of action are:

$$\frac{z \cdot R_y}{R_x} = \frac{-z \cdot R_x}{R_y} = \frac{M_{Oz} - x \cdot R_y + y \cdot R_x}{0} \to \begin{cases} z = 0 \\ -x \cdot R_y + y \cdot R_x + M_{Oz} = 0 \end{cases} \tag{12}$$

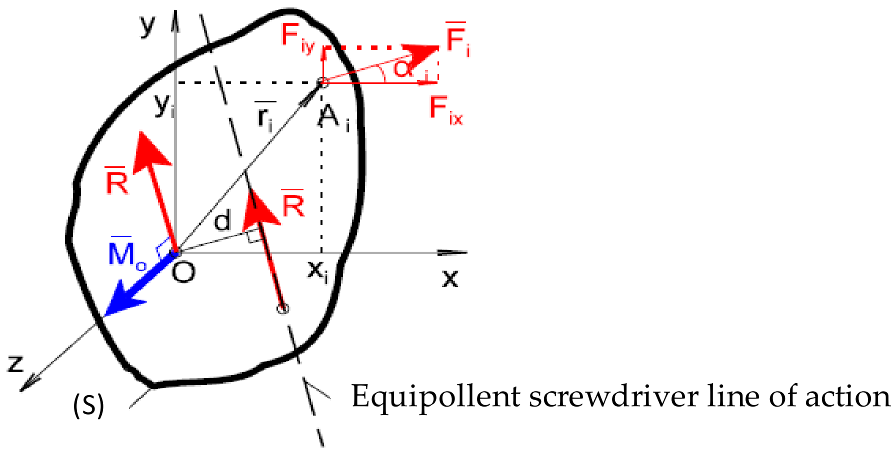


Figure 3

With respect to O, the forces system is reducible to a wrench (force-couple system) $\bar{\bar{\tau}}_O = \left(\bar{R}, \bar{M}_O\right)$, and with respect to equipollent screwdriver line of action, to a simplest resultant of a force-couple system "equipollent screwdriver" $\bar{\bar{\tau}}_{min} = \left(\bar{R}, \bar{M}^{min}\right)$.

The minimum value of the resultant moment is obtained by projecting the moment $\bar{M}_O$ on the resultant force $\bar{R}$ support axis. It results:

$$M^{min} = \frac{\bar{M}_O \cdot \bar{R}}{R} \tag{13}$$

When $\bar{M}_O \neq 0$, $\bar{R} \neq 0$ and $\bar{M}_O \cdot \bar{R} = 0$, means that $\bar{M}_O \perp \bar{R}$. The previous expression becomes: $M^{min} = 0$, resulting that the system of forces, from mechanical point of view, is equivalent with a unique resultant, whose support axis is the equipollent screwdriver line of action, defined by (10).

Hence, considering the Varignon`s Theorem; according to which: *when the system of forces is mechanically equivalent to a unique resultant, the resultant moment of a system of forces with respect to O is also equal to the moment of resultant of the system of forces determined with respect to the same pole,* it can be established analytically the distance from the point O to the equipollent screwdriver line of action as:

$$d^{an} = \frac{M_O}{R^{an}} \tag{14}$$

3. Development of the laboratory work

Each student establishes his own input data based on order number in the group and semigroup number with (1).

Using expressions (6)-(9), (12) and (14), there are computed the analytical results for resultant vector and resultant moment, the expression of equipollent screwdriver line of action, and the distance from the reference`s system origin to its line of action. The results are filled in **Table 1**.

Table 1

R_x = ...[N]	R_y = ...[N]	R^{an} = ...[N]
M_z = ...[Nm]		M_O = ...[Nm]
Equipollent screwdriver equations	z = 0 - ....x + ...y + ... = 0	
d^{an} = ...[m]		

For the **graphical determination**, (see Figure 4) the following steps are considered:

1. Using a drawing scale i.e. $k_L = 1\,cm/1\,m$, the plate is represented. Then without taking into account the magnitudes, in the points A_i are represented the forces F_i, considering the angles α_i established in (1), measured with the protractor.

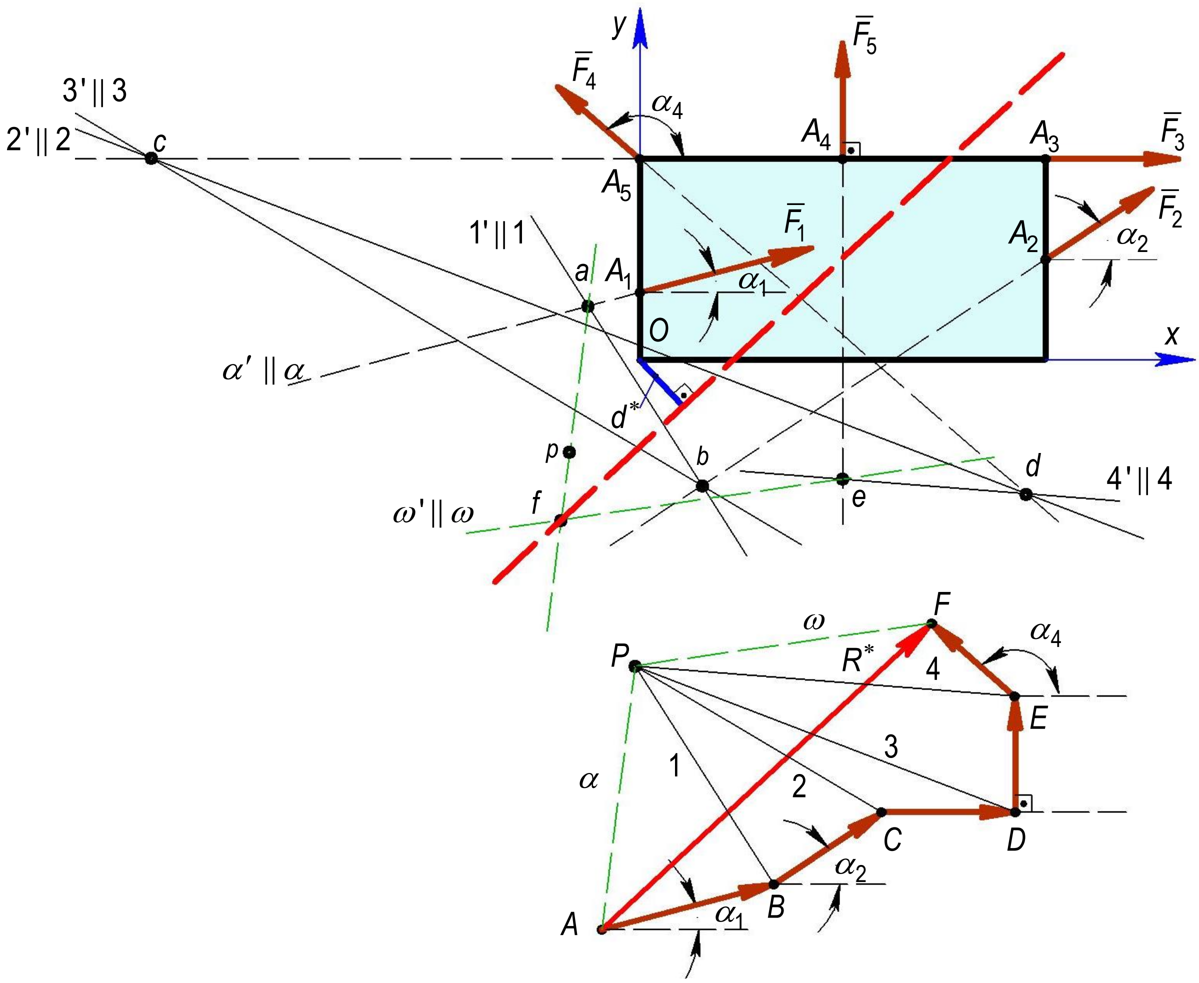


Figure 4

2. There is considered a scale for the forces, i.e. $k_F = 1\,cm/10\,N$. Using the head-to-tail method (force`s polygon) there are determined graphically the resultant vector for the five forces. There are denoted the vertices of the force's polygon with A, B, C, D, E and F. The last side of the polygon $AF = R^*$ is measured. To obtain the real magnitude for the resultant vector R^{gr}, the measured resultant vector R^* is multiplied by k_F:

$$R^{gr} = R^* \cdot k_F = ...[N] \tag{15}$$

3. There is considered an arbitrary point P near the force`s polygon. The point P merges with the polygon vertices A, B, C, D, E, F, noting the first joint α, the last ω, and the intermediates with 1, 2, 3 and 4, respecting the vertices order.

4. Is constructed the funicular polygon of the forces, characteristic to F_i as:

- ✓ There is considered an arbitrary point p located in the plate`s plane in the vicinity of $A_1$. Through point $p$ is drawn the line $\alpha` || \alpha$ which intersects the support axis of F_1 in point *a.*
- ✓ In *a* there is sketched the parallel line 1` || 1, which intersects the support axis of F_2 in point *b.*
- ✓ In *b* there is sketched the parallel line 2` || 2, which intersects the support axis of F_3 in point *c.*
- ✓ In *c* there is sketched the parallel line 3` || 3, which intersects the support axis of F_4 in point *d.*
- ✓ In *d* there is sketched the parallel line 4` || 4, which intersects the support axis of F_5 in point *e.*
- ✓ In *e* there is sketched the parallel line $\omega` || \omega$, which intersects $\alpha`$ in point *f.*

5. The point *f,* is a point belonging to the equipollent screwdriver line of action. The polygon *pabcdef*, represents the funicular polygon, and funicular polygon method allows the determination of a point belonging to the equipollent screwdriver line of action.

6. There is known that equipollent screwdriver line of action has the same orientation as the support axis of resultant vector $\bar{R}$, so that through point *f,* will be sketched a parallel line to R^*, representing the equipollent screwdriver line of action for the forces .

7. From the point O, will be drawn a perpendicular d^* to the equipollent screwdriver line of action.

8. There is measured d^*, the real distance from the origin of the reference system to the equipollent screwdriver line of action will be:

$$d^{gr} = d^* \cdot k_L = ...[m] \tag{16}$$

The relative errors are obtained by comparing the analytical values noted (*an*) with graphical values noted *(gr),* according to:

$$\varepsilon_r^R = \frac{\left|R^{an} - R^{gr}\right|}{R^{an}} \cdot 100 = ...[\%] \tag{17}$$

$$\varepsilon_r^d = \frac{\left|d^{an} - d^{gr}\right|}{d^{an}} \cdot 100 = ...[\%] \tag{18}$$

ANALYTICAL AND GRAPHICAL DETERMINATION OF THE MASS CENTER FOR A PLANE PLATE

1.The purpose of the work

A homogeneous plane plate is considered in Figure 1. The **mass center** always coincides with the **centroid** for a body with constant density.

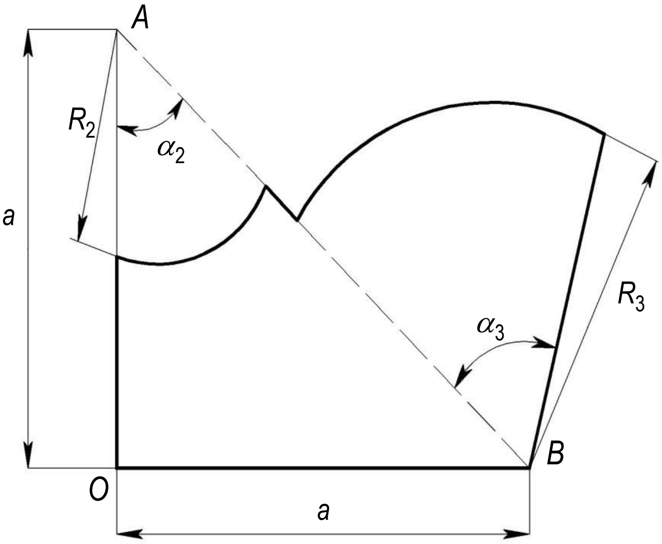


Figure 1

Determine:

a) By analytical method, the centroid of the plate (C^{an});

b) By graphical method, the centroid of the plate (C^{gr});

c) Compare the analytical results with the graphical ones by calculating the relative errors ε_{x_c} and ε_{y_c} .

The composite plate dimensions are given, as:

$a = 10 + 0.5 \cdot n \left[cm \right]$

$R_2 = 7 + 0.1 \cdot n \left[cm \right]$; $\alpha_2 = 45$ [°]

$R_3 = 8 + 0.2 \cdot n$ [cm] ; $\alpha_3 = 40 + m$ [°]

where:

n –is the student order number from the group

m – represents the number of the semigroup.

2. Theoretical considerations

A homogeneous plate having a complex shape is divided into simpler geometric parts, hence the position vector of the mass center for a complex shape plate is:

$$\overline{r}_C = \frac{\sum_{i=1}^{n} \overline{r}_{C_i} \cdot A_i}{\sum_{i=1}^{n} A_i} \tag{1}$$

where: n – is the number of parts into which the composite plate is divided, A_i are their areas, and $\overline{r}_{C_i}$ position vector of the centroid of each component part "i".

In the case of the composite plate considered in Figure 1, it will be divided into three simple geometric bodies, as can be observed in Figure 2:

1. **a rectangular triangle AOB** (solid material) (with the centroid C1 $\left(x_{C_1}, y_{C_1}, z_{C_1} = 0\right)$ at the intersection of the medians),

2. **a circle sector of radius R₂** (missing or empty material) (with centroid C2 $\left(x_{C_2}, y_{C_2}, z_{C_2} = 0\right)$ on its axis of symmetry);

3. **a circle sector of radius R₃** (filled material) (with center of weight C3 $\left(x_{C_3}, y_{C_3}, z_{C_3} = 0\right)$ on the axis of symmetry).

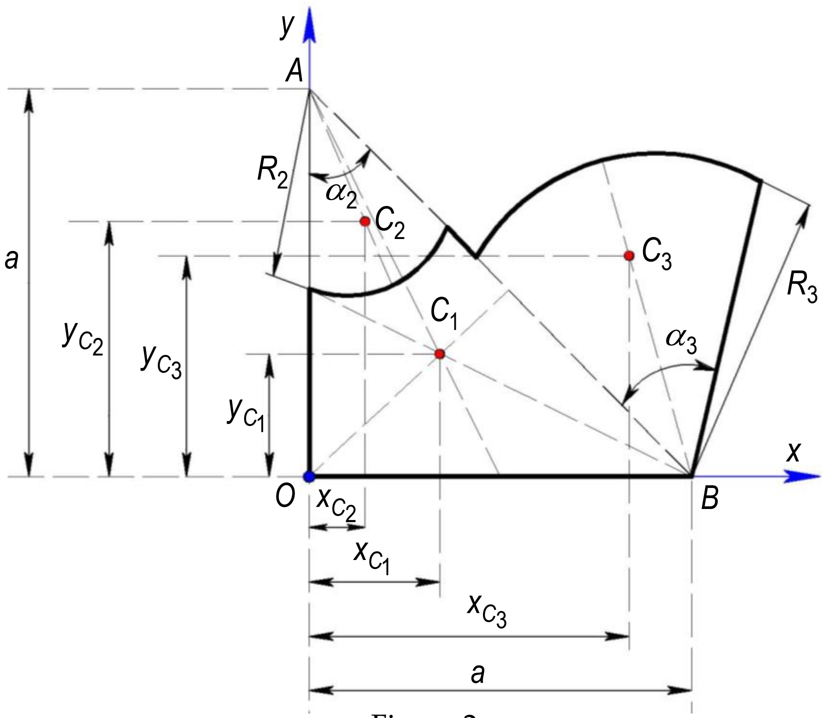


Figure 2

According to Figure 2, the composite plate being flat, the reference system to which it will refer, will also be flat, and the relation (1) will be equivalent with two scalar relations as:

$$x_C^{an} = \frac{x_{C1} \cdot A_1 - x_{C2} \cdot A_2 + x_{C3} \cdot A_3}{A_1 - A_2 + A_3} \qquad y_C^{an} = \frac{y_{C1} \cdot A_1 - y_{C2} \cdot A_2 + y_{C3} \cdot A_3}{A_1 - A_2 + A_3} \qquad (2)$$

where the coordinates of the centers of gravity and the areas of the parts are given by the relations:

Triangle AOB: $\quad x_{C1} = y_{C1} = \frac{a}{3} \qquad A_1 = \frac{a^2}{2} \qquad (3)$

Circle sector of radius R_2 (angles are taken in radians):

$$x_{C2} = \frac{2}{3} \cdot R_2 \cdot \frac{\sin\left(\frac{\alpha_2}{2}\right)}{\left(\frac{\alpha_2}{2}\right)}, \quad y_{C2} = a - \frac{2}{3} \cdot R_2 \frac{\sin\left(\frac{\alpha_2}{2}\right)}{\left(\frac{\alpha_2}{2}\right)} \cdot \cos\left(\frac{\alpha_2}{2}\right), \quad A_2 = \left(\frac{\alpha_2}{2}\right) \cdot R_2^2 \qquad (4)$$

Circle sector of radius R_3**:**

$$x_{C3} = a - \frac{2}{3} \cdot R_3 \cdot \frac{\sin\left(\frac{\alpha_3}{2}\right)}{\left(\frac{\alpha_3}{2}\right)} \cdot \cos\left(\frac{\pi}{4} + \frac{\alpha_3}{2}\right), \; y_3 = \frac{2}{3} \cdot R_3 \cdot \frac{\sin\left(\frac{\alpha_3}{2}\right)}{\left(\frac{\alpha_3}{2}\right)} \cdot \sin\left(\frac{\pi}{4} + \frac{\alpha_3}{2}\right), \; A_3 = \left(\frac{\alpha_3}{2}\right) \cdot R_3^2 \quad (5)$$

To transform the angles from degrees in radians, there is used the expression:

$$\alpha^{rad} = \frac{\pi \cdot \alpha^{\circ}}{180}.$$

3. Development of the laboratory work

The dimensions of the composite plate, given in Figure 1, are calculated by each student: in according to order number from the group, using the dimensioning relationships. Are calculated *(analytically)*, with the help of relations (2)-(5), the centers of gravity $C_i(x_i, y_i, z_i)$ (i = 1, 2, 3) of the component parts of the composite plate, respectively their coordinates, and is filled the **Table 1**, with analytical results:

Table 1

$A_1=$ [cm^2]	$x_{c1}=$ [cm]	$y_{c1}=$ [cm]
$A_2=$ [cm^2]	$x_{c2}=$ [cm]	$y_{c2}=$ [cm]
$A_3=$ [cm^2]	$x_{c3}=$ [cm]	$y_{c3}=$ [cm]
$x_c^{an}=$ [cm]	$y_c^{an}=$ [cm]	

For the ***graphic determination of the mass center***, there will be proceeded as follows (see Figure3):

1. Is considered that the gravity force of a part is proportional to its area: $Gi = kAi$. Conveniently is chosen the constant of proportionality k=1, and the numerical values of the force Gi will be taken equal to the numerical values of the areas Ai. For the missing part, the direction of force will be taken in the opposite sense. There is drawn the complex plate at a chosen scale, i.e. $k_L = 1cm/1m$.

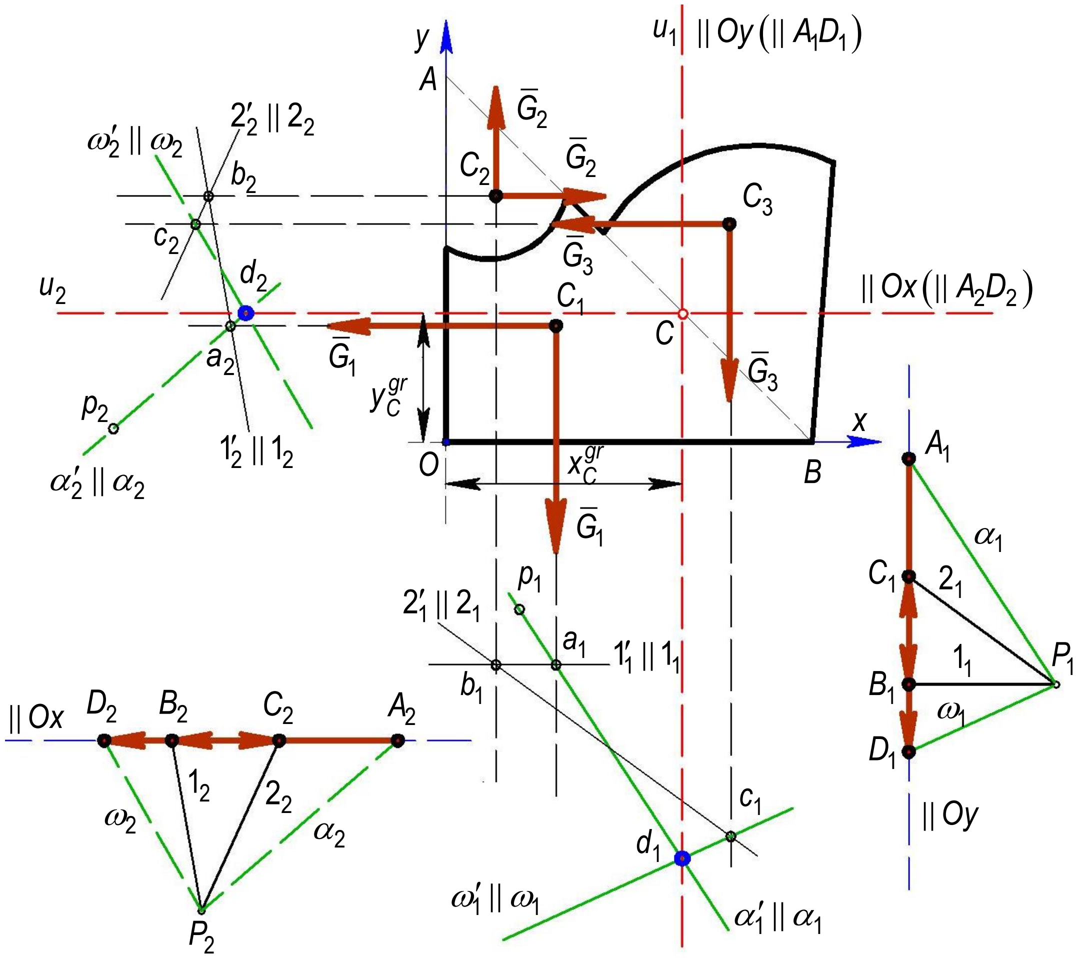


Figure 3

2. On the plate sketch, there are marked the centers of gravity *Ci* of each part and are fixed the forces *Gi*, further named gravity forces.

Note: On the drawing, the size of the gravity forces does not matter!

3. The *polygon of forces* $A_1B_1C_1D_1$ is constructed with the forces *Gi* represented on a convenient scale, i.e. $k_L = 1cm / 10N$.

Note: In the drawing of the polygon of forces the size of forces doesn`t matters!

4. Since all forces are proportional to weights, are parallel forces, and the polygon of forces becomes into points situated on the same axis. A point P_1 is chosen arbitrary and joined with the vertices of the polygon. The obtained segments are noted in order with α_1, 1_1, 2_1 and ω_1.

5. The funicular polygon will be drawn as follows:

- Is choosen, arbitrarily, a point p_1 through which we pass a straight line $\alpha` || \alpha 1$ which will intersect the gravity force support G1 in a_1.
- Through a_1 goes the line $1`||1_1$ which intersects the gravity force supporting axis of G_2 in b_1.
- Through b_1 goes the line $2`||2_1$ which intersects the gravity force supporting axis G_3 in c_1.
- Finally, it goes through c_1 the line $\omega`|| \omega_1$ that intersects the line $\alpha`$ at the point d_1 belonging to the equipollent screwdriver line of action. This is how the ***funicular polygon*** $p_1a_1b_1c_1d_1$ was built.

6. Through d_1, the equipollent screwdriver line of action parallel to the gravity forces is drawn, noted u_1.

7. The steps from the forces range and the funicular range will be resumed with a new direction of the forces. Hence, all forces will be rotated 90° clockwise. The new polygon of these forces is denoted by $A_2B_2C_2D_2$. There will be chosen a point P_2 that forms with the vertices of the polygon the segments denoted by α_2, 1_2, 2_2 and ω_2. A

new funicular polygon $p_2a_2b_2c_2d_2$ is constructed, using the same procedure as described previously, which has point d_2 on the new equipollent screwdriver line of action. Through d_2 is drawn u_2 parallel to the new direction of the gravity forces.

8. The axes (u_1) and (u_2) will intersect in the center of the parallel forces, respectively in the centroid of the complex shape plate $C\left(x_C, y_C, z_C = 0\right)$.

The coordinates x_C^{gr} and y_C^{gr} on the drawing will be strictly measured and compared with the values from the **Table 1** of analytical results, then are calculated the relative errors according to:

$$\varepsilon_{x_C} = \frac{\left|x_C^{an} - x_C^{gr}\right|}{x_C^{an}} \cdot 100 = \quad [\%] \qquad \varepsilon_{y_C} = \frac{\left|y_C^{an} - y_C^{gr}\right|}{y_C^{an}} \cdot 100 = \quad [\%] \qquad (6).$$

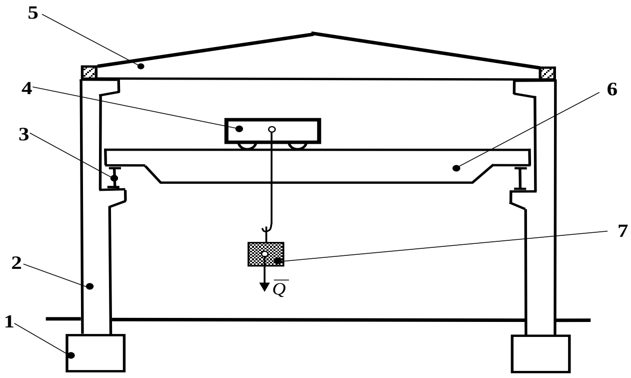
DETERMINATION OF BEAM REACTIONS SIMPLY SUPPORTED

1. The purpose of the paper

The purpose of this paper is to determine the reactions of a simply supported beam and to compare the theoretical results with the experimental ones by calculating the relative errors.

2. Theoretical considerations

A beam is the horizontal component of a structure (building), usually fixed on supports (walls or pillars), and placed at the ends or inside the beam. As example, in Figure 1, is presented the roof beam of an industrial hall or the beam of a drawbridge.



1 – Foundation;
2 – Pillar;
3 – Running beam;
4 – Trolley;
5 – Roof beam;
6 – Beam of the drawbridge;
7 – Load.

Figure 1

As presented in Figure 2, if a beam is leaning on two supports A and B and has no fixing (locking) devices, it`s said that is *simply supported*. If a localized load (F) acts on the beam, it`s said that the load is concentrated, and if the load acts on a certain portion of the length of the beam, is called *a distributed load*.

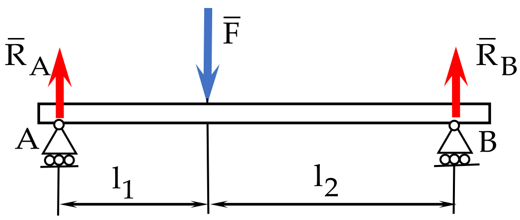


Figure 2

According to Figure 2, the forces are planar, and the beam (considered rigid body in the plane) to be in equilibrium, three scalar equilibrium equations are written: *two for forces* and *one for moments,* as follows.

$$\begin{cases} R_x + R_{lx} = 0 \\ R_y + R_{ly} = 0 \\ M_z + M_{lz} = 0 \end{cases} \tag{1}$$

To solve the system of equations (1), it should be easier if each equation contains only one unknown. This is achieved if instead of the equation of horizontal force projections (the normal reactions do not have horizontal projections), there is written a second equation of moments relative to the other support of the beam, as:

$$\begin{cases} R_A + R_B - F = 0 \\ R_B\left(l_1 + l_2\right) - F \cdot l_1 = 0 \\ R_A\left(l_1 + l_2\right) - F \cdot l_2 = 0 \end{cases} \tag{2}$$

The unknowns are the reactions in beam R_A and R_B , assuming that are known the loads. In this case, the equation of vertical force projections can be use for verification.

Customizing the relations (2) it will result eight systems of equations characteristic to the beam loading, according to the eight cases presented in **Test 1-8.**

3. Development of the laboratory work

There is considered eight experiments/tests, (according to Figure3) depending on the location of the supports and loads, denoted Test **1-8.** It must be determined for each experiment/test, the reactions R_A^{theor} and R_B^{theor} .

Note: - all lengths are in millimeters and loads in Newtons.

- the uniformly distributed load EX8A is 0.0255 N/mm over a length of 100 mm and represents a system of parallel forces that could be reduced in relation to the central axis to a single resultant placed exactly in the middle of the length;

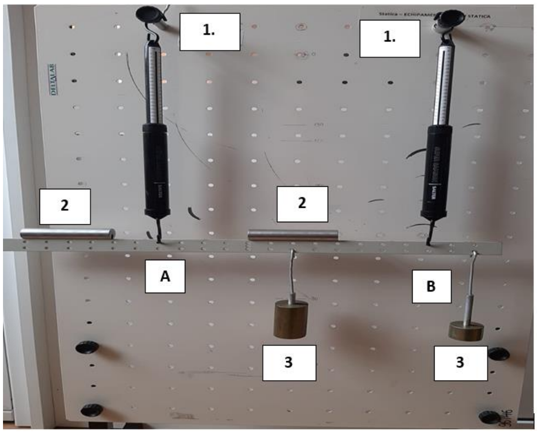


Figure 3

1 –Dynamometer, 2 – Weight with distributed loads (EX8A), 3 – Weight with a concentrated mass of 5N and 2N

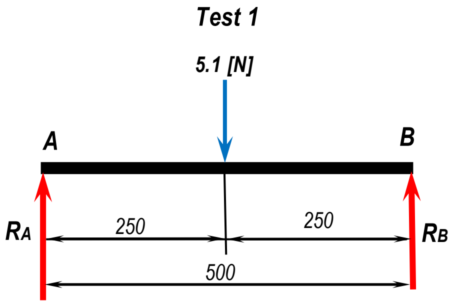
- the weight of the beam can be removed from the equations with the help of a spring device (which keeps the beam in a horizontal position, the elastic force compensating for the beam`s weight).

The equilibrium equations for moments calculated to the ends A and B of the beams, for each loading presented in Test 1-8, are as follows:

Test 1

A: $5.1 \cdot 250 - R_B^{theor} \cdot 500 = 0 \rightarrow R_B^{theor} = 5.1 \cdot 250/500 = 2.55$ N

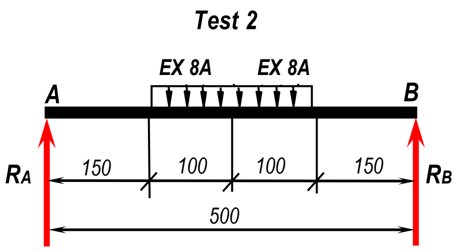
B: $5.1 \cdot 250 - R_A^{theor} \cdot 500 = 0 \rightarrow R_A^{theor} = 5.1 \cdot 250/500 = 2.55$ N



Test 2

A: $5.1 \cdot 250 - R_B^{theor} \cdot 500 = 0 \rightarrow R_B^{theor} = 5.1 \cdot 250/500 = 2.55$ N

B: $5.1 \cdot 250 - R_A^{theor} \cdot 500 = 0 \rightarrow R_A^{theor} = 5.1 \cdot 250/500 = 2.55$ N

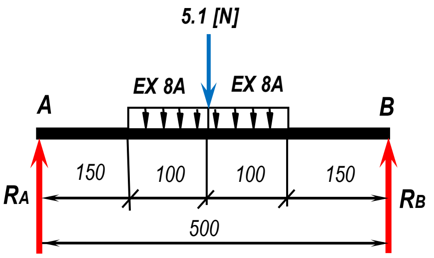


Test 3

A: $10.2 \cdot 250 - R_B^{theor} \cdot 500 = 0 \rightarrow R_B^{theor} = 10.2 \cdot 250/500 = 5.1$ N

B: $10.2 \cdot 250 - R_A^{theor} \cdot 500 = 0 \rightarrow R_A^{theor} = 10.2 \cdot 250/500 = 5.1$ N

Test 3

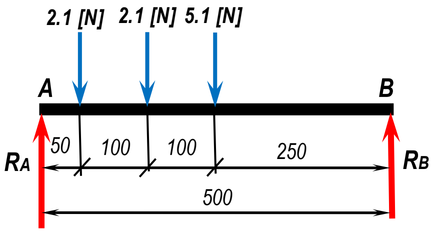


Test 4

A: $5.1 \cdot 250 + 2.1 \cdot 150 + 2.1 \cdot 50 - R_B^{theor} \cdot 500 = 0 \rightarrow R_B^{theor} = 1600.5/500 = 3.20$ N

B: $5.1 \cdot 250 + 2.1 \cdot 350 + 2.1 \cdot 450 - R_A^{theor} \cdot 500 = 0 \rightarrow R_A^{theor} = 2955/500 = 5.91$ N

Test 4

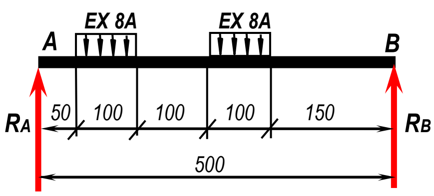


Test 5

A: $2.55 \cdot 100 + 2.55 \cdot 300 - R_B^{theor} \cdot 500 = 0 \rightarrow R_B^{theor} = 1020/500 = 2.04$ N

B: $2.55 \cdot 200 + 2.55 \cdot 400 - R_A^{theor} \cdot 500 = 0 \rightarrow R_A^{theor} = 1530/500 = 3.06$ N

Test 5

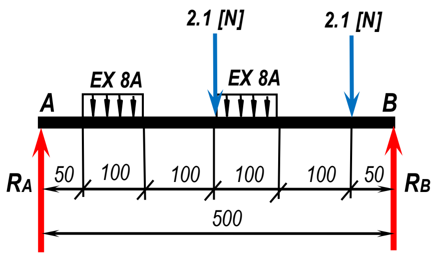


Test 6

A: $2.55 \cdot 100 + 2.55 \cdot 250 + 2.55 \cdot 300 + 2.55 \cdot 450 - R_B^{theor} \cdot 500 = 0 \rightarrow R_B^{theor} = 2805/500 = 5.61$N

B: $2.55 \cdot 250 + 2.55 \cdot 50 + 2.55 \cdot 200 + 2.55 \cdot 400 - R_A^{theor} \cdot 500 = 0 \rightarrow R_A^{theor} = 2295/500 = 4.59$ N

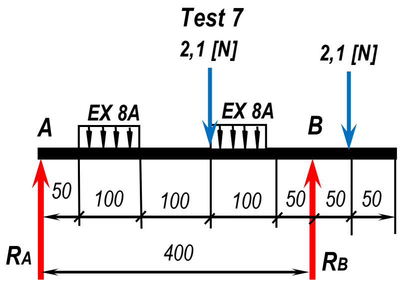
Test 6



Test 7

A: 2.55·100+2.55·250+2.55·300+2.55·450-R_B^{theor}·400 = 0 → R_B^{theor} =2805/400 = 7.01N

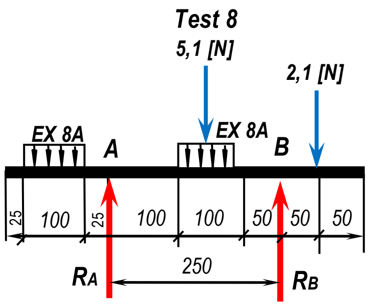
B: 2.55·300+2.55·150+2.55·100+2.55·50-R_A^{theor}·400 = 0 → R_A^{theor} = 1530/400 = 3.82 N



Test 8

A: 5.1·150+2.55·150+2.55·300-2.55·75-R_B^{theor}·250 = 0 → R_B^{theor} =2088.75/250 = 8.35 N

B: 5.1·150+2.55·150+2.55·325-2.55·50-R_A^{theor}·250 = 0 → R_A^{theor} = 2103.75/250 = 8.41 N



The **Table 1** will be fulfilled with the experimental data obtained from the tests, and the obtained values are compared by calculating the relative errors between the theoretical and experimental values as:

$$\varepsilon_{R_A} = \frac{\left|R_A^{theor} - R_A^{exp}\right|}{R_A^{theor}} \cdot 100 = \quad [\%] \qquad \varepsilon_{R_B} = \frac{\left|R_B^{theor} - R_B^{exp}\right|}{R_B^{theor}} \cdot 100 = [\%] \tag{3}$$

Table 1

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8
R_A^{exp}[N]								
R_B^{exp}[N]								
R_A^{theor}[N]								
R_B^{theor} [N]								
ε_{R_A} [%]								
ε_{R_B} [%]								

STUDY OF THE RIGID BODY PLACED ON AN INCLINED PLANE

1. The purpose of the work

The aim of the laboratory is on one hand to measure the angle of friction and to determine the coefficient of friction, and on the other hand to study the equilibrium of a body on an inclined plane.

2. Theoretical considerations

When an object (body), having a mass m is placed on a horizontal plane, all its weight presses on the plane (see Figure 1), with a force called gravitational force $\bar{G} = m \cdot \bar{g}$. To force $\bar{G}$, is opposing a normal force, $\bar{N}$ equal in modulus, but with opposite sense.

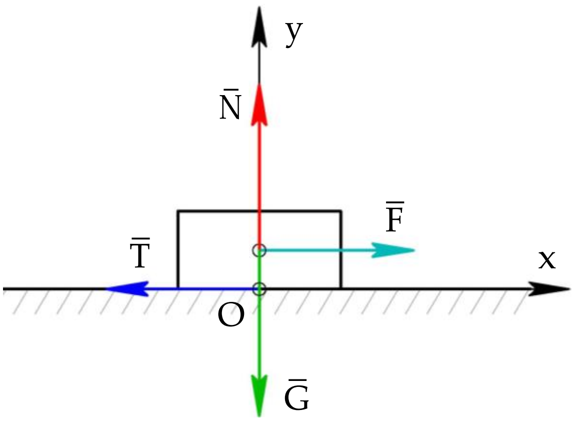


Figure 1

If the horizontal plane is rough, to a horizontal force $\bar{F}$, is opposing a resistance force, called frictional force, generally denoted $\bar{T}$, or $\bar{F}_f$. It is found that initially the body is in equilibrium on the surface until is reached a maximum value of the force $\bar{F}$ at which the body goes out of equilibrium and starts moving. At first the body stands still due to friction with the surface (frictional force).

The equilibrium equations on the axes of the reference system are:

$$\begin{cases} F - T = 0 \\ N - G = 0 \end{cases} \quad (1)$$

At those expressions, it must be added the condition at the limit of sliding friction: $T \le \mu \cdot N$, where μ is called *the coefficient of sliding friction*, a dimensionless parameter, which takes values in $[0,...,1)$ interval.

Coulomb's laws or laws of dry friction:

-The maximum friction force does not depend on the size of the surfaces in contact.

-The maximum friction force is proportional to the value of the normal reaction.

-The maximum friction force depends on the nature of the surfaces in contact and on the degree of their processing.

-The maximum frictional force does not depend on the relative speed of the surfaces in contact.

Mathematically, the *Coulomb's laws or laws of dry friction* can be expressed as:

$$F_{f\,max} = \mu \cdot N \tag{3}$$

and are highlighting that the coefficient of proportionality between the maximum friction force (T_{max} or F_{fmax}) and the normal reaction N is μ called *the coefficient of sliding friction*.

In practice, there are built devices based on a inclined plane used for lifting weights or bringing them down. An example is shown in Figure 1 with a boat pulled out from water.

Figure 2

When a rigid body is placed on an inclined plane at a certain angle (see Figure 3), a part of the weight of the rigid will act parallel to the plane (noted G_t called tangential component), and another part will generate pressure on the plane (noted G_n called normal component).

The plane reacts on the body, according to the principle of action and reaction, with a force called reaction denoted $\bar{N}$. Another force parallel and opposite to the motion on the plane is called frictional force $\overline{T}$, or $\overline{F}_f$. It is denoted by α, the angle of inclination of the plane at which the sliding starts, it will be shown that the angle α is the angle of friction.

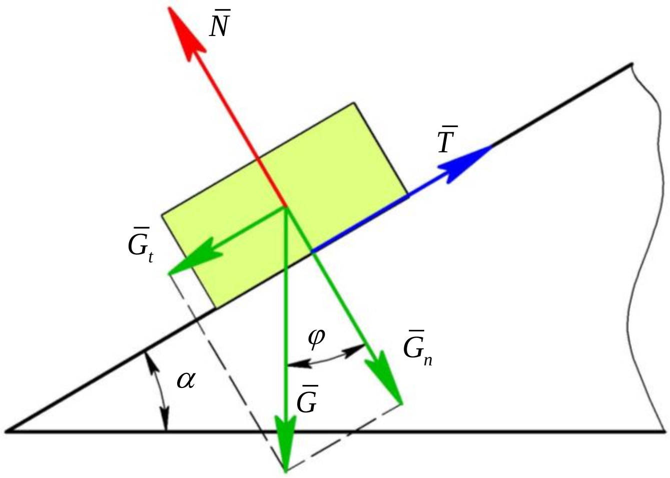


Figure 3

In the case of equilibrium, the following expressions are obvious:

$$\begin{cases} N = G_n = G \cdot \cos\varphi \\ T = G_t = G \cdot \sin\varphi \\ T = \mu \cdot N \end{cases} \tag{4}$$

When the angle of inclination of the plane reaches the maximum value, for which is possible the equilibrium $(\alpha = \varphi)$, the equalities contained in (4) are becoming:

$$\begin{matrix} N = G_n = G \cdot \cos\varphi \\ T_{max} = G_t = G \cdot \sin\varphi \end{matrix} \tag{5}$$

According to Coulomb's law, there can be established the coefficient of sliding friction as:

$$G \cdot \sin\varphi = \mu \cdot G \cdot \cos\varphi \;\rightarrow\; \mu = \frac{\sin\varphi}{\cos\varphi} = tg\varphi \tag{6}$$

The angle φ whose tangent is equal to the coefficient μ is called ***friction angle***.

There can be observed, that the body is sliding freely downwards if $\alpha > \varphi$.

If a certain weight must be climbed on an inclined plane, it is necessary to apply a traction force (F) that overcomes both the friction force (T) as well as the component of the weight parallel to the plane G_t, see Figure 4.

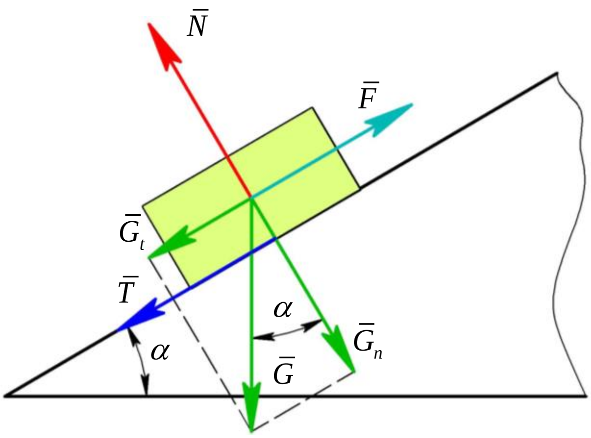


Figure 4

According to Figure 4, when ascending, the equilibrium equations at the limit of friction are:

$$F - G_t - T = 0\,;\quad N - G_n = 0;\quad T = \mu \cdot N \tag{7}$$

where: $$G_t = G \cdot \sin\alpha \text{ and } G_n = G \cdot \cos\alpha \tag{8}$$

Replacing (8) in (7) results that:

$$F = G \cdot (\sin\alpha + \mu \cdot \cos\alpha) \tag{9}$$

When descending, also in the case of equilibrium at the limit, the direction of the force of friction (T) is reversed and the same expression (9) will result, but with the minus sign between the components.

3. Development of the laboratory work

It is used the assembly from Figure 5 composed of: plate, inclined plane, wire and hook with a weight of 0.1N .

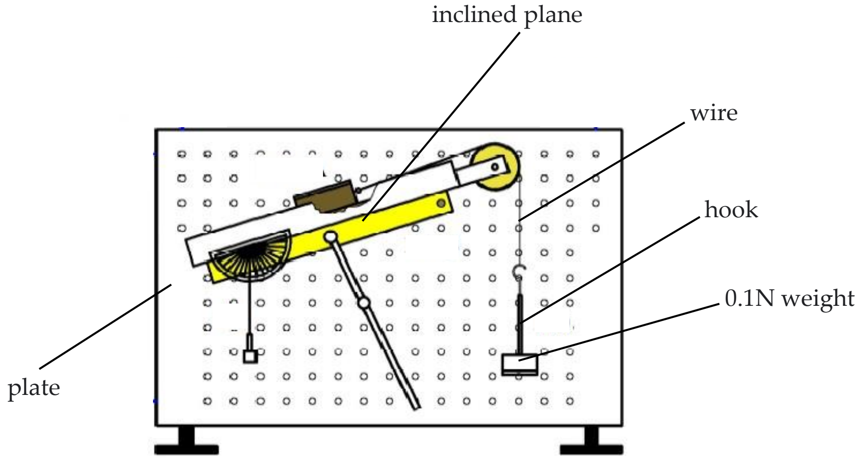


Figure 5

A body having the mass "m*" is placed on the inclined plane. The angle of the plane is changed starting from the horizontal, slightly pushing the body to overcome the static (sticking) friction, until the body starts to slide. It is read on the protractor, mounted on the plane, the angle of inclination of the plane, which in this case is the friction angle $\varphi = \alpha$. Three tests are performed for each of the possible combinations of rigid material and inclined plane material. The measurements are filled in **Table 1.**

Table 1

Surface of contact	**φ[°]**	**φ^{medium}[°]**	**$\mu = tg\,\varphi^{medium}$**
Aluminum-Steel			
Aluminum – Aluminum			
Wood – Steel			
Wood – Aluminum			
Wood– Wood			
Aluminum – Rubber			
Wood - Rubber			

In the specialized literature, the adhesion and sliding friction coefficients have the values presented in **Table 2.**

Table 2

Surface of contact	Adhesion (μ)	Sliding (μ)
Metal - Metal	0.15-0.25	0.09-0.15
Metal - Wood	0.4	0.3-0.4
Wood - Wood	Longitudinal on the fiber 0.62 Transverse on the fiber 0.43	0.48
Rubber – Brass	0.65 Smooth wheel 0.49 Notched wheel 0.76	0.19-0.34 Smooth wheel 0.28 Notched wheel 0.55

A wire is attached to the body that has at the other end a hook. It is chosen one of the combinations of materials from **Table 1**. Are set different values for the inclination angle of the plane, taking care that one of them is exactly the value of the friction angle. The hook is loaded with various weights, pushing the body gently to overcome static (sticking) friction, until it begins to go up. Three experiments are made for each value of the angle of plane. The weight of the loaded hook is filled in **Table 3** at F^{exp}. With relation (9), the theoretical value of the force of traction F^{theor} is determined and it is also filled in **Table 3**. Then the relative error (ε_r) between the analytically and the experimental values, is determined with the expression:

$$\varepsilon_r = \frac{\left|F^{theor} - F^{\exp}\right|}{F^{theor}} \cdot 100 = ...[\%] \tag{10}$$

Table 3

MATERIALS					
Nr.	$\alpha[°]$	F^{exp} [N]	F^{medium}[N]	F^{theor}[N]	ε_r[%]
	φ°				
	20°				
	30°				
	...				

* The mass of the body is: m = 71g = 0.071kg.

DETERMINATION OF SLIDING AND ROLLING FRICTION COEFFICIENTS

1. The purpose of the paper

The aim of the paper is, on the one hand, the experimental determination of the *dry sliding friction coefficient (μ)* for different materials in contact, and on the other hand, the determination of *the rolling friction coefficient (s).*

2. The Experimental Stand

The friction study device, shown in Figure 1, can be used for:

- determining the friction force, respectively the normal reaction and the sliding friction coefficient between two surfaces in contact;
- comparing the sliding friction coefficient values between two surfaces in contact using different materials;
- comparing the values obtained for the sliding friction coefficient for dry or lubricated surfaces;
- comparing the values obtained for the friction forces in the case of sliding friction with the values obtained in the case of rolling friction.

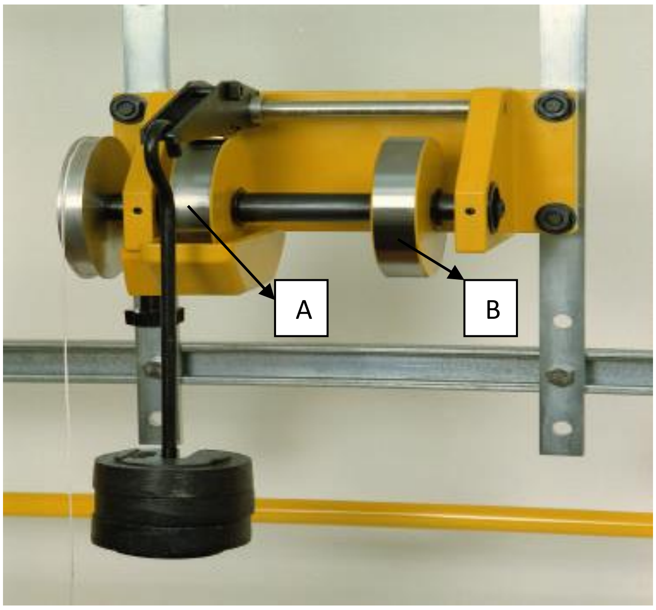


Figure 1

The stand used in experimental determinations of friction coefficients consists of two steel discs with equal diameters, denoted by A and B. Disc A is used for the experiments intended to study wet friction (oil bath), and disc B is used in the study of dry friction. To perform the experiments, the so-called friction clogs, made of different materials (eg cast steel, copper, iron, nylon and rubber) will be used.

The sketch of the device used for experiments, is presented in the Figure 2.

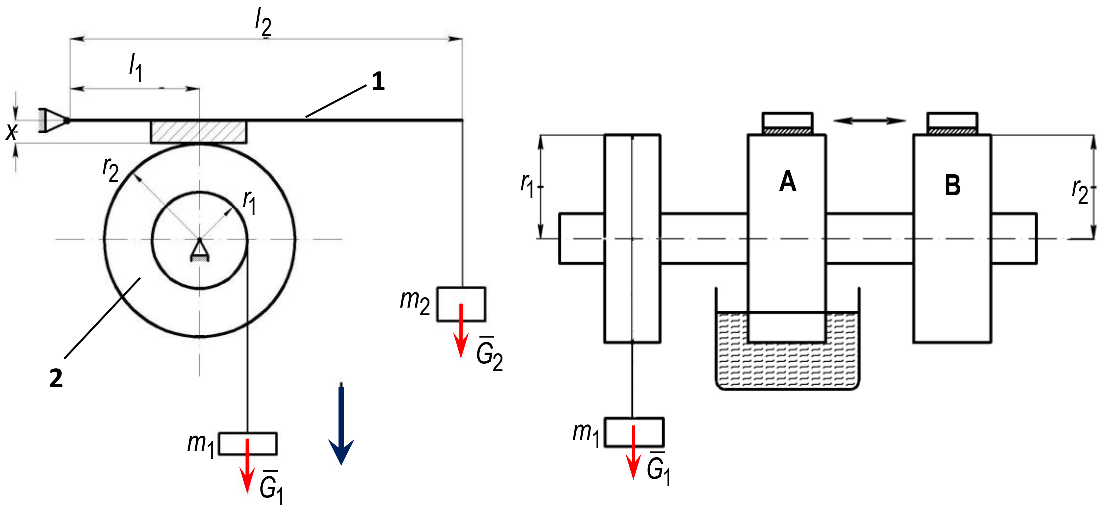


Figure 2

3. Theoretical considerations

When a body having the mass m, is placed on a horizontal plane, located in a gravitational field, the force of gravity $G = m \cdot g$ acts on it, whose orientation is towards the center of the Earth. According to the principle of action and reaction, to force of gravity, it opposes the reaction force of the plane, noted N, opposite in direction, but equal in modulus, as shown in Figure 3.

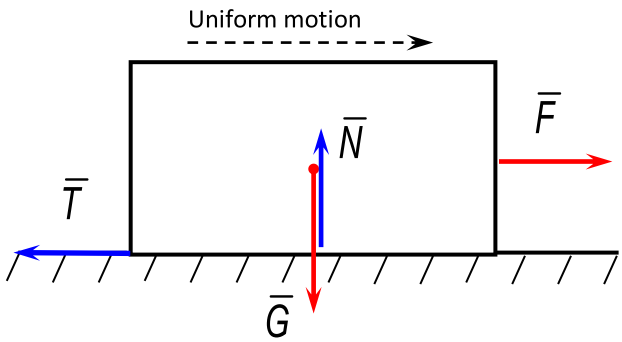


Figure 3

If a constant (horizontal) force acts on the body, parallel to the plane, the rigid body will move uniformly in the direction of the force $\bar{F}$. If the surface of the plane is rough, the movement of the body is opposing frictional force denoted by $\bar{T}$.

From theoretical point of view, the sliding friction coefficient (μ) is defined as the ratio between the friction force and the normal reaction occurring at the contact surface between the bodies, as:

$$\mu = \frac{T}{N} \tag{1}$$

In the case of the system of bodies from Figure 2, to determine *the normal reaction (N)* and the sliding friction force (T) between the two bodies in contact, *the method of separation of bodies* is applied (in the analyzed case, body 1 and 2). Therefore, the bodies are separated (the lever with the brake are considered body 1 and disc B is body 2), the linkage forces are introduced, after which three equilibrium equations are written, two for forces and one for moments (relative to point O - for the body 1, and relative to point O_1 - for the body 2) (see Figure 4).

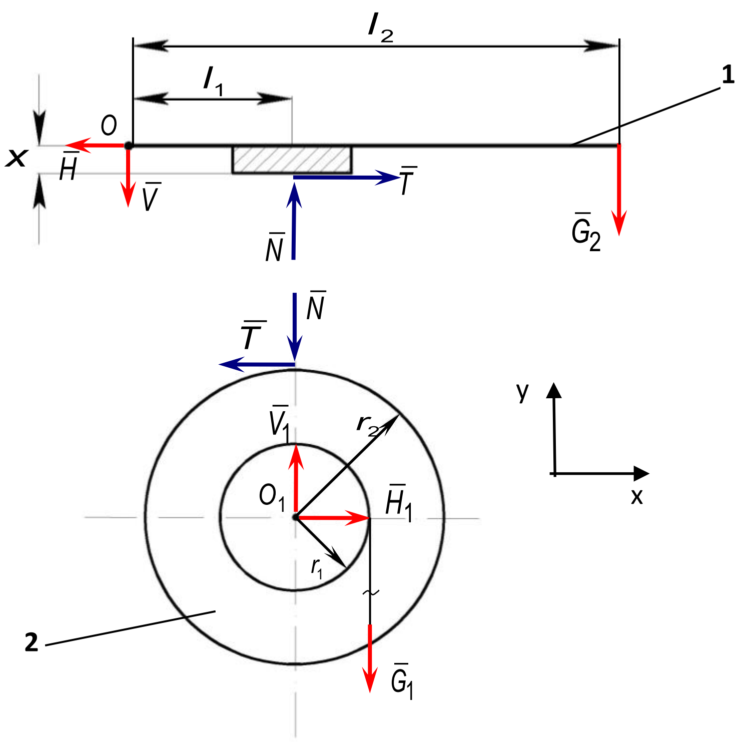


Figure 4

Note: *In study was neglect the lever weight.*

Body 1:
$$\begin{aligned} &Ox: T - H = 0; \\ &Oy: N - V - G_2 = 0; \\ &O: N \cdot l_1 + T \cdot x - G_2 \cdot l_2 = 0 \end{aligned} \tag{2}$$

$$T = \mu \cdot N$$

Body 2:
$$\begin{aligned} &Ox: H_1 - T = 0 \\ &Oy: V_1 - N - G_1 = 0 \\ &O_1: T \cdot r_2 - G_1 \cdot r_1 = 0 \end{aligned} \tag{3}$$

In keeping with (2) and (3) it results the forces N , T and sliding friction coefficient (μ), according to:

$$N = \frac{G_2 \cdot l_2 - \dfrac{G_1 \cdot r_1 \cdot x}{r_2}}{l_1}; \qquad T = \frac{G_1 \cdot r_1}{r_2} \tag{4}$$

$$\mu = \frac{G_1 \cdot r_1 \cdot l_1}{G_2 \cdot r_2 \cdot l_2 - G_1 \cdot r_1 \cdot l_1 \cdot x} \tag{5}$$

The constructive dimensions of the laboratory equipment, are:

$$l_1 = 6 \cdot 10^{-2}\ [m],\ \ l_2 = 18 \cdot 10^{-2}\ [m],\ r_1 = r_2 = 6 \cdot 10^{-2}\ [m],\ x = 10^{-2}\ [m],\ r\ =\ 37 \cdot 10^{-3}\ [m]$$

using the same stand, the rolling friction can be also studied, by mounting a roller instead of the sliding shoe. The scheme of the device in this case is shown in Figure 5.

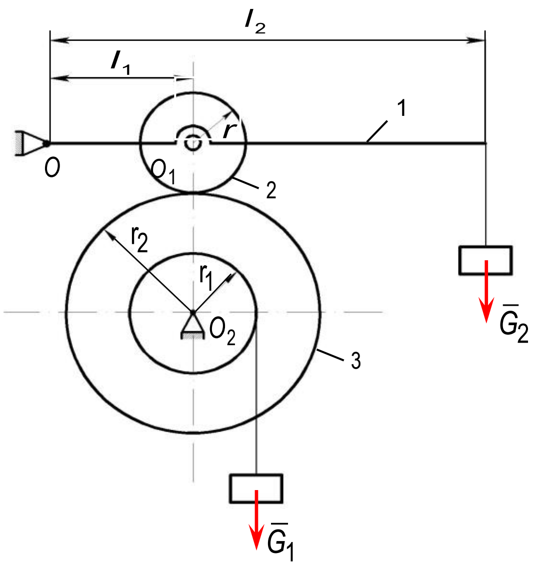


Figure 5

To determine the coefficient of rolling friction (s) between the roller and the disc, the separation of bodies method is applied again. According to Figure 6, there are three bodies: ***the lever (1), the roller (2)*** *and* ***disc B (3)***. There are represented the mechanical sketches of forces and are written the equilibrium equations at the limit of sliding and rolling friction in conformance of Figure 6.

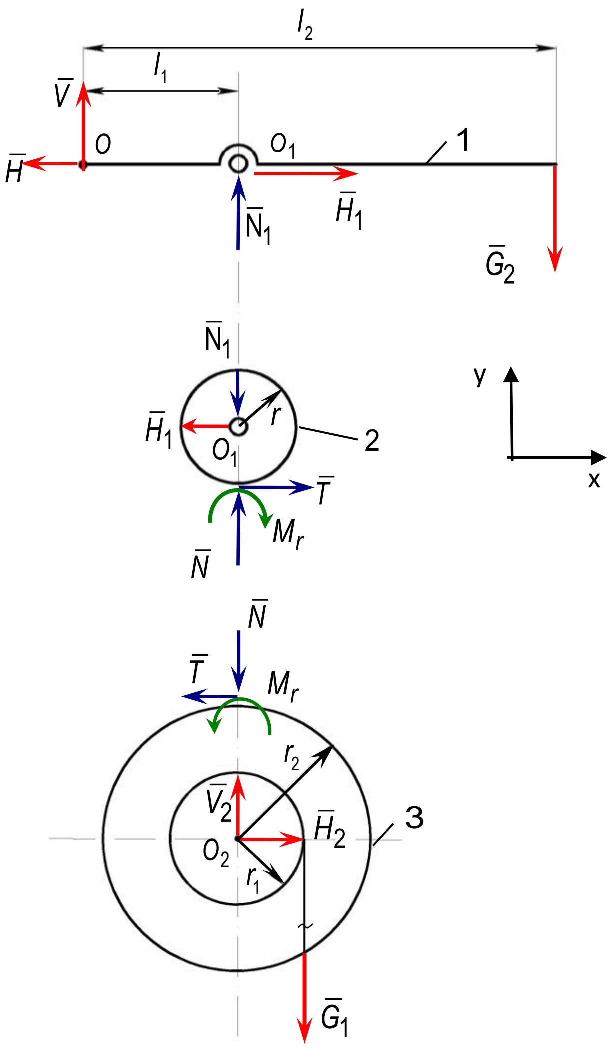


Figure 6

Note: Rolling is assumed to be without sliding: $F_f < \mu \cdot N$

The equilibrium equations are written as:

Body 1

$$\begin{aligned} &Ox: H_1 - H = 0 \\ &Oy: V + N_1 - G_2 = 0 \\ &O: -N_1 \cdot l_1 + G_2 \cdot l_2 = 0 \end{aligned} \tag{6}$$

Body 2:

$$\begin{aligned} &Ox: -H_1 + T = 0 \\ &Oy: -N_1 + N = 0 \\ &O_1: \ T \cdot r - M_r = 0 \\ &T = \mu \cdot N; \quad M_r = s \cdot N \end{aligned} \tag{7}$$

Body 3:

$$\begin{aligned} &Ox: H_2 + T = 0 \\ &Oy: V_2 - N - G_1 = 0 \\ &O_2: -G_1 \cdot r + T \cdot r - M_r = 0 \end{aligned} \tag{8}$$

According to expressions (6)-(8) the sliding friction and rolling friction coefficients are:

$$\mu = \frac{G_1 \cdot r_1 \cdot l_1}{G_2 \cdot l_2 \cdot (r_2 + r)} \qquad s = \frac{G_1 \cdot r_1 \cdot l_1 \cdot r}{G_2 \cdot l_2 \cdot (r_2 + r)} \tag{9}$$

3. Development of the laboratory work

A series of experiments will be carried out, with the scope of determining the relationship that is established between the friction force (T) and the normal force (N) that characterizes the surfaces in contact. The values obtained for the friction forces and the corresponding normal forces using different materials will be compared.

Before performing any experiment must be checked the condition of the disc **B** and the shoe to be used (their surfaces must be dry and clean). If at the beginning of the experiment the mass m_2 is not attached, the normal force will be the result of the action of the gravity force of the shoe, and for equilibrium, the corresponding mass m_1 must be determined.

To determine the coefficients of rolling friction (*s*), instead of the shoes, an aluminum or rubber roller is mounted, and then the methodology from the previous experiment will be repeated.

It is found that $s_{steel-steel} < \mu_{steel-steel}$ and $s_{steel-steel} < s_{rubber-steel}$.

In the case of determining the coefficient of sliding friction for each set of materials used, three measurements will be made, and are noted the related gravity forces in **Table 1**. Then with the help of expressions (4) and (5) the value of the sliding friction coefficient is determined, the normal reaction and the frictional force in each case, the values are filled in **Table 1.**

Table 1

Materials	$G_1 [N]$	$G_2 [N]$	μ	μ_{medium}	N [N]	T [N]
Steel-Steel						
Brass-Steel						
Plastic-Steel						
Ferodo-Steel						
Rubber-Steel						

The graphic representation of the friction force as a function of the normal reaction, highlights a linear variation, like the example from Figure 7. With the values of the two forces, from **Table 1,** there are five curves for each combination of materials on the same graph.

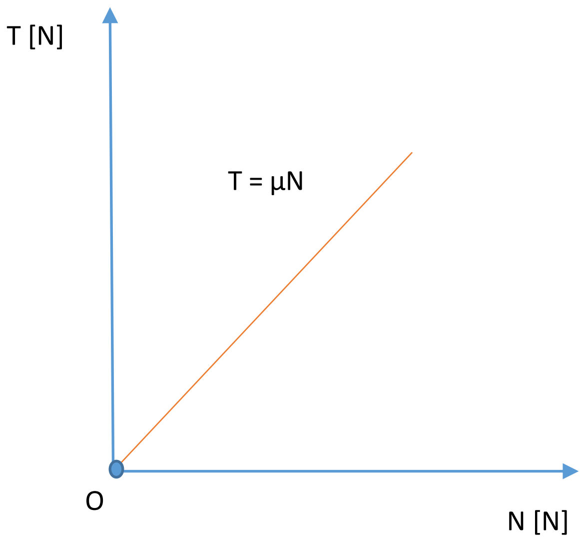


Figure 7

In the case of determining the rolling friction coefficient, three sets of measurements are performed for each set of materials, noting the gravitational forces values in **Table 2.** With relations (9), are calculated the sliding friction and rolling friction coefficients, the obtained values, being filled in the same **Table 2.**

Table 2

Materials	$G_1\,[N]$	$G_2\,[N]$	μ	$\mu_{average}$	s	$s_{average}$
Steel-Steel						
Brass-Steel						

DETERMINATION OF THE MECHANICAL ADVANTAGE OF LEVERS

1. The purpose of the work

In the paper will be determined the mechanical advantage (MA) by calculating the ratio between the resisting force and the active force (Fr/Fa)) for different types of levers.

2. Theoretical considerations

The **lever** – is a rigid bar that rests on a fixed point (fulcrum) and on which an active force (effort) and a resistant force (load) are exerted (assuming friction is neglected). In practice, levers are frequently used to lift weights or are part of mechanisms that lift weights (see Figure1).

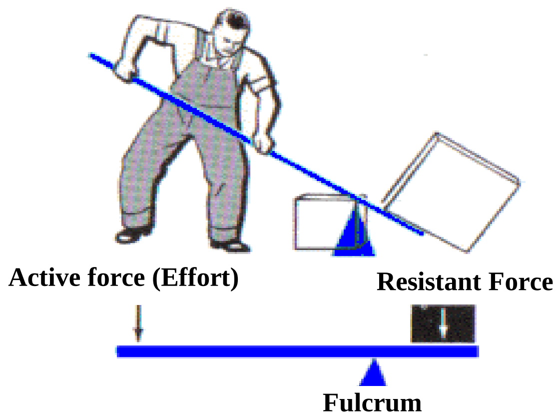


Figure 1

There is considered a force $\overline{F}$ applied on a point A and a point O, arbitrarily chosen. The moment vector of the force $\overline{F}$, related to O, is denoted with $\overline{M}_O$ and expressed vectorial by the relation:

$$\overline{M}_O = \overline{r} \times \overline{F} \tag{1}$$

where $\overline{r}$ is the position vector of force $\overline{F}$ related to point O.

Being a vectorial quantity $\overline{M}_O$ is perpendicular on the plane determined by position vector $\overline{r}$ and force $\overline{F}$, the direction being given by right-hand rule (on the point where $\overline{r}$ and $\overline{F}$ meet the drill is placed perpendicular on the plane of the two vectors and is rotated so that, the first vector will overlap the second vector, on the shortest path). The direction of the drill is the direction of the polar moment, and the modulus is given by the product of the force and the force arm.

In the following applications, it is considered that an ***active force*** $\overline{F}_a$, acts on the lever, which rotates the lever around a *point O* and which is opposed by a ***resisting or loading force*** $\overline{F}_r$. ***The positions of the application points*** of the forces (A for the active force, and B for the resisting force) and ***the position of point O (fulcrum),*** are determining the three types of levers (Figure 2):

- ***Lever of type I*** -fulcrum, is located between $\overline{F}_r$ and $\overline{F}_a$ (AOB). Examples: balance with equal arms, crowbar, nail pliers, scissors, etc.

- ***Lever of type II*** - the load $\overline{F}_r$ is located between fulcrum and the active force $\overline{F}_a$ (OBA). Examples: wheelbarrow, nutcracker, nut wrench, etc.

- ***Lever of type III***- the force $\overline{F}_a$ is located between fulcrum, and the load $\overline{F}_r$ (OAB). Examples: tweezers, tongs, steam boiler safety valve, sewing machine pedal, etc.

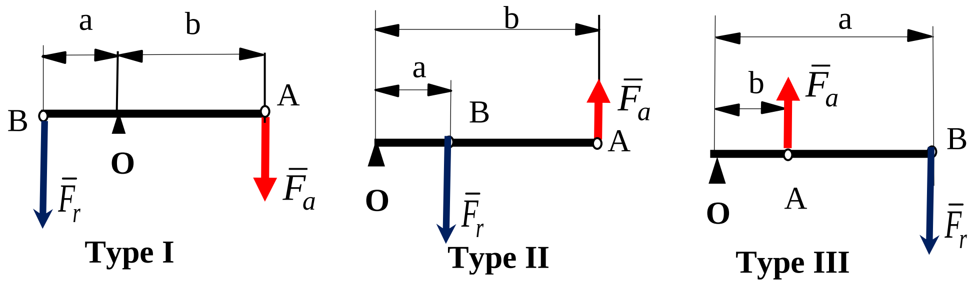


Figure 2

In keeping with the definition of a moment (1), for the mechanical system to be in **equilibrium**, the sum of the **moments** that the **two forces** (active and resisting) are giving, related to point, must be zero.

According to Figure 2, there is noted with "a" the distance from the fulcrum (point O) to the resistant force (load) $\overline{F}_r$, and by "b" the distance the active force $\overline{F}_a$ to point O ("a" and "b" are called arms of the lever). Hence, there can be written:

$$F_r \cdot a = F_a \cdot b \tag{3}$$

The ratio F_r / F_a is named ***active force multiplication factor*** or ***the mechanical advantage (MA)***, of the lever:

$$MA = \frac{F_r}{F_a} = \frac{b}{a} \tag{4}$$

which can be calculated either as a *ratio of forces*, or as a *ratio of arms*.

If MA >1 there is ***amplification*** of forces/lever arms ratio;

If MA<1 there is ***reduction*** forces/lever arms ratio.

3. Development of the laboratory work

In the following experiments, there are considered known the resistive forces (i.e. 1,1; 2,1; 3,1; 4,1; 5,1 [N]), and the distances "a" and "b". There is requested to determine the active forces, the ***mechanical advantage,*** and to calculate the relative errors for the mechanical advantage and fill them in the same table.

Lever of type I (see Figure 3a*)*

Two hooks are attached to the lever in the left and the right of the support point O. There is measured the distance "a" and "b" and is reminded that the weight of a hook is 0.1N. On the left hook, there are attached weights of 1-5[N], and in each case, on the right hook there are attached weights to get balance ***(the lever should be horizontal).*** The obtained values are filled in **Table 1.**

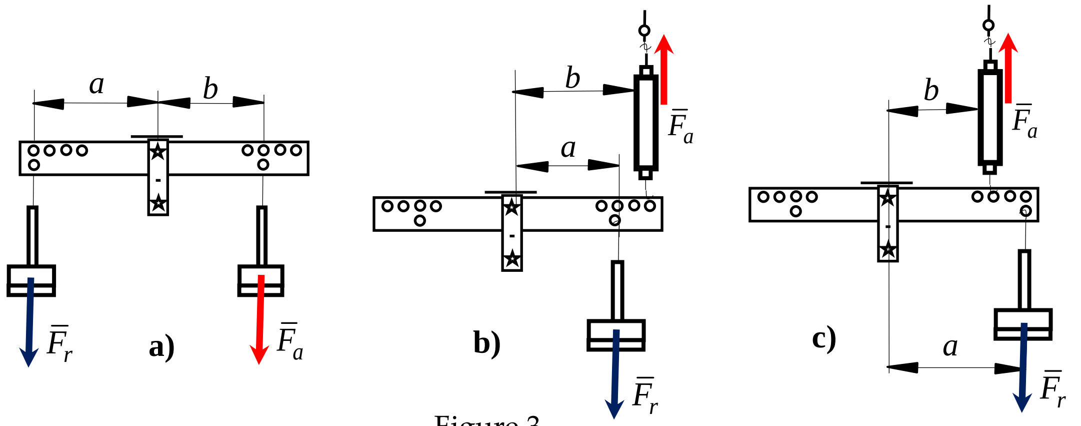


Figure 3

Table 1

Lever of type I		a [mm] =		b [mm] =
F_r [N]	F_a [N]	MA		$\varepsilon_r = \frac{\lvert MA^b - MA^f \rvert}{MA^b} \cdot 100 \; [\%]$
		$MA^f = F_r / F_a$	$MA^b = b/a$	
1.1				
2.1				
3.1				
4.1				
5.1				

Lever of type II (see Figure 3b)

A hook is inserted in a lower hole located to the right of fulcrum at a distance "a" and a dynamometer is also mounted in an upper hole also to the right at a distance $b > a$. There are measured the distances "a" and "b". The hook is loaded with a weight of 1 [N], representing the loading/resistive force. The position of the dynamometer is adjusted until the lever reaches a horizontal position. The measurement is filled in **Table 2.** The measurements are repeated for weights of 2-5[N], all the values of the active forces being registered in **Table 2.**

Table 2.

Lever of type II		a [mm] =		b [mm] =
F_r [N]	F_a [N]	MA		$\varepsilon_r = \frac{\lvert MA^b - MA^f \rvert}{MA^b} \cdot 100 \; [\%]$
		$MA^f = F_r / F_a$	$MA^b = b/a$	
1.1				
2.1				
3.1				
4.1				
5.1				

Lever of type III (see Figure 3c)

In the experiment shown in Figure 3c, a hook is inserted into a hole in the lower part of the extremity of the right arm of the lever, at a distance "a", representing the load. A dynamometer is mounted, in an upper hole, also on the extremity of the right arm of the lever, at distance b, so that $b < a$. Adjust the position of the dynamometer, on an adjustable hook, so that the lever reaches a horizontal position. The values of the active forces F_a, given by the dynamometer, and the arms "a" and "b" of the forces, values measured from the point of the lever's support to the hanging hooks, are recorded. The experiment is repeated for different resistance forces, and the values are filled in **Table** ***3***.

Table 3

Lever of type III	a [mm] =		b [mm] =	
F_r [N]	F_a [N]	MA		$\varepsilon_r = \frac{\lvert MA^b - MA^f \rvert}{MA^b} \cdot 100 \quad [\%]$
		$MA^f = F_r / F_a$	$MA^b = b/a$	
1.1				
2.1				
3.1				
4.1				
5.1				

DETERMINATION OF EFFORTS

FROM THE BARS OF A TRUSS

1. The purpose of the paper

There is considered in Figure 1, a truss, on which are applied the forces F_1 and F_2, under the given angles α_1 and α_2.

There is required:

a) To determine if the beam is rigid and statically determined.

b) To determine the efforts in the truss bars by **the method of joints**.

c) To determine the efforts in trusses 4, 5 and 6 by **the method of sections/Ritter**.

d) To determine the graphical reactions and compare them analytically with the graphical ones.

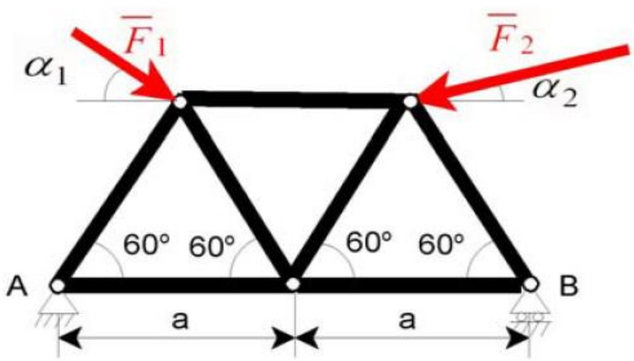


Figure 1

The input data are:

$F_1 = 100m + 10n$ [daN] $\alpha_1 = 45 + n$ [°]

$F_2 = 300m + 10n$ [daN] $\alpha_2 = 45 - n$ [°] a = 6.5+n [m]

where: *n* – order number of the student in the group

m – semigroup number.

2. Theoretical considerations

A truss is **a truss that spans a roof or other structure and is designed to carry a higher load than other trusses in the same construction**. A truss is a structural member whose components are **only** ever in tension or compression but **not bending**. Additional plates and posts, in addition to those in standard trusses spanning the same distance, provide added strength against bending and shear.

The simple truss is a plane structure constituted from members (**bars)** joined in **pins**. The simplest non-deformable (rigid truss) has a triangle shape form by three bars and three pins (see Figure 2). If there is added another node/pin to keep it rigid, two more bars are needed and so on.

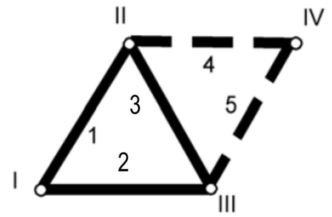


Figure 2

Denoting with N the number of nodes/pins and with b the number of bars, it results **the condition of rigidity (in plane)**, as: **b = 2N-3.**

The unknowns are the efforts in the bars when it is subject to the action of external forces. Connections with the exterior are materialized through planar joints and through planar simple supports. Denoting with r_a the number of joints (two unknowns/joint) and with r_s the number of simple support (one unknown/simple support) **the static determination condition** can be expressed (number of equations = number of unknowns): $2 \cdot N = b + 2 \cdot r_a + r_s$, because for each pin it can be written two scalar equilibrium equations.

• ***Convention:*** If the bar is on tension stress, the effort in the bar comes out of the node!

The nodes are symbolized with ***Roman numerals*** and bars with ***Arabic numerals*** (see Figure 3).

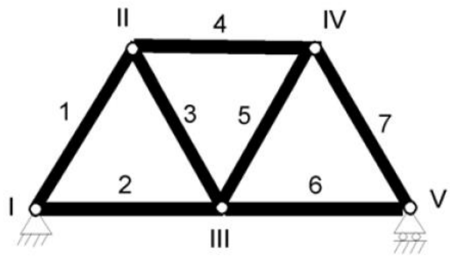


Figure 3

a. Method of joints

Step: I. There are determined the reaction forces from the external links, releasing the truss by external links, and introducing the linkage forces in their place (Figure 4) and writing the equilibrium equations as follows: *an equation of force projections on horizontally, and two equations of moments reported to points A and B.*

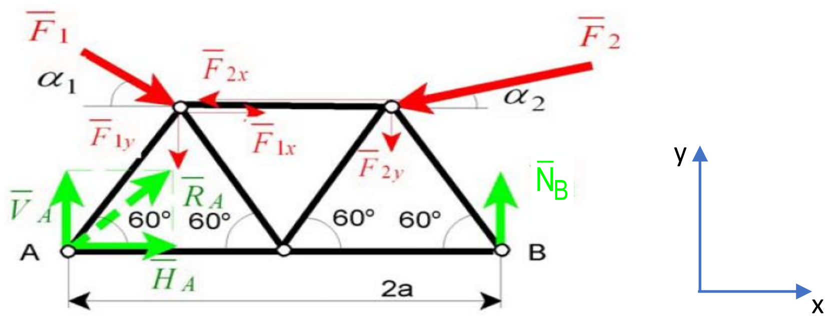


Figure 4

In equilibrium equations for moments, the positive sense can be adopted arbitrarily, but once chosen, it remains the same for the entire equations. Hence, in concordance with Figure 4, it can be written:

$$H_A + F_1 cos\alpha_1 - F_2 cos\alpha_2 = 0$$

$$B{:}V_A \cdot 2a + F_1 cos\alpha_1 \cdot a\ sin60 - F_1 sin\alpha_1 \cdot \frac{3a}{2} - F_2 cos\alpha_2 \cdot a\ sin60 - F_2 sin\alpha_2 \cdot \frac{a}{2} = 0 \qquad (1)$$

$$A{:}N_B \cdot 2a - F_1 cos\alpha_1 \cdot a\ sin60 - F_1 sin\alpha_1 \cdot a\ cos60 + F_2 cos\alpha_2 \cdot a\ sin60 - F_2 sin\alpha_2 \cdot \frac{3a}{2} = 0$$

The equilibrium equations (1) were considered such that, to have only one unknown in an equation to simplify the solution of system (1). There are resulting the unknowns: H_A, V_A and N_B.

Step II. The stresses in the bars are determined starting from one node and then passing through all the other nodes, hence that are resulting: S_1, S_2, S_3, S_4, S_5, S_6 and S_7.

Step II.1 Thus, there is considered a node where are only two bars (two unknowns), for example, node I (see Figure 5) and are written two equilibrium equations for the node, from which the efforts S_1 and S_2 are resulting as:

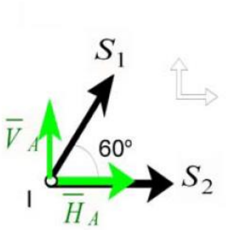


Figure 5

$$S_1 cos60 + S_2 + H_A = 0 \tag{2}$$

$$V_A + S_1 sin60 = 0 \quad \rightarrow S_1, S_2$$

Step II.2 There is considered the next node with only two unknowns (node II) (see Figure 6). Solving the system of equations resulting S_3 and S_4.

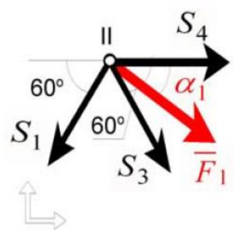


Figure 6

$$F_1 cos\alpha_1 - S_1 cos60 + S_3 cos60 + S_4 = 0 \tag{3}$$

$$F_1 sin\alpha_1 + S_1 sin60 + S_3 sin60 = 0 \quad \rightarrow S_3, S_4$$

Step II.3 The next node with only two unknowns is node (III) (the efforts S_2, S_3 being already known) (see Figure 7). There are written the following equilibrium equations, resulting S_5 and S_6.

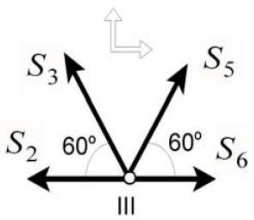
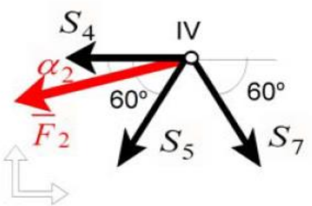


Figure 7

$$S_6 + S_5 cos60 - S_3 cos60 - S_2 = 0 \tag{4}$$

$$S_3 sin60 + S_5 sin60 = 0 \quad \rightarrow S_5, S_6$$

Step II.4 The effort S7 remains to be determined. There is considered node IV , equivalent in number of unknowns to node V, see Figure 8. There can be written two equilibrium equations, which contains the unknown S7:



$$S_7 cos60 - F_2 cos\alpha_2 - S_4 - S_5 cos60 = 0 \qquad (5)$$

$$F_2 sin\alpha_2 + S_5 sin60 + S_7 sin60 = 0 \qquad \rightarrow S_7$$

Figure 8

Step II.5 It is found that three equations remained unused (one from node IV and two from node V), which we can use to check the results. Their non-using is explained by the prior determination of the reactions (three unknowns).

b. Method of sections (Ritter method)

Step: I. There are determined the reactions in A and B, respectively H_A, V_A, N_B.

Step II. The stresses in only certain bars are determined (as opposed to the previous method which gives the stresses in all bars).

Step II.1 There is sectioned the truss from Figure 1, respecting the following conditions: ***do not cut more than three bars, and the bars must not be concurrent in the same node or parallel to each other*** (as seen in Figure 9).

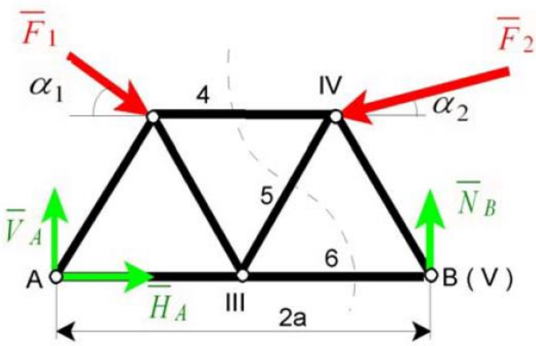


Figure 9

Step II.2 There is isolated a part of the sectioned truss (for example the right side of Figure 10), introducing instead of the sectioned bars the stresses in the bars.

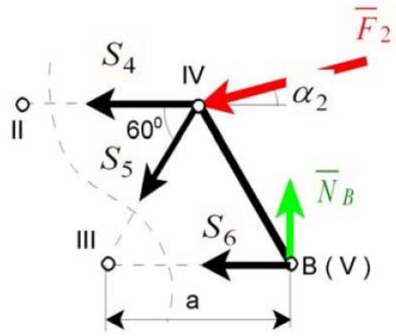


Figure 10

There are written three scalar equilibrium equations for this part of the beam, taking care that from the considered equations to be chosen only those that contain only one unknown per equation (the equation of forces on vertical direction, and the moment equations with respect to nodes III and IV).

$$N_B - S_5 sin60 - F_2 cos\alpha_2 = 0$$

$$S_4 \cdot a\ sin60 + N_B a - F_2 sin\alpha_2 \cdot a\ sin60 + F_2 cos\alpha_2 \cdot a\ sin60 = 0 \tag{6}$$

$$S_6 \cdot a\ sin60 - N_B \cdot a\ cos60 = 0$$

Step II.3 The solving of the three equations, conduct to establishing of efforts S_4, S_5 and S_6 , without the need to traverse the entire beam, from node to node, as in the method of joints.

3. Development of the laboratory work

- From the topology analysis of the truss from Figure 1. It results that the system has b = 7 (bars), and n = 5 (pins/nodes), $r_a = 1$ joint, and $r_s = 1$ simple support.
- The truss rigidity is to be determined as:

$b = 2 \cdot N - 3$ namely: $7 = 2 \cdot 5 - 3$ results the truss is **rigid** (in plane)

- The condition of static determination is to be verified:

$2 \cdot N = b + 2 \cdot r_a + r_s$ namely: $7 + 2 \cdot 1 + 1 = 2 \cdot 5$ results that the truss is **statically determined**.

Analytical part

The **method of joints** is applied and the stresses in the bars S_1, S_2, S_3, S_4, S_5, S_6, S_7 of the truss are calculated, and then with the **Ritter method** are checked the stresses S_4, S_5, S_6 .

- It is filled **Table 1**, with the obtained results.

Table 1

Input data	**Truss reactions**
$n =$ (order number of student in group) $m =$ (semigroup number) $F_1 =$ $[daN]$ $F_2 =$ $[daN]$ $\alpha_1 =$ $[°]$ $\alpha_2 =$ $[°]$ $a =$ $[m]$	$H_A^{an} =$ $[daN]$ $V_A^{an} =$ $[daN]$ $R_A^{an} = \sqrt{(H_A^{an})^2 + (V_A^{an})^2} =$ $[daN]$ $N_B^{an} =$ $[daN]$
Method of joints	**Method of sections**
$S_1 =$ $[daN]$ $S_2 =$ $[daN]$ $S_3 =$ $[daN]$ $S_4 =$ $[daN]$ $S_5 =$ $[daN]$ $S_6 =$ $[daN]$ $S_7 =$ $[daN]$	$S_4 =$ $[daN]$ $S_5 =$ $[daN]$ $S_6 =$ $[daN]$

Graphical part

The reactions will be determined graphically, neglecting the weights of the truss bars, as follows:

- The truss is considered as a rigid solid which will be in equilibrium, if the vectorial conditions of forces and moments are satisfied:

$$\bar{R} + \bar{R}_l = 0$$
$$\bar{M}_O + \bar{M}_{lO} = 0 \quad (7)$$

According to (7), there results:

$$\bar{F}_1 + \bar{F}_2 + \bar{H}_A + \bar{V}_A + \bar{N}_B = 0 \;\rightarrow\; \bar{R} + \bar{R}_A + \bar{N}_B = 0 \quad (7)$$

where: $\bar{F}_1 + \bar{F}_2 = \bar{R}$

$$\bar{H}_A + \bar{V}_A = \bar{R}_A \quad (8)$$

$\bar{R}$ *and* $\bar{R}_l$ – represents the resultant of external forces ($\bar{F}_1 + \bar{F}_2$) and of the linkage forces ($\bar{H}_A + \bar{V}_A + \bar{N}_B$);

$\bar{M}_0$ *and* $\bar{M}_{lo}$ – are the resulting momentum of output forces and the binding forces.

The geometric image presented in Figure 11, of the relation (7) is a triangle, which is built as follows:

- The beam is drawn to the length scale k_L, i.e. $k_L = 1\text{cm} / 1\text{m}$;
- The forces F_1 and F_2 are sliding (drawn on the force scale k_f, i.e. $k_f = 1\text{cm} / 500\ \text{daN}$) on their support until they are intersecting;
- These two forces are composed according to the parallelogram rule, obtaining their resultant $\bar{R}$ (see Figure 11) on the drawing of the beam and on the forces polygon *abc*;
- The direction of $\bar{R}$ intersects the direction of the normal reaction $\bar{N}_B$, at a point O on the beam drawing.
- The moments of the two forces (R and N_B) are zero with respect to point O (both supports pass through point O). In this case, for the truss to be in equilibrium, and the second condition for equilibrium of moments to be fulfilled, the reaction $\bar{R}_A$ must pass through the same point O, which leads to the determination of the support of $\bar{R}_A$.
- The triangle of forces *acd* knowing one side and the directions of the other two, can be sketched.
- The reaction force R_A can be projected horizontally and vertically, obtaining the H_A and V_A components (points *f* and *e* in the force polygon).

- Measuring the segments *cd, ad, ce* and *cf* and multiplying them with the force scale k_f, the real reaction forces values are obtained.
-

The analytical values are compared with the graphical ones, calculating the relative errors between the values determined analytical and the one determined graphical: $$\varepsilon_r = \frac{|V^{an} - V^{gr}|}{V^{an}} \cdot 100 = \quad [\%]$$

Has to be fulfilled **Table 2** as:

Table 2

Graphically results	Relative errors
$R_A^{gr} = \quad [daN]$ $H_A^{gr} = \quad [daN]$ $V_A^{gr} = \quad [daN]$ $N_B^{gr} = \quad [daN]$	$\varepsilon_{rR_A} = \frac{\lvert R_A^{an} - R_A^{gr} \rvert}{R_A^{an}} \cdot 100 = \quad [\%]$ $\varepsilon_{rH_A} = \frac{\lvert H_A^{an} - H_A^{gr} \rvert}{H_A^{an}} \cdot 100 = \quad [\%]$ $\varepsilon_{rV_A} = \frac{\lvert V_A^{an} - V_A^{gr} \rvert}{V_A^{an}} \cdot 100 = \quad [\%]$ $\varepsilon_{rN_B} = \frac{\lvert N_B^{an} - N_B^{gr} \rvert}{N_B^{an}} \cdot 100 = \quad [\%]$

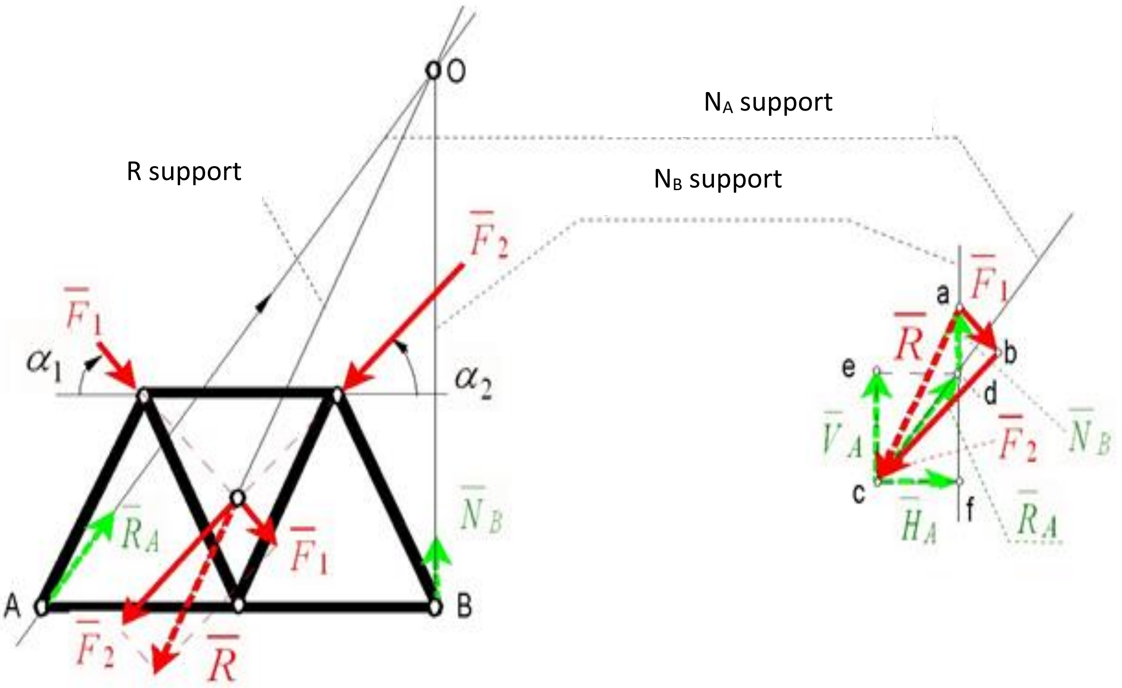


Figure 11

FIXED AND MOVABLE PULLEY SYSTEMS

1. The purpose of the paper

The aim of the paper is to understand the functioning of simple and mobile pulleys and their combinations. It is analyzed also, the mechanical advantage obtained in case of using different types of pulleys.

2. Theoretical considerations

A pulley is a simple device consisting of a disk/wheel that can rotate around its own axis, on the periphery of which a cable is wired that allows a change in the direction of a force or an economy of force.

The pulley can be ***fixed*** (*its own axis does not change its position with related to a fixed coordinate system*) or it can be ***mobile*** (*its own axis is mobile*). The mobile pulleys are supported by the wires that pass under them.

By combining fixed and mobile pulleys, heavy weights can be lifted with less effort, which leads to a ***mechanical advantage (MA).*** Such combinations are found in lifting equipment and systems (see Figure 1).

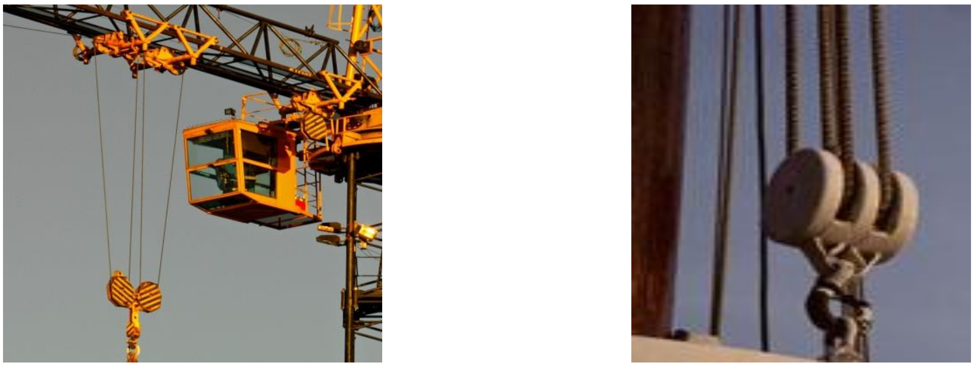


Figure 1

The **pulley system** is a combination of fixed and mobile pulleys to exploit the properties of fixed pulleys to change the direction of the resistive force (F_r) and those of the movable pulleys to reduce the driving force (F_m).

In the case of the **fixed pulley** (Figure 2a), if the stiffness of the wires and the friction from the pulley axis are considered, then the driving force required to overcome the resisting force is:

$$F_m = k \cdot F_r \tag{1}$$

where: $k < 1$ is a coefficient also called the *multiplication factor of resistive force.*

$$k = 1 + \lambda + \frac{2 \cdot \mu \cdot r}{R} \tag{2}$$

and $\lambda = \frac{(\varepsilon_1 + \varepsilon_2)}{R} = k_1 \cdot d_{wire}$, is due to the stiffness of the wire/cord/cable

(k_1 = 0.002...0.006 m^{-2} for hemp cords and k_1 = 0.03...0.09 m^{-2} for steel cables), where ε_1 , ε_2 are the eccentricities of the wires. The friction from the pulley axis is given by the parameter $2\mu r/R$ where μ is the coefficient of sliding friction; r is the radius of the spindle, and R represents the radius of the pulley.

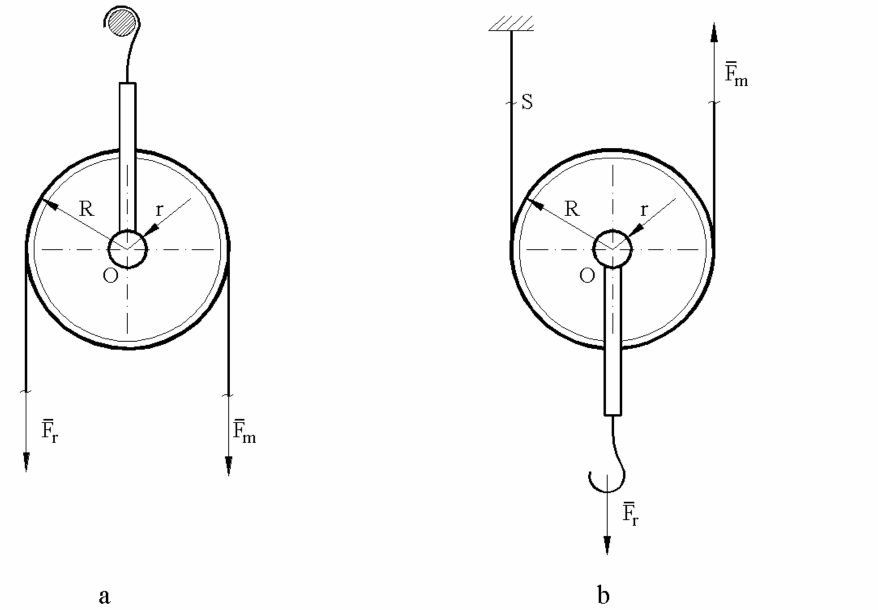


Figure 2

In general case, the performance of the pulley is:

$$\eta = \frac{F_0}{F_m} \tag{3}$$

where F_0 is the driving force in the ideal case, the mechanical advantage is calculated with:

$$MA = \frac{F_r}{F_m} \tag{4}$$

In case of *fixed pulley* $\eta = 1/k$ and mechanical advantage $MA = 1/k = \eta$. It is observed that $k \geq 1$ and that is a multiplication of the resistive force.

If the *fixed pulley* is *ideal* the wire/cable stiffness and shaft friction are neglected, then k = 1, and the driving force is equal to the resistive force.

For the **mobile pulley** (Figure 2b): $$F_m = \frac{1}{1+\frac{1}{k}} \cdot F_r \qquad (5)$$

The coefficient being subunitary, it is about a demultiplication of the driving force, meaning that a lower driving force is needed to overcome the resistive force. The pulley will move the same way with the motion of the point of application of the driving force. It can be seen that in this case the displacement of the resistive force is smaller compared to the displacement of the driving force.

If the *mobile pulley* is *ideal* then k = 1; $F_m = 0{,}5 \cdot F_r$, the mechanical advantage is AM = 2.

The most common pulley systems are **hoists** or **pulley blocks** , see Figure 3.

Figure 3

The hoists are used for lifting heavy weights, being operated either manually or with motors (for cranes). In order to have a high mechanical advantage (MA), combinations of yoke pulleys are used.

The **yoke pulley** is a combination of several pulleys fixed to the same fork. Fixation on the fork can be done on the same axis (the pulleys have the same diameter) or on parallel axes (the pulleys have different diameters).

The hoist is made up of two yoke pulleys, one fixed and one mobile. Each yoke pulley contains an equal number of pulleys mounted on the same fork. The hoist can be done in two versions (Figure 4) in both of them, the calculation of the driving force and the multiplication coefficient being made in the same way.

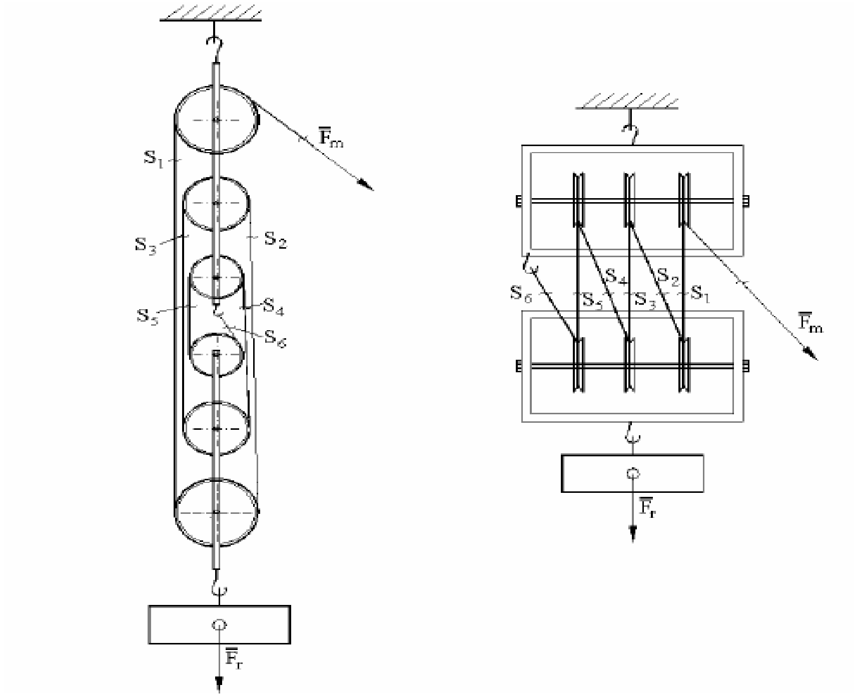


Figure 4

For example, for the hoist with six pulleys having two yoke pulleys, the driving force is:

$$F_m = \frac{k^6}{1 + k + k^2 + k^3 + k^4 + k^5} \cdot F_r = \frac{k^6 \cdot (k-1)}{k^6 - 1} \cdot F_r \tag{6}$$

In general case, with two pulleys and two yoke pulleys driving force is:

$$F_m = \frac{k^{2n} \cdot (k-1)}{k^{2n} - 1} \cdot F_r \tag{7}$$

In the ideal situation (k = 1) an indeterminate form appears, which will be solved applying *l'Hospital's Rule*:

$$\begin{aligned} F_m &= \lim_{k=1} \frac{k^{2n} \cdot (k-1)}{k^{2n-1}} \cdot F_r = \lim_{k=1} \left[\frac{\frac{d}{dk}\left(k^{2n+1} - k^{2n}\right)}{\frac{d}{dk}\left(k^{2n} - 1\right)} \right] \cdot F_r = \\ &= \lim_{k=1} \frac{(2n+1) \cdot k^{2n} - 2n \cdot k^{2n-1}}{2n \cdot k^{2n-1}} \cdot F_r = \lim_{k=1} \left[\frac{(2n+1) \cdot k}{2n} - 1 \right] \cdot F_r = \frac{1}{2n} \cdot F_r \end{aligned} \tag{8}$$

Note: It can be observed that, the coefficient of the resistive force is subunitary, meaning that is about demultiplication, results that a lower driving force is needed to overcome the resistive force.

3. Experimental stand description

The experimental stand consists of the panel (EX10) mounted in a vertical position with different components:

- Four fixing nuts (P1);
- Two teflon pulleys (P12), one fixed (80g) and one mobile (30/43g);
- A simple pulley hook (P15);
- Two hooks for weights (P10);
- A 10 N dynamometer (P8);
- A set of weights (P7);
- Two hoists (P5) one with two pulleys (172g) and one with three pulleys (214g).

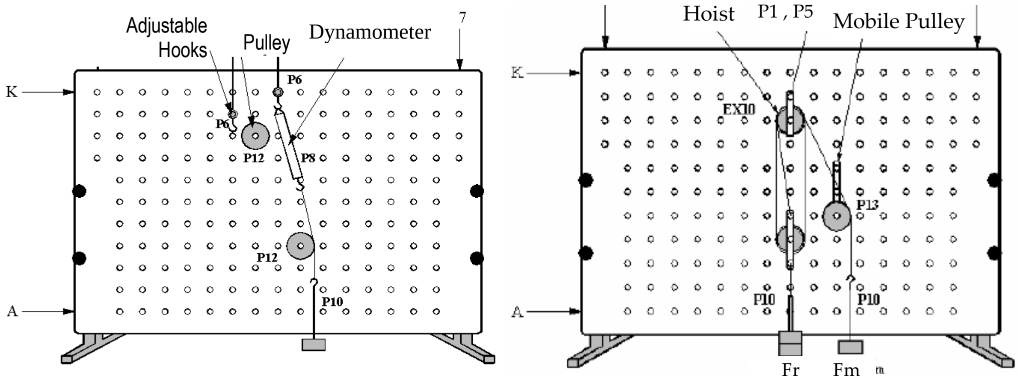


Figure 5

4. Development of the laboratory work

The stand from the Figure 5 is used, and are made three assemblies:

- Fixed pulley.

- Mobile pulley.

- Hoist.

For each assembly, there is considered the resistive force, as in **Table 1**. There are measurements of the driving force (F_m), which are read on the dynamometer and filled in **Table 1.** In each case the mechanical advantage is calculated with (4).

The relative percent error between the theoretical and experimental mechanical advantage is calculated with the relation:

$$\varepsilon_r = \frac{\left|MA^{theor} - MA^{exp}\right|}{MA^{theor}} \cdot 100 \tag{9}$$

Table 1

	F_r [N]	F_m [N]	MA^{exp}	MA^{theor}	ε_r [%]
Fixed pulley	0,5			1	
	2,1				
	5,1				
Mobile pulley	1,1			0,5	
	2,1				
	5,1				
Hoist equipment	2,1			0,125	
	3,1				
	5,1				

KINEMATICS

THE KINEMATIC STUDY
OF THE SLIDER - CRANK MECHANISM

1. Purpose of the work

In Figure 1 there is considered the slider - crank mechanism. Considering that the driving crank OA rotates with a constant revolution n, it is required:

a) To determine the motion law of the piston, as well as the variation laws of velocity (v) and acceleration (a).

b) To analyze the functions s(φ), v(φ) and a(φ) established in the previous point, by determining all the characteristic points and, to represent graphic all these functions.

c) Determine from the diagram the values for s, v and a for the given angle φ_0 and verify them analytically.

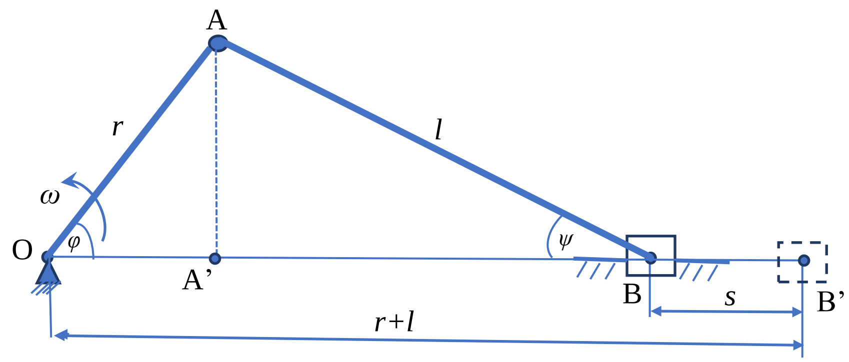


Figure 1

In the Figure 1 are used the followings notations:

r = OA – the crank length, r = 100+5i [mm],

where i is the number of each student in the group;

l = AB – connecting rod length [mm];

$$\frac{r}{l} = \frac{1}{3+0.1\,i}$$

ω – angular velocity of the crank [rad/s]

n = 200 + 2ij [RPM] , j – number of the semigroup;

s –the position of the piston relative to the outer "dead" point (the elongation of piston B);

φ – the angle of the crank OA related to the direction of movement of the piston B (initially $\varphi_0 = 25 + 5i$ [°]);

a [m/s^2], v [m/s] – the velocity and acceleration in point B for piston.

2. Theoretical considerations

a) The position of the mechanism in the Figure 1 is unique determined by the position of the angle *φ(t)* as: $\varphi = \omega t = \frac{\pi n}{30} t$ (1)

The position *s* of the piston B reported at the "dead" point outside B′, in conformity with the figure it is:3

$$s = r + l - OB = r + l - (r cos\varphi + l cos\psi) \quad (2)$$

Expressing AA' with the *sin function* from the two rectangular formed, conducts to:

$$\sin\psi = \frac{r}{l}\sin\varphi \quad (3)$$

Knowing that: $\sin^2(\alpha) + \cos^2(\alpha) = 1$ results:

$$cos\psi = \sqrt{1 - \left(\frac{r}{l}\right)^2 sin^2\varphi} = 1 - \frac{1}{2}\left(\frac{r}{l}\right)^2 sin^2\varphi + \frac{1}{8}\left(\frac{r}{l}\right)^4 sin^4\varphi - \cdots \quad (4)$$

Considering the experimental values of the ratio $\left(\frac{r}{l}\right)$, it can be approximated that sum of the first two terms of the development of power series (4) as:

$$cos\psi = 1 - \frac{1}{2}\left(\frac{r}{l}\right)^2 sin^2\varphi \quad (5)$$

Using the above approximation, the displacement *s* of the relation (2) becomes:

$$s(\varphi) = r\left(1 - cos\varphi + \frac{1}{2}\left(\frac{r}{l}\right) \quad sin^2\varphi\right) \quad (6)$$

Hence, the moving law s(t) of the piston B is:

$$s(t) = r\left(1 - \cos(\omega t) + \frac{1}{2}\left(\frac{r}{l}\right) \quad sin^2(\omega t)\right) \quad (7)$$

To find the variation moving law of the velocity v(t) and acceleration $a(t)$, the relation (7) is derived as:

$$\mathrm{v} = \frac{ds}{dt} = \frac{d\varphi}{dt}\cdot\frac{ds}{d\varphi} = \omega\cdot\frac{ds}{d\varphi} \quad (8)$$

$$\mathrm{v}(\varphi) = \omega r\left(sin\,\varphi + \frac{1}{2}\left(\frac{r}{l}\right) \quad sin2\varphi\right) \quad (9)$$

$$\mathrm{v}(t) = \omega r\left(\sin\,(\omega t) + \frac{1}{2}\left(\frac{r}{l}\right)\;\; sin2\,(\omega t)\right) \tag{10}$$

and, for determining of the acceleration a(t) of the piston B, in keeping with (8)-(10) results:

$$a = \frac{d\mathrm{v}}{dt} = \frac{d\varphi}{dt}\cdot\frac{d\mathrm{v}}{d\varphi} = \omega\cdot\frac{d\mathrm{v}}{d\varphi} \tag{11}$$

$$a(\varphi) = \omega^2 r\left(cos\varphi + \frac{r}{l}cos2\varphi\right) \tag{12}$$

$$a(t) = \omega^2 r\left(cos\omega t + \frac{r}{l}cos2\omega t\right) \tag{13}$$

3. Development of the work

a) For the analysis of the functions on the interval $[0, 2\pi]$, s(φ), v(φ) and $a(\varphi)$, it is observed that the motion is periodical, having the period:

$$T = \frac{2\pi}{\omega} \tag{14}$$

the piston having the elongation 2r.

There are determined the extreme values of the function a(φ) in keeping with:

$$\frac{da}{d\varphi} = -\omega^2 r\; sin\varphi(1 + 4\frac{r}{l}cos\varphi) = 0 \tag{15}$$

where: $\frac{da}{d\varphi} = 0$ if $\varphi_8=0$, $\varphi_9 = \pi$, $\varphi_{10} = 2\,\pi$ and $\varphi_{11,12} = \arccos\left(\frac{-l}{4r}\right)$, if $\frac{r}{l} > \frac{1}{4}$.

The variation of the functions $s(\varphi)$, v(φ), $a(\varphi)$ and $\frac{da}{d\varphi}$ are given the **Table 1** as:

Table1

φ	**0**	φ_6	φ_{11}	π	φ_{12}	φ_7	2π
$s(\varphi)$	Min (0)			Max (2r)			Min (0)
v(φ)	0	Max		0		Min	0
$a(\varphi)$	Max	0	(Min)	(Max) Min	(Min)	0	Max
$\frac{da}{d\varphi}$	0		(0)	0	(0)		0

b) There are established the theoretical values for s_{theor}, v_{theor}, and a_{theor}, with (6), (9), (12), replacing φ with angle φ_0.

c) There are calculated the functions $s(\varphi)$, $v(\varphi)$, $a(\varphi)$, computed in the interval 0°-360°, with step of 10°. The functions are graphically represented as in Figure 2.

d) For a given angle φ_0, the parameters $s(\varphi_0)$, $v(\varphi_0)$, $a(\varphi_0)$, are determined from graphic, as:

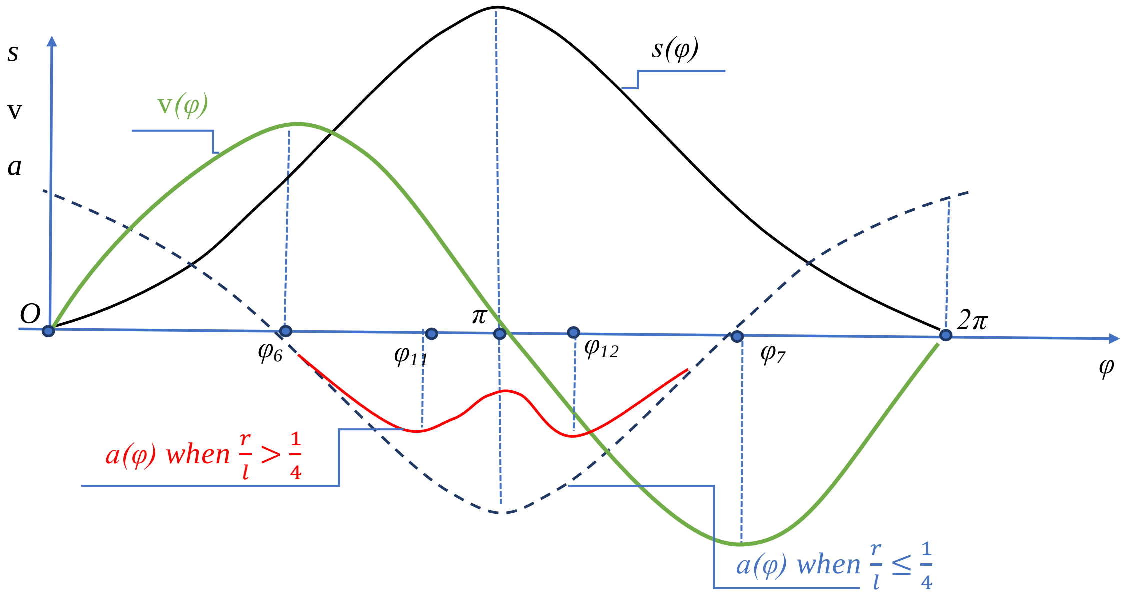


Figure. 2

- From abscissa $\varphi = \varphi_0$ the ordinates (y_s, y_v, y_a) are measured in millimeters;

- The values of s_{gr}, v_{gr}, and a_{gr}, in the diagram will be:

$$s_{gr} = y_s \cdot r,\; v_{gr} = y_v \cdot \omega_r,\; a_{gr} = y_a \cdot \omega_r^2, \tag{16}$$

e) The values obtained by relation (16) will be compared with those calculated in relations (6), (9) and (12) replacing the angle φ with φ_0. The relative errors ε_r will be calculated by:

$$\varepsilon_{r_s} = \frac{|s - s_{gr}|}{s} \cdot 100\ [\%]; \qquad \varepsilon_{r_v} = \frac{|v - v_{gr}|}{v} \cdot 100\ [\%]; \qquad \varepsilon_{r_a} = \frac{|a - a_{gr}|}{a} \cdot 100\ [\%] \tag{17}$$

CENTRODES OF THE PLANAR MOVEMENT

1. Purpose of the work

In the paper there will be determined analytical and graphical the centroids of the planar motion of two bars.

2. Theoretical considerations

The plane-parallel motion of a rigid (C) is defined in terms of the displacement of three non-collinear points remaining in a fixed plane during motion. The three points are determining a mobile plane, a section of the rigid (Γ), which moves in the fixed plane, called the director plane see Figure 1.

Therefore, all sections of the rigid body parallel to (Γ), will move in parallel planes to the director plane, hence the movement is called plane-parallel motion. If is taken a line (Δ) belonging to the rigid body, perpendicular to the plane (Γ), then during the movement the line will also move, remaining all the time normal to (Γ), respectively to the director plane and at the same time parallel to himself. Therefore, the line (Δ) will perform a pure translational motion. All its points will have the same instantaneous velocities and acceleration, and their trajectories will be identical curves, located in planes parallel to each other, but also with the section (Γ), corresponding to the director plane.

Space centrode on these considerations, the plane-parallel motion study can be reduced to the study of motion of the section (Γ) in the director plane. From the mechanical point of view, the section (Γ) is a plate, and its motion in the director plane is call plane motion (motion of a plate in a plane).

In the Figure 1, there is considered a plate (Γ) moving in the fixed plane $O_1x_1y_1$, identical to the director plane. A mobile Oxy system is attached to the plate. In planar motion, the plate (Γ) has three degrees of freedom because its position in the fixed plane is determined by three independent parameters: x and y coordinates in the plane and a rotation angle φ. On the plate, there is also considered a point M.

According to the same Figure 1, there can be written:

$$\bar{r}_1 = \bar{r}_0 + \bar{r} \tag{1}$$

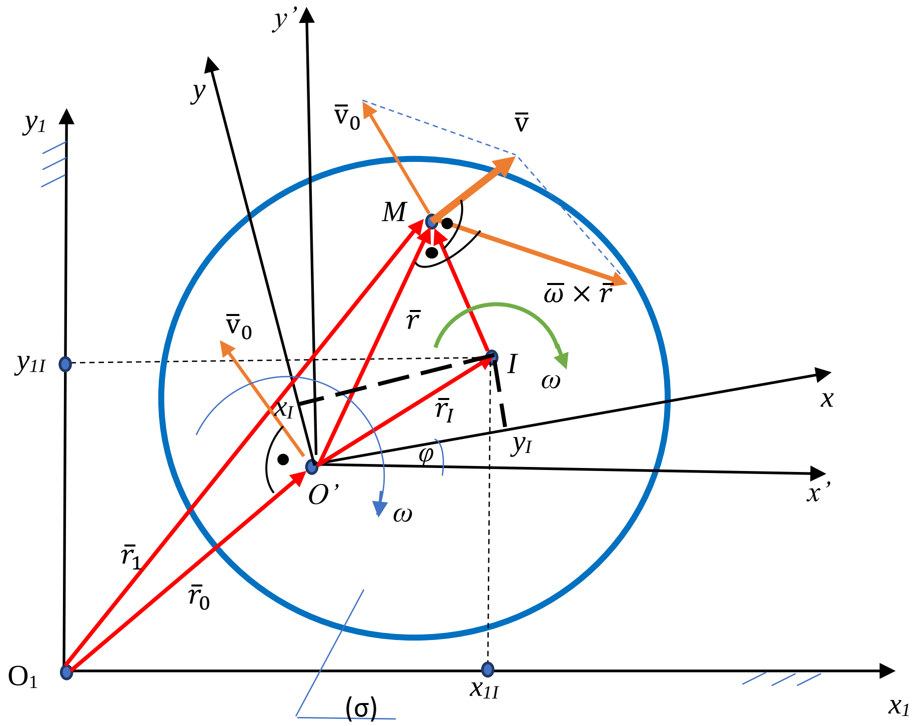


Figure 1

By deriving the relation (1) with respect to time, it will be obtained:

$$\dot{\bar{r}}_1 = \dot{\bar{r}}_0 + \dot{r} \tag{2}$$

where $\dot{\bar{r}}_1 = \bar{v}$ şi $\dot{\bar{r}}_0 = \bar{v}_0$ (3)

are the absolute velocities of the points M and O relative to the fixed system O_1x_1y,

and $\dot{r} = \bar{\varpi} \times \bar{r}$ (4)

represents the velocity of point M due to the change in the orientation of the position of the vector r, constantly.

The angular velocity of the plane is:

$$\bar{\varpi} = \omega \bar{k} = \dot{\varphi} \bar{k} \tag{5}$$

Substituting (3) and (4) in relation (2), results:

$$\bar{\mathrm{v}} = \bar{\mathrm{v}}_0 + \bar{\omega} \times \bar{r} \tag{6}$$

The relation (6) allows the determination of the velocity for any point belonging the plate (Γ), knowing the velocity of point O and the plate angular velocity ω. Relation (6) is called Euler's relation for velocities and describes the distribution (field) of velocities in plane-parallel motion. The velocity, or distribution/field is called the totality of the instantaneous velocities vectors of the points belonging to the plate or a rigid body.

Properties of the velocity distribution

a. The velocity distribution in plane-parallel motion is composed of two terms, one specific to translational motion $\bar{\mathrm{v}}_0$ and one specific to pure rotational motion $\bar{\omega} \times \bar{r}$ with $\bar{\omega} \perp \bar{\mathrm{v}}_0$.

b. The instantaneous value of the angular velocity $\bar{\omega}$ is an invariant of plane motion, in other words, all points of the plate (Γ) at a given moment have the same instantaneous value of the angular velocity $\bar{\omega}$.

c. In plane motion there is a point having zero instantaneous velocity, a point called the **Instantaneous Center of Velocity (ICV)** denoted by **I**. To prove this statement, is considered (6), projected it onto the mobile reference frame Oxy. Substituting $\bar{\mathrm{v}} = \bar{\mathrm{v}}_{\mathrm{I}} = 0$, results:

$$\begin{cases} \mathrm{v}_{\mathrm{ox}} - \omega y_I = 0 \\ \mathrm{v}_{\mathrm{oy}} - \omega x_I = 0 \end{cases} \tag{7}$$

There is obtained a system with the two equations and two unknowns. The condition for the existence of a unique solution is:

$$\Delta \neq 0 \tag{8}$$

$$\begin{vmatrix} 0 & -\omega \\ \omega & 0 \end{vmatrix} = \omega^2 \neq 0 \tag{9}$$

In these conditions, the system (7) validates the relation (9), it is compatible determinate, having the unique solutions.

$$x_I = -\frac{\mathrm{v}_{oy}}{\omega}; \qquad y_I = \frac{\mathrm{v}_{ox}}{\omega} \tag{10}$$

The relations contained in (10) represents the coordinates of point I, so there is, at a given time, only one point that has zero velocity.

d. The velocities distribution law with respect to I, can be obtained successively applying relation (6) to points M and I and taking into account that $\mathrm{v}_I = 0$. Hence:

$$\bar{\mathrm{v}}_I = 0 = \bar{\mathrm{v}}_0 + \bar{\omega} \times \bar{r}_I \tag{11}$$

where: $\bar{r}_I$ is the position vector of the point I reported of the mobile plane.

Subtracting the relations (6) and (11) results the distribution of relative velocity law with respect to point I:

$$\bar{\mathrm{v}} = \bar{\omega} \times (\bar{r} - \bar{r}_I) = \bar{\omega} \times \overline{IM} \tag{12}$$

Space centrode on (12), it can be stated that relative to point I, the velocity distribution is identical to the velocity distribution in rotational motion, as if the plate rotates around I. In addition, relation (12) becomes space centrode on ICV the method of determining the velocities in planar movement (graphic-analytical method).

e. Planar motion centrodes

In (10), the velocity components of point O are dependent of time:

$$x_I = x_I(t); \qquad y_I = y_I(t) \tag{13}$$

The relations (13) are called parametric equations (with parameter t) of a curve. They represent the geometric locus of the successive positions of the ICV relative to the mobile system and are called the **Body Centrode (R)**. The equation of the trajectory for (R) is obtained by eliminating the parameter (t) from (13), respectively:

$$(R): \qquad f(x_I, y_I) = 0 \tag{14}$$

Applying the relation (1) of the I point, will result:

$$\bar{r}_{1I} = \bar{r}_0 + \bar{r}_I \tag{15}$$

The relation (15) is projected on the fixed reference frame, and this will lead of the relation where all the coordinates are time functions:

$$\begin{cases} x_{1I} = x_0 + x_I cos\varphi - y_I sin\varphi \\ y_{1I} = y_0 + x_I sin\varphi + y_I cos\varphi \end{cases} \tag{16}$$

$$x_{1I} = x_{1I}(t); \qquad y_{1I} = y_{1I}(t) \tag{17}$$

The relations (17) are the parametric equations of another curve. They represent the geometric place of the successive positions of the ICV referred to the fixed system and is called the **Space Centrode** noted **(B**). The equation of the Space Centrode trajectory is obtained by removing the time parameter (t) from relations (17), respectively: (B): $f(x_{1I}, y_{1I}) = 0$ (18)

f. The **Body Centrode** rolls without slipping on the **Space Centrode**.

3. Development of the laboratory work

There are considered two bars as presented in Figures 2, and 3. The free end of the bars, denoted A, moves on the fixed axis O_1x_1 with the constant velocity (v).

As input data, according to Figures 2 and 3, there are considered:

$l = 5 + 0.1n$ [cm]

$h = 6 – 0.1n$ [cm], where n – is the ordering number of the student in group.

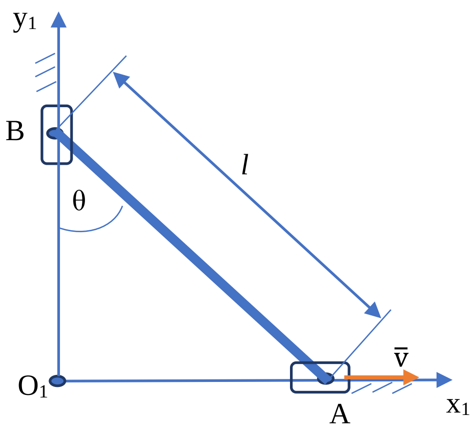


Figure 2 - A, B slides.

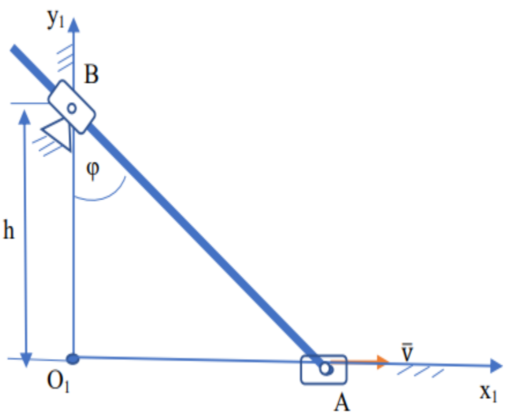


Figure. 3 - A slide, B slide joint.

There is asked to determine:

a) Analytical the moving of centrodes regarding the two bars from Figures 2 and 3.

b) Graphical the motion of centrodes regarding the two bars of Figures 2 and 3.

3.1 Analytical determination of Centrodes of the bar

a. For the bar presented in Figure 2, the *Instantaneous Center of Velocity (I/ICV)* is determined at the intersection of the perpendiculars in points A and B on the support axis of the velocities at these points (see Figure 4). To bar AB, will be attached the reference mobile system Axy. With respect to the fixed $O_1x_1y_1$, and mobile Axy, reference frames, the point I has the coordinates:

$$\begin{cases} x_{1I} = AB\ sin\theta = l\ sin\theta \\ y_{1I} = AB\ cos\theta = l\ cos\theta \end{cases} \tag{19}$$

$$\begin{cases} x_I = IA\ sin\theta = l\ cos\theta\ sin\theta = \frac{1}{2} sin2\theta \\ y_I = IA\ cos\theta = l\ cos^2\theta = \frac{1}{2}(1 + cos2\theta) \end{cases} \tag{20}$$

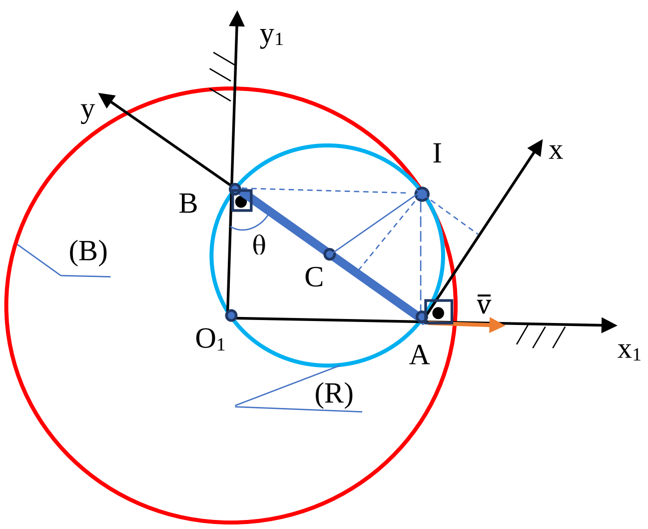


Figure 4

Considering the angle θ as a parameter, the expressions (19) and (20) are the parametric equations of the space centrode (B) and body centrode (R). Their Cartesian equations are obtained by removing the parameter θ. Hence, there are obtained:

(B): $$x_{1I}^2 + y_{1I}^2 = l^2 \qquad (21)$$

$$sin2\theta = \frac{2x_I}{l}\,; \qquad cos2\theta = \frac{2y_I}{l} - 1 \qquad (22)$$

$$\left(\frac{2x_I}{l}\right)^2 + \left(\frac{2y_I}{l} - 1\right)^2 = 1 \qquad (23)$$

(R): $$x_I^2 + \left(y_I - \frac{1}{2}\right)^2 = \left(\frac{l}{2}\right)^2 \qquad (24)$$

From the relations (21) and (24) corresponding to the Space centrode and Body centrode, it can be remarked the fact that there are circles of radius (l) and center at O_1; and a circle of radius ½ with center at C (center of gravity of bar AB).

b. For the bar from Figure 3, the *Instantaneous Center of Velocity* (I) is established at the intersection of the perpendiculars taken in points A and B on the support axis of the velocities in these points. According to Figure 5, to the bar AB, will be attached the reference mobile frame Axy. Related to the fixed $O_1x_1y_1$, and mobile Axy, reference frames the point I has the coordinates:

$$\begin{cases} x_{1I} = O_1A = h\,tg\varphi \\ y_{1I} = IA = \frac{h}{cos\varphi}\frac{1}{cos\varphi} = \frac{h}{cos^2\varphi} \end{cases} \qquad (25)$$

$$\begin{cases} x_I = IA\,sin\varphi = \frac{h\,sin\varphi}{cos^2\varphi} = \frac{h}{cos\varphi}tg\varphi \\ y_I = AB = \frac{h}{cos\varphi} \end{cases} \qquad (26)$$

Considering the angle φ as a parameter, relations (25) and (26) are the parametric equations of the space centrode (B) and the body centrode (R) of the bar from Figure 3. Their Cartesian equations are obtained by removing the parameter φ from (25) and (26) (see Figure 5).

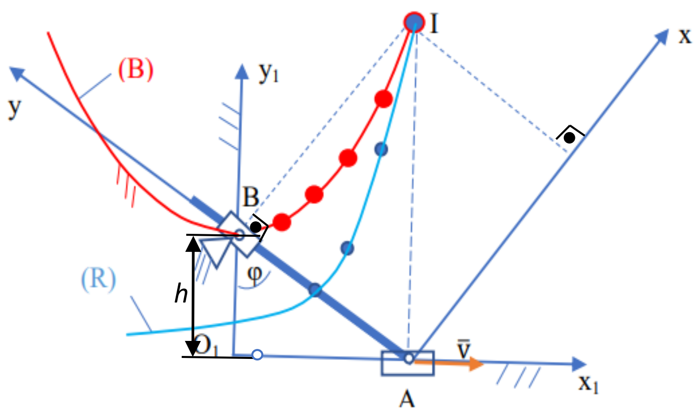


Figure 5

$$\begin{cases} tg\ \varphi = \frac{x_{1I}}{h} \\ \frac{1}{cos^2\varphi} = \frac{y_{1I}}{h} \end{cases} \tag{27}$$

$$1 + \frac{x_{1I}^2}{h^2} = \frac{y_{1I}}{h} \tag{28}$$

(B): $$y_{1I} = \frac{x_{1I}^2}{h^2} + h \tag{29}$$

$$\begin{cases} tg\ \varphi = \frac{x_I}{y_I} \\ \frac{1}{cos^2\varphi} = \frac{y_I^2}{h^2} \end{cases} \tag{30}$$

$$1 + \frac{x_I^2}{y_I^2} = \frac{y_I^2}{h^2} \tag{31}$$

(R): $$h^2 x_I^2 - y_I^2(y_I^2 - h^2) = 0 \tag{32}$$

From (29) and (32) it can be seen that the space centrode (B) is a parabola symmetrical to the axis O_1y_1 with the peak in B, and the body centrode (R) is a fourth - degree spatial curve in y_I.

3.2 Determination of centrodes by graphical method

The space centrodes, graphically are determined as follows:

- The bars are considered in various positions, for each being determined the ICV at the intersection of the normal on the velocities, hence is obtained for each bar a string of positions I_1, I_2, ...I_n. Joining the points I_i from each set of strings, the two space centrodes should be obtained as presented in Figure 6a and Figure 6b.

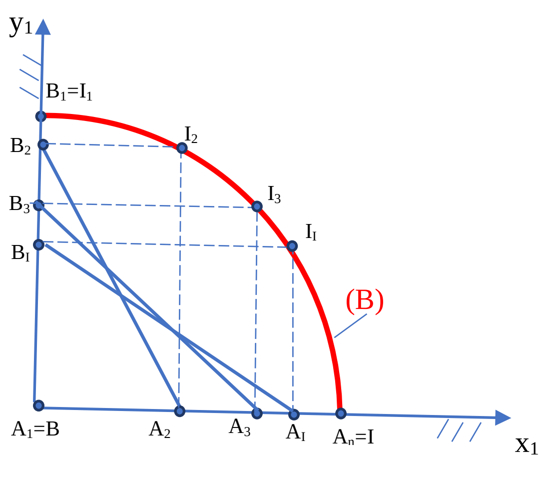


Figure 6a.

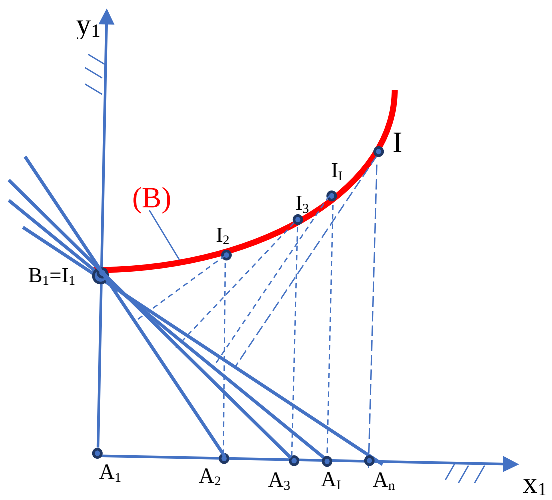


Figure 6b.

To determine graphically the body centrodes, there is used tracing paper for each bar.

On tracing paper, the bars and mobile reference systems are sketched. The tracing paper will be overlapped over the bar so that it coincides for each A_iB_i position, marking the I_i points on the tracing paper (for each bar, around 20 positions of point I will be marked). In this way, a string of I_i points will be obtained on each tracing paper sheet. The above obtained points will be joined resulting the two mobile (rolling) centroids for the bars of Figures 2 and 3 (Figure 7a and Figure 7b).

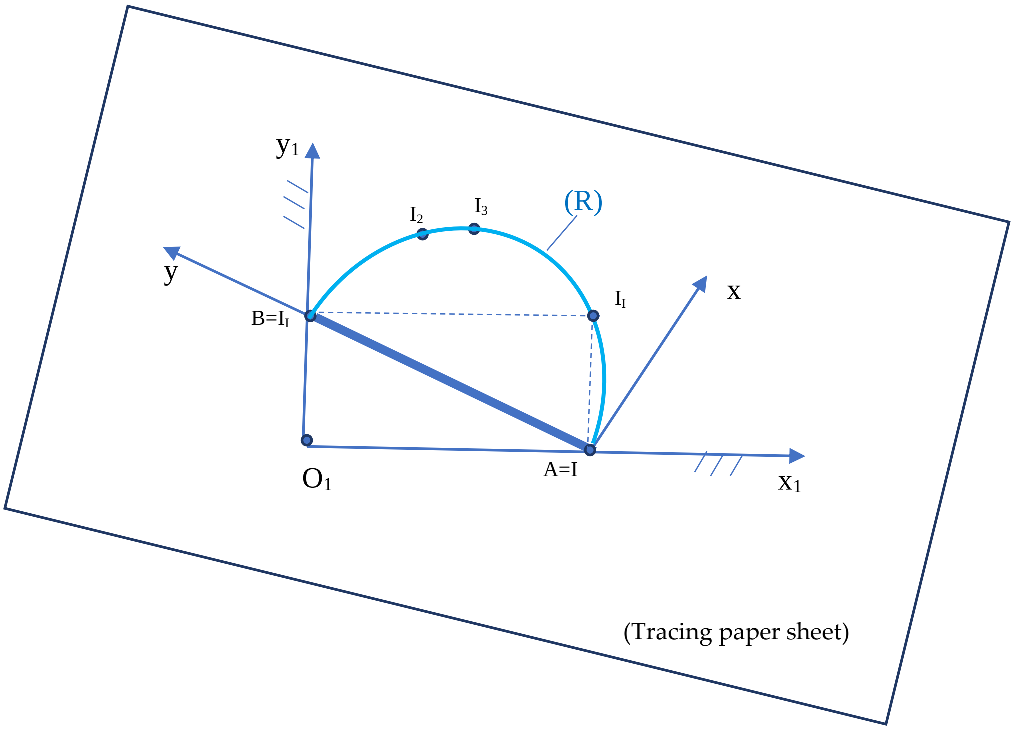


Figure 7a.

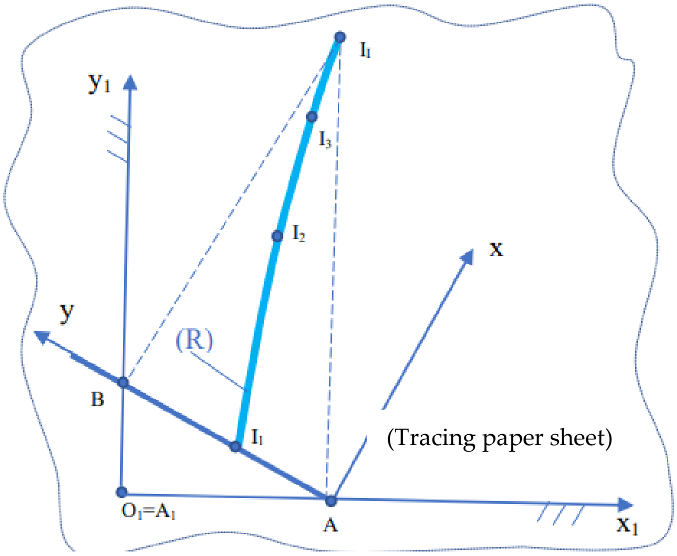


Figure. 7b.

STUDY OF VELOCITY DISTRIBUTION IN CARDANIC MOVEMENT OF A BAR

1. The purpose of the paper

There is considered a straight bar AB, having the length *l*, which ends A and B are moving on the axes of a Cartesian reference frames Ox and Oy , as can be seen in Figure 1). It is required to determine the velocities of points A, B, C, of the bar AB (V_A, V_B, V_C).

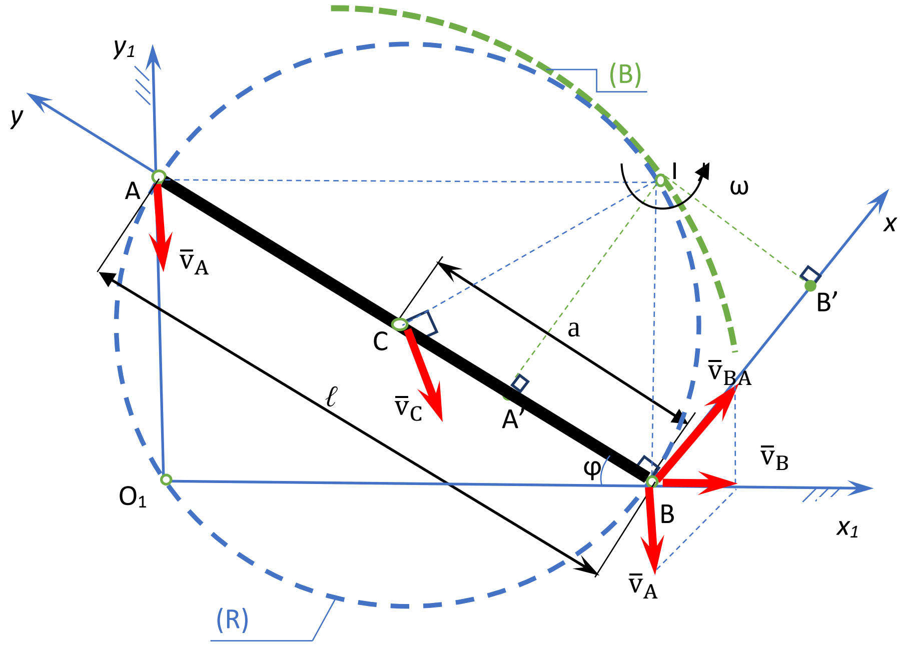


Figure 1

2. Theoretical considerations

The **parallel-plane motion** is the movement, in which three non-collinear points of a rigid are permanently in a fixed plane, known as **movement plane**. The three points also defining a plane, known as **mobile plane**, hence, the mobile plane is moving in the fixed plane. If the rigid body is a plate, the motion is called plane motion.

A rigid which is performing a **parallel-plane motion,** has three degrees of freedom:

$$x_{1A} = x_{1A}(t); \quad y_{1A} = y_{1A}(t); \quad \theta = \theta(t) \tag{1}$$

The movement of the bar is a plane-parallel movement in the xOy plane, hence between the velocities $\bar{v}_A$ and $\bar{v}_B$ of points A and B, there is established the relationship:

$$\bar{v}_B = \bar{v}_A + \bar{v}_{BA} \tag{2}$$

where: $\bar{v}_{BA}$ represents the relative velocity of point B to A. Expression (2) is known as ***Euler's formula for velocities*** and represents the velocities distribution law in the planar movement of the bar AB. The velocity distribution in the planar movement of the bar AB is reducible to a velocity distribution specific to a rotational movement, around the Instantaneous Center of Velocity. Due to this property, the velocity $\bar{v}_C$ of a point C belonging to the bar (representing the center of gravity of the bar AB), can be calculated:

$$\bar{v}_C = \bar{\omega} \times \overline{IC} \qquad\qquad v_C = \omega \cdot IC \tag{3}$$

There is notated with φ the angle between bar AB and O_1x_1 axis. In conformity with the Figure 1, it can be written:

$$\omega = \frac{v_A}{l \cdot \cos\varphi} = \frac{v_B}{l \cdot \sin\varphi} \tag{4}$$

The *Instantaneous Center of Velocity (I/ICV)* is determined intersecting the perpendiculars from the application points of the velocities, on the direction of velocities. The geometrical locus of ICV related to the fix reference frame $O_1x_1y_1$ is called **Space Centrode** (**B**), and the geometrical locus of ICV related to a mobile reference frame xBy, attached to the bar AB, is called **Body Centrode (R). Space Centrode** and **Body Centrode** are tangent, the **Body Centrode is rolling on Space Centrode** without slipping. The parametric expressions for **Space Centrode** and **Body Centrode** are:

$$\begin{cases} x_{1I} = O_1B = l \cdot \cos\varphi \\ y_{1I} = O_1A = l \cdot \sin\varphi \end{cases} \tag{5}$$

$$\begin{cases} x_I = IB \cdot \cos\varphi = O_1A \cdot \cos\varphi = l \cdot \cos\varphi \cdot \sin\varphi = \dfrac{1}{2} \cdot \sin 2\varphi \\ y_I = IB \cdot \sin\varphi = O_1A \cdot \sin\varphi = l \cdot \sin^2\varphi = \dfrac{1}{2}(1 - \cos 2\varphi) \end{cases} \tag{6}$$

From (5) and (6), by eliminating the parameter φ, results:

$$x_{1I}^2 + y_{1I}^2 = l^2 \tag{7}$$

- the equation of the **Space Centrode**, representing a circle having the center in O_1 of radius l;

$$x_I^2 + \left(y_I - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2 \tag{8}$$

- the equation of the **Body Centrode**, representing a circle having the center in bar mass center and radius $\frac{1}{2}$.

3. Description of the Experimental Stand

For the study of bar motion, there is used the device shown in Figure 2 .

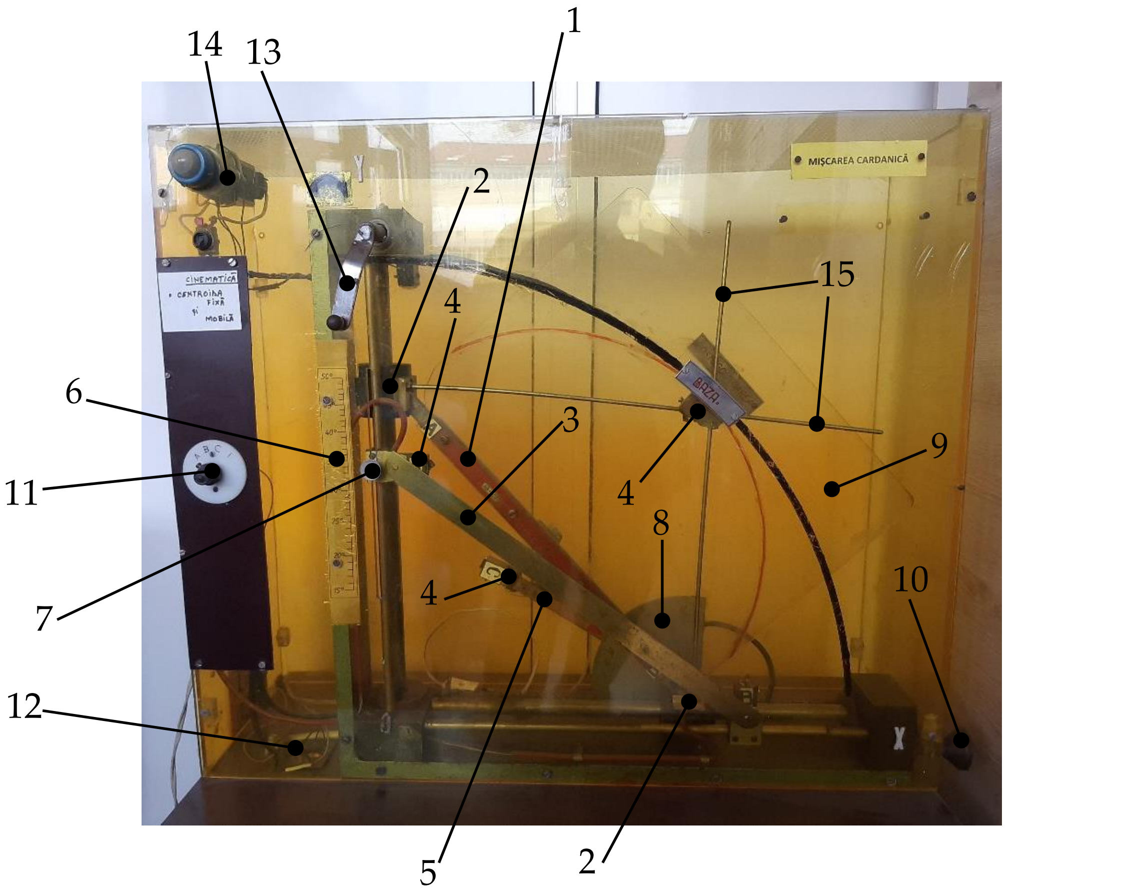


Figure 2

The components of the experimental stand are:

1- The bar AB, having the length l=400 [mm];

2- The sliders A and B;

3- Device for adjusting the position of the bar AB (adjusting the angle φ);

4- Optical sensors;

5- Device for adjusting the position of the bar's angle for establishing the position of point C (the mass center position); a=205 [mm];

6,8 – Indicators of the angle φ;

7 – Screw for adjustment of device 3;

9- Mobile plan fixed on bar AB;

10- Adjustment screw for 5;

11- Selector for setting of measuring points (A,B or C);

12- Electronic Assembly.

13- Crank for the bar AB;

14- Transformer 220V/24V for powering the optical sensors 4;

15- Bars symbolizing the normal velocities of points A and B.

The experimental stand is represented symbolically in Figure 3.

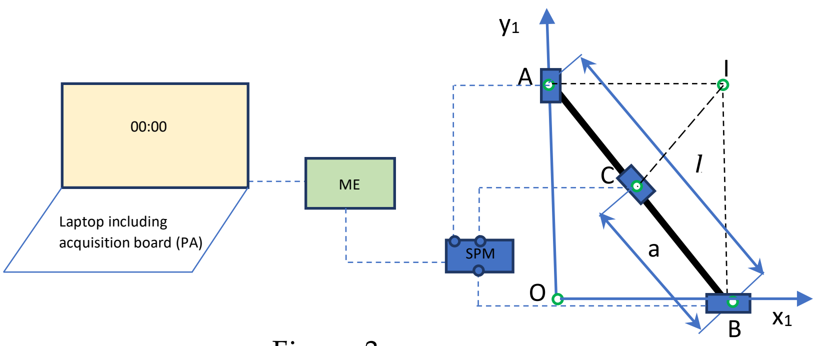


Figure 3

4. Development of the laboratory work

With the help of crank 13, the bar AB is raised to a certain height. With 11, is set up the measuring point A, B, or C (is connected the optical sensor 4, at the Electronic Assembly 12). With the same 13, there is unlocked the bar, which falls under the action of its own weight, executing a parallel-plane movement in a vertical plane. When the bar passes through the position given by the angle φ (selected by the devices 3 and 5), the Electronic Assembly 12 starts the timer, for the period when the light beam of the optical sensor 4 is interrupted. The timer displays the time Δt. Knowing the length Δl, there can be established the chosen point A,B, or C with the expression:

$$v = \frac{\Delta l}{\Delta t} \quad [\mathrm{mm/s}] \tag{9}$$

where: $\Delta l = 20\ [mm]$, is the length of the lamella which interrupts the light beam at the sensors; Δt is the interrupting time of the light beam at the sensors, displayed by timer.

Hence, there are experimental determined the velocities of the points A,B and C:

$$v_A^{exp} = \frac{\Delta l}{t_A}; \qquad v_B^{exp} = \frac{\Delta l}{t_B}; \qquad v_C^{exp} = \frac{\Delta l}{t_C} \tag{10}$$

There can be calculated the experimental value of the angle φ, as:

$$\tan\varphi^{exp} = \frac{t_A}{t_B}, \text{ or } \varphi^{exp} = \arctan\frac{t_A}{t_B} \tag{11}$$

The velocities for the points A,B and C can be theoretically determined using the ICV Method, by:

$$\bar{v}_A = \bar{\omega} \times \overline{IA} \qquad v_A^{theor} = \omega \cdot IA \tag{12}$$

$$\bar{v}_B = \bar{\omega} \times \overline{IB} \qquad v_B^{theor} = \omega \cdot IB \tag{13}$$

In keeping with (3), the theoretical value for the velocity of point C, is established as:

$$\bar{v}_C = \bar{\omega} \times \overline{IC} \qquad v_C^{theor} = \omega \cdot IC \tag{14}$$

where:
$$IC = \sqrt{IB^2 + BC^2 - 2 \cdot IB \cdot BC \cdot \cos(\frac{\pi}{2} - \varphi)} = \tag{15}$$
$$= \sqrt{(l \cdot \sin\varphi)^2 + a^2 - 2 \cdot l \cdot \sin\varphi \cdot a \cdot \sin\varphi} = \sqrt{l \cdot (l - 2 \cdot a) \cdot \sin^2\varphi + a^2}$$

The same velocity of point C, can be calculated, using the coordinates of the point:

$$\begin{cases} x_{1C} = O_1B\text{-}BC \cdot \cos\varphi = (l\text{ -}a) \cdot \cos\varphi \\ y_{1C} = BC \cdot \sin\varphi = a \cdot \sin\varphi \end{cases} \tag{16}$$

Applying the time derivative on (16), there is obtained:

$$\begin{cases} \dot{x}_{1C} = -\omega \cdot (l\text{ -}a) \cdot \sin\varphi \\ \dot{y}_{1C} = \omega \cdot a \cdot \cos\varphi \end{cases} \tag{17}$$

Hence, based on expressions contained in (16), there is obtained:

$$v_C^{teor} = \sqrt{\dot{x}_{1C}^2 + \dot{y}_{1C}^2} = \omega \cdot \sqrt{(l\text{ -}a)^2 \cdot \sin^2\varphi + a^2 \cdot \cos^2\varphi} = \omega \cdot \sqrt{l \cdot (l\text{ -}2 \cdot a) \cdot \sin^2\varphi + a^2} \tag{18}$$

From (12) there can be determined ω, which is introduced in (13), resulting the expression:

$$v_B = v_A \cdot \frac{IB}{IA} = v_A \cdot \frac{l \cdot \sin\varphi}{l \cdot \cos\varphi} = v_A \cdot \tan\varphi \tag{19}$$

which conducts to:
$$\tan\varphi^{teor} = \frac{v_B}{v_A} \tag{20}$$

Considering (11) with (20), there is determined the relative error for φ, with:

$$\varepsilon_{r_\varphi} = \frac{\left|\tan\varphi^{theor} - \tan\varphi^{exp}\right|}{\tan\varphi^{theor}} \cdot 100[\%] \tag{21}$$

The above calculated results are registered in **Table 1**.

Table 1

Experiment	φ^{theor}	Δt			φ^{exp}	ε_{r_φ}
		t_A [s]	t_B [s]	t_C [s]		
1						
2						
3						

Having determined the velocities for points A, B and C, there will be filled **Table 2,** and then there is calculated the relative error for velocity v_C, with:

$$\varepsilon_{r_{v_C}} = \frac{\left| {v_C}^{theor} - {v_C}^{exp} \right|}{{v_C}^{theor}} \cdot 100[\%] \tag{22}$$

Table 2

Experiment	φ^{theor}	v_A^{theor}	v_B^{theor}	v_C^{theor}	v_C^{exp}	$\varepsilon_{r_{v_C}}$
1						
2						
3						

GRAPHICAL DETERMINATION OF THE VELOCITIES AND ACCELERATIONS
IN THE MOVEMENT OF A PLANAR MECHANISM

1. The purpose of the work

The aim of the paper is the graphical determination of the velocities and accelerations in the case of a planar mechanism, based on Euler's laws for velocities and accelerations.

2. Theoretical considerations

It is considered a rigid body in motion, (see Figure 1) so that three non-collinear points of it (related to the body) remains in a fixed plane throughout the body motion, this type of motion being called *plane-parallel motion*. The three points are defining a mobile plane (a section of the rigid) (π) that moves in a fix plane, named *directional plane*.

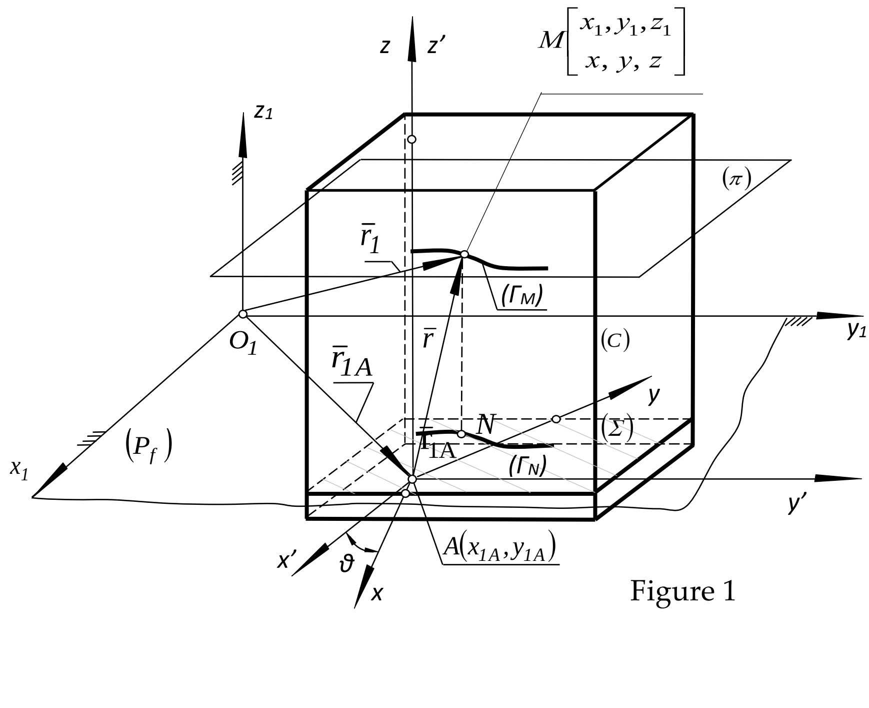


Figure 1

Therefore, all sections of the rigid that are parallel to (π) will move in planes that are parallel to the directional plane, fact suggested by the name of plane-parallel motion.

In the plane-parallel motion the rigid has three degrees of freedom, the coordinates of a point inside the plane and the rotation angle.

If a straight line (Δ) belonging to the rigid is taken, perpendicular to the plane (π), then during plane parallel motion the line (Δ) it will also move, remaining all time normal to the (π), respectively to the directional plane, and parallel to itself in the same time. So, the straight line will perform a pure translational motion. All its points will have the same velocity and the same instantaneous acceleration, and their trajectories will be identical curves but located in parallel planes with each other and with the section (π), respectively with the directional plane.

On these considerations the study of plane-parallel motion can be reduced to the study of section (π) motion in the directional plane. From the mechanical point of view, section (π) is a plate, and its movement in the directional plane is called planar motion (the movement of a plate in a plane).

Let be a material point *M*, belonging to the rigid. Between the coordinates x_1, y_1, z_1 and x, y, z of this point recorded in relation to a fixed system, respectively to a mobile system, solitary with the rigid one, there is the relationship matrix:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & x_{1A} \\ \sin\theta & \cos\theta & 0 & y_{1A} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{1}$$

From relation (1) it is observed that z_1 does not depend on time, but only on the height of point M, which confirms the three degrees of freedom and the fact that the trajectories of the points on the straight line (Δ) are identical curves located in parallel planes.

Examples of parallel-plane motions include: the movement of the connecting rod in a connecting slider-crank mechanism, the movement of a vehicle wheel on a straight road, the cardanic movement of a straight bar, the movement of cam and gear mechanisms.

3. Development of the laboratory work

There is considered the mechanism from Figure 2, composed by:

1. Equalizer bar ;
3. Crank driven;
4. Triangular plate;
5. Connecting rod;
6. Piston, in a configuration given by angle φ.

There are known as input data the following:

- the angle $\varphi = 35 + 2n\ [°]$;
- the rotation of the Driving crank O_1A, $n_1 = 300 + 10n$ = const., n being the order number of the student in group.

$- O_1A = 30\ cm, \quad O_1O_2 = 30\ cm, \ O_2B = 30\ cm\ \ AB = 40\ cm, \ \ AC = 20\ cm,$

$CD = 20\ cm$

It is required to determine graphically the configuration of the mechanism defined by the angle φ, the velocity and acceleration of point D, by using the velocity and acceleration plane method.

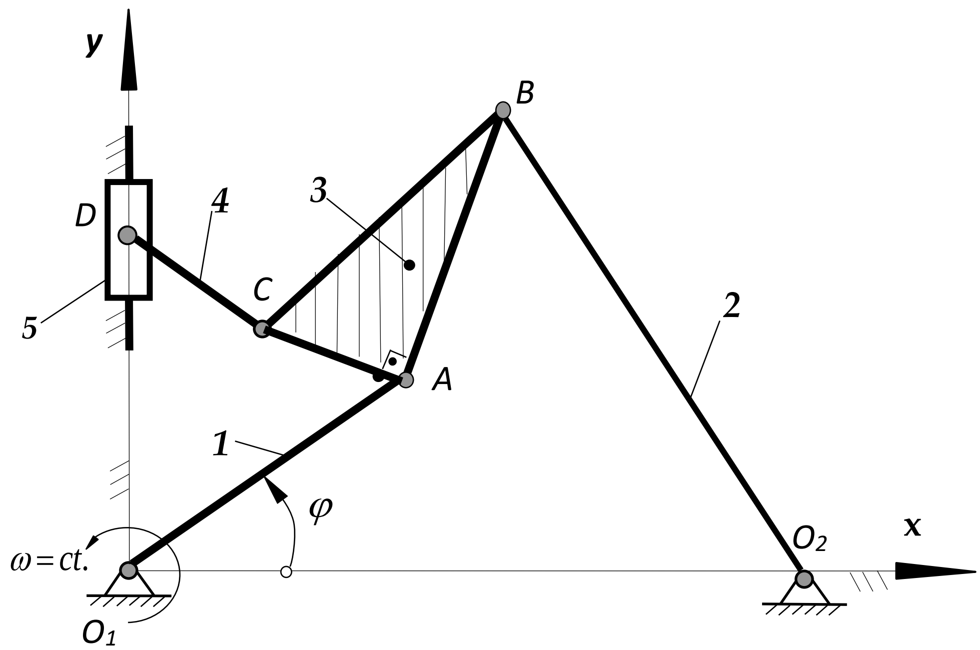


Figure 2

a. Determination of the velocity $\bar{v}_D$

The velocity of the piston will be determined by the **velocity plane method**. In this sense, after determining the velocity of the point A belonging to the crank, the velocities of the points B, C, D are successively determined, by writing the Euler`s relations for the velocities, for the pairs of points belonging to the same element. The order of operations is as follows:

- There is calculated the angular velocity:

$$\omega = \frac{\pi \cdot n_1}{30} \quad [\frac{rad}{s}] \tag{2}$$

- It is calculated the velocity of the point A ($\bar{v}_A$), belonging to the crank O_1A by:

$$v_A = \omega \cdot O_1A. \tag{3}$$

The vector $\bar{v}_A$ has a normal direction on the O_1A and the sense given by ω.

- It is represented the mechanism at a scale, into configuration given by the angle φ .

It is noted with k_L ***the length scale coefficient,*** represented as a ratio between the given length and the drawn length:

$$k_L = \frac{L_{real}\,(cm)}{L_{drawn}(cm)} = \frac{O_1A}{O_1A_{drawn}} \tag{4}$$

- There is represented the velocity of point A on the mechanism plane, as being equal with drawn length of the crank: $$v_{A_{drawn}} = O_1A_{drawn} \tag{5}$$

Hence, results the scale coefficient of the velocities $\boldsymbol{k_v}$:

$$k_v = \frac{v_{real}}{v_{drawn}} = \frac{\omega \cdot O_1A}{O_1A_{drawn}} = \omega \cdot k_L. \tag{6}$$

-there is determined the velocity of point B, taking in account the relations:

$$\bar{v}_B = \bar{v}_A + \bar{v}_{BA}, \qquad \bar{v}_{BA} = \bar{\omega}_3 \times \overline{AB} \qquad \bar{v}_B = \bar{\omega}_2 \times \overline{O_2B}. \tag{7}$$

The vector $\bar{v}_A$ is known, as magnitude and direction, and the vectors $\bar{v}_{BA} \perp \overline{BA}$, and $\bar{v}_B \perp \overline{O_2B}$ are known only as direction.

*For the graphical building of the velocity plane, is considered an arbitrarily point **o**, (see Figure 3). In point **o**, the velocity $\bar{v}_A$ is represented to scale. It is noted by **a** the extremity of the vector $\bar{v}_A$. Through the extremity **a** of $\bar{v}_A$ a perpendicular is drawn to AB, and through **o** a* *perpendicular is drawn to O_2B. The point of intersection of these perpendiculars, denoted by* ***b****, represents the extremity of the absolute velocity vector $\bar{v}_B$.*

Note: As can be seen from the sketch, a relation for velocities has as image a triangle.

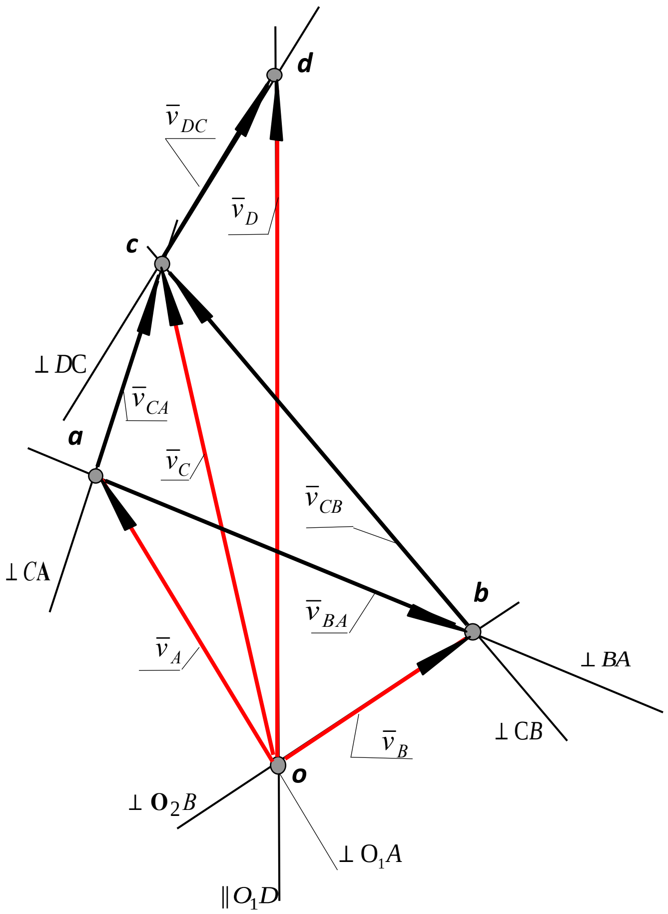


Figure 3

Based on following Euler`s expressions for velocities, there is determined the velocity of point C, noted $\bar{v}_C$, expressed related to $\bar{v}_A$ and $\bar{v}_B$:

$$\bar{v}_C = \bar{v}_A + \bar{v}_{CA}, \bar{v}_{CA} = \bar{\omega}_2 \times \overline{AC}, \bar{v}_{CA} \perp \overline{AC}. \tag{8}$$

$$\bar{v}_C = \bar{v}_B + \bar{v}_{CB}, \bar{v}_{CB} = \bar{\omega}_3 \times \overline{BC}, \bar{v}_{CB} \perp \overline{BC}. \tag{9}$$

In the velocities plane, a perpendicular to AC is drawn through point ***a****, respectively through* ***b*** *another one perpendicular to BC. These perpendiculars intersect in point* ***c****, which represents the extremity of the absolute velocity vector $\bar{v}_C$.*

- On basis of the expressions:

$$\bar{v}_D = \bar{v}_C + \bar{v}_{DC}, \bar{v}_{DC} = \bar{\omega}_4 \times \overline{CD}, \bar{v}_D \perp \overline{CD}. \tag{10}$$

$$\bar{v}_D \| \overline{O_1 D}. \tag{11}$$

is determined the velocity of point D.

In the velocity plane, a perpendicular to CD is drawn through point ***c****, which intersects in* ***d*** *the parallel taken through* ***o*** *to O_1D. The point* ***d*** *represents the extremity of the absolute velocity vector$\bar{v}_D$.. Following relations (7)-(11) and Figure 3, the polygons starting from* ***o*** *to* ***b, c*** *and* ***d*** *are closed, resulting the vectors $\bar{v}_{BA}, \bar{v}_B, \bar{v}_{CA}, \bar{v}_{CB}, \bar{v}_C, \bar{v}_{DC}, \bar{v}_D$.*

The value of the velocity $\bar{v}_D$ is calculated as:

$$v_D = k_V \cdot v_{D_{drawn}} \,. \tag{12}$$

- It can be calculated the angular velocities of the **elements 2, 3, 4,** as:

$$\omega_2 = \frac{v_B}{O_2 B} = \frac{v_B}{BO_2}, \quad \omega_3 = \frac{v_{BA}}{AB} = \frac{v_{CA}}{AC}, \quad \omega_4 = \frac{v_{DC}}{CD}. \tag{13}$$

or:

$$\omega_2 = \omega \frac{v_{B_{drawn}}}{O_2 B_{drawn}}, \quad \omega_3 = \omega \frac{v_{BA_{drawn}}}{AB_{drawn}} = \omega \frac{v_{CA_{drawn}}}{AC_{drawn}}, \quad \omega_4 = \omega \frac{v_{DC_{drawn}}}{CD_{drawn}}. \tag{14}$$

The directions of the angular velocities are given by the direction in which the relative rotational velocities rotate the segments to which they refer.

b) Determination of the acceleration $\bar{a}_D$ of the point D

The acceleration of point D will be determined by the **acceleration plan method**. In this sense, after determining the absolute acceleration of point A belonging to the O_1A, the accelerations of points B, C, D are successively determined, writing Euler relations for accelerations at pairs of points belonging to the same element.

The steps for building the acceleration plan are as follows:

- The acceleration of point A, belonging to the crank is calculated, in keeping with the fact that $\varepsilon = 0$, hence it has only the normal component with the sense from A to O_1.

$$a_A = a_A^{\nu} = \omega^2 \cdot O_1 A \tag{15}$$

Note: $\bar{a}_A$ is determined graphically by the Rectangular Triangle Method. (see Figure 4)

*There is represented to scale the velocity of point A, then the point O_1 is joined with the extremity of $\bar{v}_A$, and in **a** there is drawn a normal on O_1a which intersects O_1A in **b.** The segment **Ab** is folded 180° on O_1A, hence resulting the point **c**. the segment Ac is the vector $\overline{a_A^{\nu}}$ of the point A.*

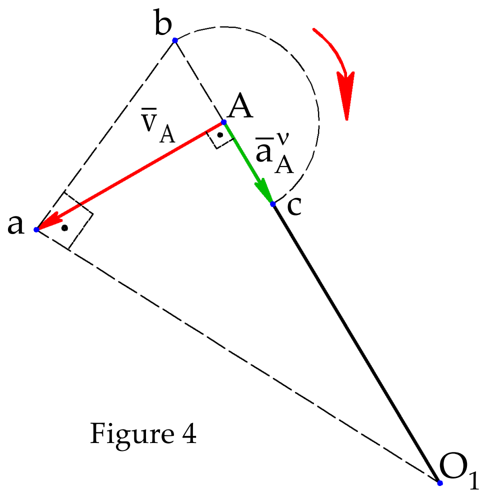


Figure 4

*In the right-angle triangle O_1Ab, Aa is a height, and according to the **Altitude/Height Theorem**, results:*

$$Aa^2 = O_1A \cdot Ab \ , \tag{16}$$

hence:

$$Ab = \frac{Aa^2}{O_1A} = \frac{v_A^2}{O_1A} = a_a^{\nu} \ , \tag{17}$$

On the mechanism plane there is represented the acceleration of point A, such that to be equal with the length of the crank, respectively:

$$a_{A_{drawn}} = O_1A_{drawn} \tag{18}$$

Results the accelerations scale coefficient k_a:

$$k_a = \frac{a_{real}}{a_{drawn}} = \frac{\omega^2 \cdot O_1A}{O_1A_{drawn}} = \omega^2 \cdot k_L \tag{19}$$

- There is determined the acceleration of point B, that belongs to the connecting rod, by the Euler`s formula for accelerations, as:

$$\bar{a}_B = \bar{a}_A + \bar{a}_{BA} = \bar{a}_A + \bar{a}_{BA}^{\tau} + \bar{a}_{BA}^{\nu} \tag{20}$$

$$\bar{a}_B = \bar{a}_B^\tau + \bar{a}_B^\nu \tag{21}$$

where:

- $\bar{a}_A$ is known as magnitude and direction;
- $\bar{a}_{BA}^\nu$ and $\bar{a}_B^\nu$ are ***normal components of accelerations***, determined by **Rectangular Triangle Method** using the velocities $\bar{v}_{BA}$ and $\bar{v}_B$, known as magnitude and direction;
- $\bar{a}_{BA}^\tau$ and $\bar{a}_B^\tau$ are ***tangential components of accelerations*** having known directions – perpendicular on AB and O_2B. ($\bar{a}_{BA}^\tau \perp \overline{AB}$, $\bar{a}_B^\tau \perp \overline{O_2 B}$)

Note: *The Euler expression for accelerations has as geometrical image a quadrilateral in acceleration plane.*

To create the acceleration plane, there is arbitrarily chosen a point $\boldsymbol{o'}$ *where is represented the acceleration* $\bar{a}_A$ *of point A, it`s extremity being noted* $\boldsymbol{a'}$*. Through the extremity* a' *is drawn at acceleration scale from B towards A, the vector* $\bar{a}_{BA}^\nu$*. Thru its extremities is drawn a perpendicular to AB then, through of* $\boldsymbol{o'}$*, is drawn, with the direction from B towards* O_2*, to the scale the vector* $\bar{a}_B^\nu$*. Through its extremity* $\bar{a}_B^\nu$ *goes a perpendicular to* O_2B*, which meets in* $\boldsymbol{b'}$ *the perpendicular to AB. The point* $\boldsymbol{b'}$ *thus established, is the extremity of the absolute acceleration vector* $\bar{a}_B$*.*

Following the relations (20)-(21) and Figure 5, are closed the polygons starting from $\boldsymbol{o'}$ *to* $\boldsymbol{b'}$*, thus being obtained the vectors:* a_{BA}^τ, a_B^τ *,* a_{BA}*,* a_B *.*

• The acceleration of point C, belonging to element 3 can be determined on the basis of Euler`s expressions for accelerations, as:

$$\bar{a}_C = \bar{a}_A + \bar{a}_{CA} = \bar{a}_A + \bar{a}_{CA}^\tau + \bar{a}_{CA}^\nu \tag{22}$$

$$\bar{a}_C = \bar{a}_B + \bar{a}_{CB} = \bar{a}_B + \bar{a}_{CB}^\tau + \bar{a}_{CB}^\nu \tag{23}$$

where:

- $\bar{a}_A$ and $\bar{a}_B$ are known as magnitude and direction;

- $\bar{a}^{\nu}_{CA}$ and $\bar{a}^{\nu}_{CB}$ are ***normal components of accelerations***, determined by **Rectangular Triangle Method** using the velocities $\bar{v}_{CA}$ and $\bar{v}_{CB}$, known as magnitude and direction;
- $\bar{a}^{\tau}_{CA}$ and $\bar{a}^{\tau}_{CB}$ are ***tangential components of accelerations*** having known directions – perpendicular on AC and BC. ($\bar{a}^{\tau}_{CA} \perp \overline{CA}$, $\bar{a}^{\tau}_{CB} \perp \overline{CB}$ $\bar{a}^{\tau}_{B} \perp \overline{O_2B}$).

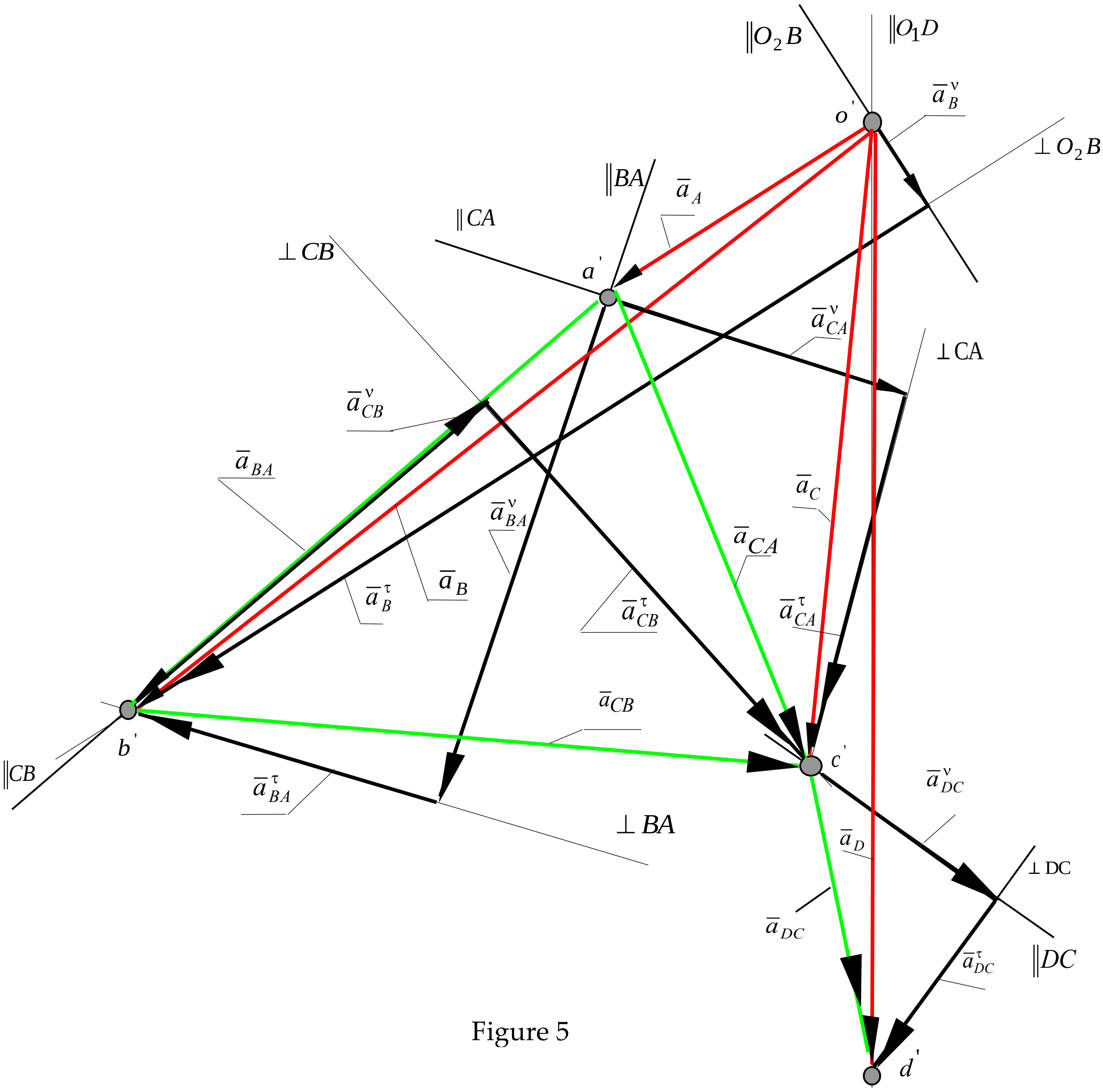


Figure 5

To determine the acceleration $\bar{a}_C$ of the point C, is drawn through $\boldsymbol{a'}$, *with the direction from C towards A, the vector $\bar{a}_{CA}^{\nu}$ obtained by the* ***Rectangular Triangle Method****. Through its extremity goes a perpendicular to AC. Then, through* $\boldsymbol{b'}$, *with the direction from C towards B, the vector $\bar{a}_{CB}^{\nu}$ is drawn on the acceleration scale. Through the extremity of $\bar{a}_{CB}^{\nu}$ is drawn a perpendicular ob BC, which meets in* $\boldsymbol{c'}$ *the perpendicular to AC. Hence the point* $\boldsymbol{c'}$ *represents the extremity of the absolute acceleration vector $\bar{a}_C$.*

Following relations (22) and (23) and Figure 5, the polygons are closed starting from $\boldsymbol{o'}$ *towards* $\boldsymbol{c'}$*, resulting in the vectors:* $a_{CA}^{\tau}, a_{CB}^{\tau}, a_{CA}, a_{CB}, a_C$.

- The acceleration of point D is obtain based on the relations:

$$\bar{a}_D = \bar{a}_C + \bar{a}_{DC} = \bar{a}_C + \bar{a}_{DC}^{\tau} + \bar{a}_{DC}^{\vartheta} \ ; \ \bar{a}_D || O_1 D \tag{24}$$

where:

- $\bar{a}_C$ is known as magnitude and direction;
- $\bar{a}_{DC}^{\nu}$ is a ***normal component of acceleration***, determined by **Rectangular Triangle Method** using the velocities $\bar{v}_{DC}$, known as magnitude and direction;
- $\bar{a}_{DC}^{\tau}$ is ***tangential component of acceleration*** having known direction – perpendicular on CD. ($\bar{a}_{DC}^{\tau} \perp \overline{DC}$);
- $\bar{a}_D$ is known as direction; (the point D is moving along O_1y)

To graphically determine the acceleration $\bar{a}_D$, through the point $\boldsymbol{c'}$ *is drawn, on the acceleration scale the vector $\bar{a}_{DC}^{\nu}$. Through the extremity of $\bar{a}_{DC}^{\nu}$ is drawn a perpendicular to CD, which meets in* $\boldsymbol{d'}$ *the parallel taken through* $\boldsymbol{o'}$ *to O_1y. Point* $\boldsymbol{d'}$ *represents the extremity of the absolute acceleration vector $\bar{a}_D$.*

Following relations (24) and Figure 5, are closed the polygons starting from $\boldsymbol{o'}$ *towards* $\boldsymbol{d'}$*, resulting the acceleration vectors:* $a_{DC}^{\tau}, a_{DC}, a_D$.

The acceleration of point D is obtained by transposing the vector $\bar{a}_D$ from the acceleration plane to point D in the plane of the mechanism. The real value of this acceleration will be:

$$a_D = k_a \cdot a_{D_{drawn}} \tag{25}$$

- The angular accelerations of the **elements 2, 3, 4** can be determinate by:

$$\varepsilon_2 = \frac{a_{BA}^{\tau}}{BA}, \quad \varepsilon_3 = \frac{a_{BC}^{\tau}}{BC} = \frac{a_B^{\tau}}{BC}, \qquad \varepsilon_4 = \frac{a_{DC}^{\tau}}{DC} \tag{26}$$

or

$$\varepsilon_2 = \frac{a_B^{\tau}}{O_2B} = \omega^2 \cdot \frac{a_{B_{drawn}}^{\tau}}{O_2B_{drawn}}, \quad \varepsilon_3 = \frac{a_{BA}^{\tau}}{AB} = \omega^2 \cdot \frac{a_{BA_{drawn}}^{\tau}}{AB_{drawn}}, \qquad \varepsilon_4 = \frac{a_{DC}^{\tau}}{DC} = \omega^2 \cdot \frac{a_{DC_{drawn}}^{\tau}}{CD_{drawn}} \tag{27}$$

The accelerations a_{BA}^{τ}, a_B^{τ}, a_{DC}^{τ} from the above expressions on scale representation are taken from the accelerations plane.

Note: *According to Mehmke Theorem for velocities and accelerations, the polygon formed by points of the same element of the mechanism is similar as the polygon formed by homologous points from the acceleration and velocities plans. They allow the establishing of velocities and acceleration poles of the elements 3 and 4 of the mechanism being in planar movement.*

GRAPHICAL DETERMINATION OF THE VELOCITIES AND ACCELERATIONS FOR THE POINTS OF A PLANE MECHANISM WITH ELEMENTS IN RELATIVE MOTION

1. The purpose of the paper

The paper studies the movement of a mechanism, having elements which are executing relative motion. There will be established graphical the velocities and accelerations, based on the laws of composition of velocities and accelerations in relative movement.

2. Theoretical considerations

The movement of a point relative to a fixed reference system is called **absolute movement**, characterized by the absolute trajectory(C_a), the absolute velocity ($\bar{v}_a$) and the absolute acceleration ($\bar{a}_a$).

The movement of a point related to a mobile reference system is called **relative movement**, characterized by the relative trajectory (C_r), the relative velocity ($\bar{v}_r$) and the relative acceleration ($\bar{a}_r$).

If imaginary, is considered that the material point is fixed to the mobile reference system, the movement performed together with the mobile system, is called **transport movement**, characterized by the transport trajectory (C_t), the transport velocity ($\bar{v}_t$) and transport acceleration ($\bar{a}_t$).

On these three types of movements characterized by corresponding velocities and accelerations, are based the following laws of composition for the kinematic parameters:

$$\bar{v}_a = \bar{v}_r + \bar{v}_t \qquad \bar{a}_a = \bar{a}_r + \bar{a}_t + \bar{a}_C \tag{1}$$

where:

- $\bar{v}_a$ and $\bar{a}_a$ are the absolute velocities and accelerations.
- $\bar{v}_t$ and $\bar{a}_t$ are the transport velocities and accelerations.
- $\bar{a}_C$ is the Coriolis acceleration, expressed as:

$$\bar{a}_C = 2 \cdot \bar{\omega}_t x \bar{v}_r \tag{2}$$

3. Development of the work

It is considered the mechanism from Figure 1, at which the crank OA rotates with the constant, clockwise angular velocity ω_1. There is required to determine graphically the velocities and accelerations of the points A and B, for the position of the crank OA given by the angle related to the vertical direction $\varphi_1 = 30 + n\,[°]$, "n" being the student order number in the group.

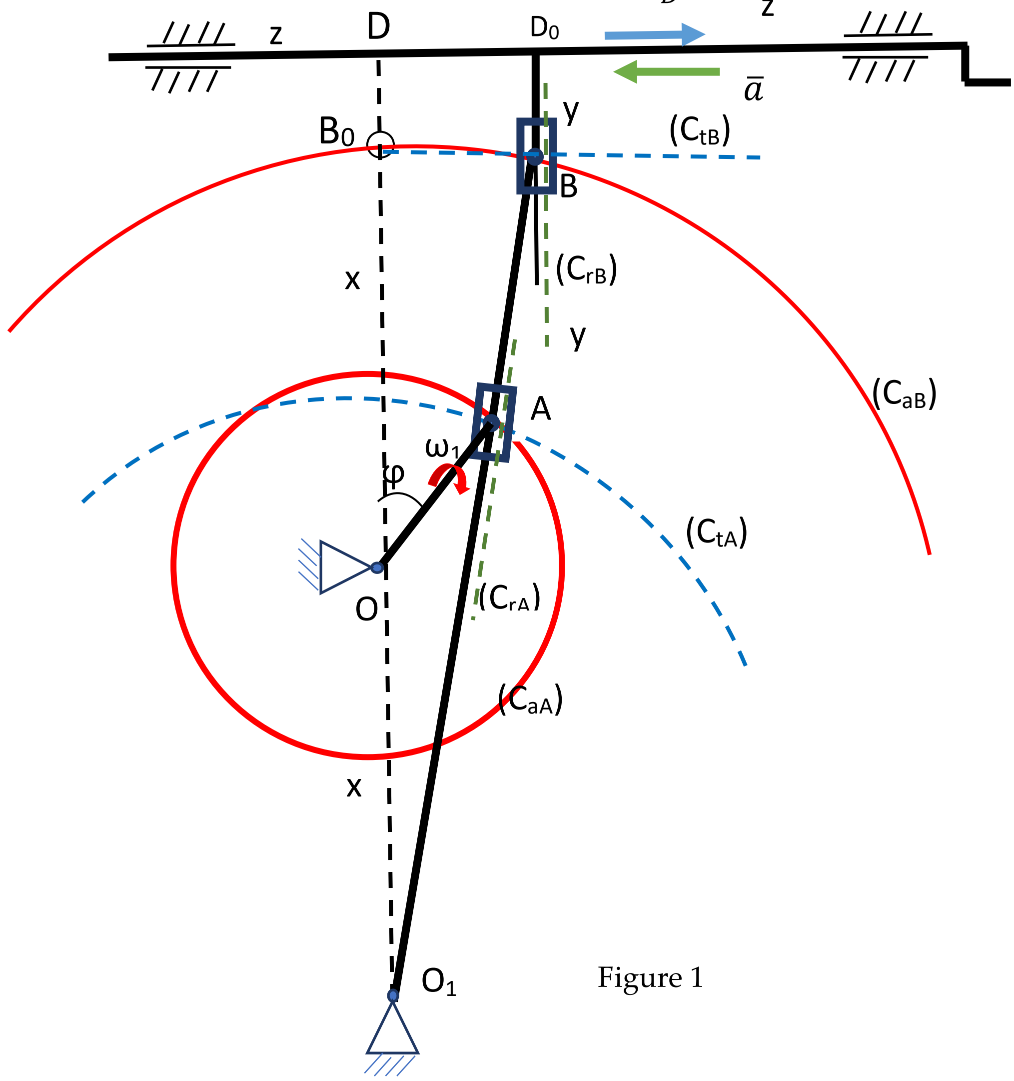


Figure 1

The elements of the mechanism are given by:

$$\omega_1 = 0.5 + \frac{n}{20}\left[\frac{rad}{s}\right],$$

$$OA = 2 + 0.25 \cdot n \text{ [cm]},$$

$$OO_1 = 5 + 0.30 \cdot n \text{ [cm]},$$

$$O_1D = 11 + 0.5 \cdot n \text{ [cm]},$$

$$O_1B = 10 + 0.5 \cdot n \text{ [cm]}.$$

Each student establishes own input data based on order number in the group, and after, calculates the input data with the above relations.

Based on previous calculated input data, the mechanism considered in Figure 1, is drawn, for the configuration given by angle φ.

There is introduced the notation k_L representing the scale of length coefficient, which is a ratio between the real and the drawn length: $k_L = \frac{L_{real}}{L_{drawn}}[cm] = \frac{O_1A}{O_1A_{drawn}}$.

On the mechanism sketch, there is represented the velocity of point A equal with the drawn length of the crank, respectively: $v_{A_{drawn}} = O_1A_{drawn}$. Hence, there is introduced the scale coefficient of velocities: $k_v = \frac{v_{real}}{v_{drawn}} = \frac{\omega \cdot O_1A}{O_1A_{drawn}} = \omega \cdot k_L$.

a) **Distributions of accelerations for points A, B**

Point A

The absolute trajectory (C_{aA}) of point A is a circle, centered in O having the radius OA. Its circulary movement is with the constant angular velocity ω_1. The absolute angular velocity is normal to OA, and has the sense given by ω_1. The magnitude of the absolute velocity vector of point A is:

$$v_{aA} = \omega_1 \cdot OA \tag{4}$$

The relative trajectory (C_{rA}) is rectilinear, along the linear guidance O_1B ($\bar{v}_{rA} \| xx$).
The transport trajectory (C_{tA}) is obtained by considering point A fixed to the linear guidance O_1B, thus obtaining a circle with the center in O_1 of radius O_1A. The velocity will be perpendicular to the guidance ($\bar{v}_{tA} \perp xx$).

To determine the relative $\bar{v}_{rA}$ and transport $\bar{v}_{tA}$ velocities of point A, the absolute velocity $\bar{v}_{aA}$ is decomposed into two directions: one parallel and the other perpendicular to the *xx* axis (see Figure 2), hence is written:

$$\bar{v}_{aA} = \bar{v}_{rA} + \bar{v}_{tA} \tag{3}$$

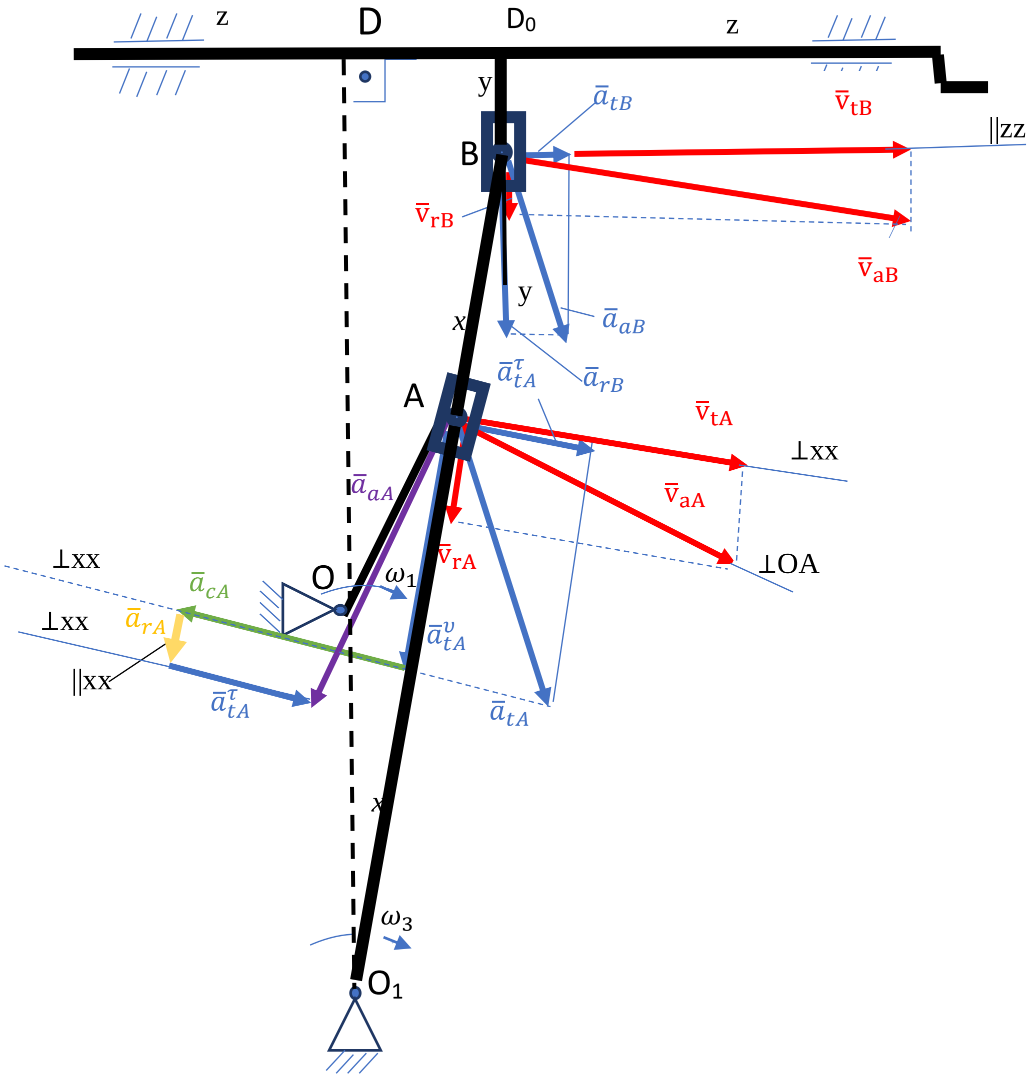


Figure 2

Point B

The absolute trajectory (C_{aB}) is a circle, centered at O_1, with the radius O_1B. The velocities distribution on bar O_1B is proportional to the distance from the considered point to O_1, so the absolute velocity of the point B is proportional to the transport velocity $\bar{v}_{tA}$.

The relative trajectory (C_{rB}) is on the guidance *yy* direction ($\bar{v}_{rB} \| yy$), and **the transport trajectory (C_{tB}) of point B** is parallel to the guidance *zz* ($\bar{v}_{tB} \| zz$). To obtain the relative ($\bar{v}_{rB}$) and the transport ($\bar{v}_{tB}$) velocities, the absolute velocity of point B ($\bar{v}_{aB}$) is decomposed along two directions, one parallel to *yy* and one parallel to *zz* direction.

Hence, the velocities distribution for point B, is:

$$\bar{v}_{aB} = \bar{v}_{rB} + \bar{v}_{tB} \tag{5}$$

b) **Distribution of accelerations for points A, B**

For the point A the acceleration distribution law is:

$$\bar{a}_{aA} = \bar{a}_{rA} + \bar{a}_{tA} + \bar{a}_{CA} \tag{6}$$

where: $$\bar{a}_{aA} = \bar{a}_{aA}^{\tau} + \bar{a}_{aA}^{\upsilon} \tag{7}$$

But, ω = ct., ε = 0, and $\bar{a}_{aA}^{\tau} = 0$ and $\bar{a}_{aA} = \bar{a}_{aA}^{\upsilon} = \omega_1^2 \cdot \overline{OA}$ (8)

Hence, the modulus of $\bar{a}_{aA}$ has the same expression with (8), the direction parallel to OA and the sense from A to O. On the mechanism sketch, is represented the acceleration of point A equal with the drawn length of the crank, respectively: $a_{A_{drawn}} = O_1A_{drawn}$. Hence, there is introduced the scale coefficient of accelerations:

$$k_a = \frac{a_{real}}{a_{drawn}} = \frac{\omega^2 \cdot O_1A}{O_1A_{drawn}} = \omega^2 \cdot k_L .$$

The relative trajectory of point A is on the guidance xx, therefore $\bar{a}_{rA}$ is parallel with *xx* direction, and the transport acceleration is.

$$\bar{a}_{tA} = \bar{a}_{tA}^{\tau} + \bar{a}_{tA}^{\upsilon} \tag{9}$$

The normal component of acceleration is: $a_{aA}^{\upsilon} = \frac{v_{tA}^2}{O_1A}$, (10)

the direction is parall with *xx* (O_1A), and the sense from A to O_1.

The tangential component is perpendiculat to *xx*.

Note: All the normal (perpendicular) components of the accelarations can be determined by using of *Rectangular Triangle Method*.

To establish the Coriolis component of acceleration, on a_{tA}^{υ} is scketched a similar triangle with the velocities triangle for point A (see expression (3)), having $\mathrm{v_{tA}}$ as homologous side.

There is noted by u, the homologous side of $\mathrm{v_{rA}}$. In keeping with the similarity of triangles of velocities and accelerations, from one of the similarity ratios, will result:

$$\frac{\mathrm{u}}{\mathrm{v_{rA}}} = \frac{a_{tA}^{\upsilon}}{\mathrm{v_{tA}}} = \frac{\omega_t^2 \cdot O_1A}{\omega_t^2 \cdot O_1A} = \frac{\mathrm{v_{rA}}}{a_{tA}^{\tau}} = \omega_t \tag{11}$$

Hence: $$\mathrm{u} = \omega_t \cdot \mathrm{v_{rA}} = \frac{1}{2} a_{cA} \rightarrow a_{CA} = 2u = 2\omega_t \cdot \mathrm{v_{rA}} \tag{12}$$

The sense of $\bar{a}_{CA}$ is given by: $$\bar{a}_{CA} = 2\bar{\omega}_t \times \bar{\mathrm{v}}_{\mathrm{rA}} \tag{13}$$

where, $$\omega_{tA} = \frac{\mathrm{v_{tA}}}{O_1A} \tag{14}$$

It has the meaning given by $\bar{\mathrm{v}}_{\mathrm{tA}}$ and is perpendicular to the plane of the figure, $\bar{a}_{cA}$ is perpendicular to the plane determined by the vectors $(\bar{\omega}_t \times \bar{\mathrm{v}}_{\mathrm{rA}})$ by rotating $\bar{\omega}_t$ over $\bar{\mathrm{v}}_{\mathrm{rA}}$.

In this way, $$\bar{a}_{aA} = \bar{a}_{rA} + \bar{a}_{tA}{}^{\tau} + \bar{a}_{tA}{}^{\upsilon} + \bar{a}_{CA} \tag{15}$$

Graphic, the expression(15) is a polygon of forces (see Figure 2) where are known: the sides $\bar{a}_{tA}{}^{\upsilon}$, $\bar{a}_{CA}$ and only the directios for $\bar{a}_{rA}$ and $\bar{a}_{tA}{}^{\tau}$. The polygon is obtained if to $\bar{a}_{tA}{}^{\upsilon}$, is added $\bar{a}_{CA}$ through which extremity is drawn a paralel axis to xx direction, and to the extremity of $\bar{a}_{aA}$ a perpendicula axis to xx direction. The two segments are closing the polygon allowing the establishment of the sizes for the known edges only as direction $\bar{a}_{rA}$ and $\bar{a}_{tA}{}^{\tau}$. It can be composed $\bar{a}_{tA}{}^{\tau}$ and $\bar{a}_{tA}{}^{\upsilon}$, thus resulting $\bar{a}_{tA}$.

For the point B the acceleration distribution law is:

$$\bar{a}_{aB} = \bar{a}_{rB} + \bar{a}_{tB} + \bar{a}_{cB} \tag{16}$$

For the distribution of accelerations along the bar O_1B, the theorem of similarity between the absolute acceleration $\bar{a}_{aB}$ and the transport acceleration of $\bar{a}_{tA}$ of points B and A. Because $\omega_{tB} = 0$, $\bar{a}_{cB} = 2\bar{\omega}_{tB} \times \bar{v}_{rB} = 0$, the absolute acceleration of point B ($\bar{a}_{aB}$) have only two components: $\bar{a}_{rB}$ parallel to yy direction and $\bar{a}_{tB}$ parallel to zz. Hence:

$$\bar{a}_{aB} = \bar{a}_{rB} + \bar{a}_{tB} \tag{17}$$

DYNAMICS

DETERMINATION OF GRAVITATIONAL ACCELERATION THROUGH THE SIMPLE PENDULUM METHOD

1. The purpose of the work

It will be shown that the period of oscillations of a mathematical/simple pendulum depends only on its length, in the case of small oscillations ($\alpha <= 5°$), and in the case of large oscillations ($\alpha > 5°$) on its amplitude, both cases not being influenced by pendulum mass. The value of the gravitational acceleration will also be determined.

2. Theoretical considerations

The ***mathematical pendulum*** or ***simple pendulum*** consists of a material point with mass (*m*), which is suspended by an ideal wire and allowed to oscillate in a vertical plane under the action of its own weight (see Figure 1).

By measuring the period of oscillation of a pendulum, the value of the gravitational acceleration can be determined.

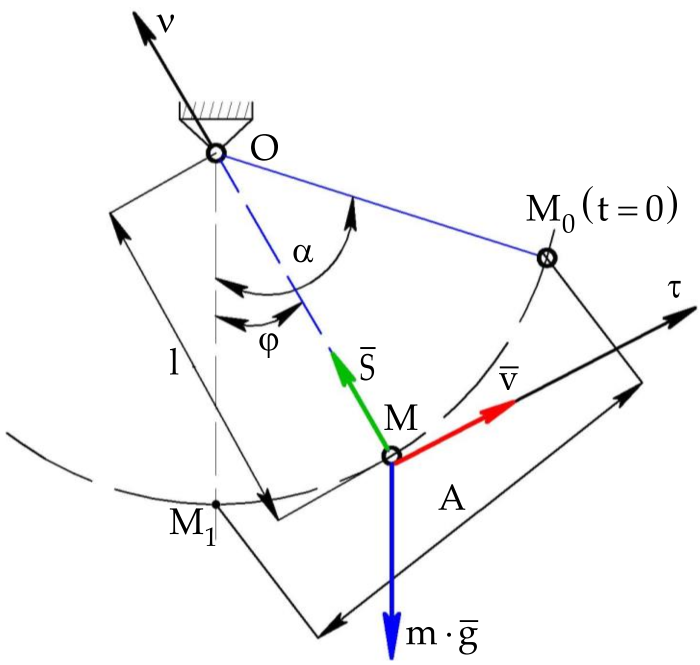


Figure 1

There is considered in Figure 1 a mathematical pendulum of mass (m) and length (l). The angle of rotation of the pendulum on the vertical axis is denoted by φ, considering that the pendulum starts from the angle α without initial velocity.

It is desired to obtain the law of motion of the simple pendulum, as well as to determine the period (T) of its oscillation. The forces acting on the material point M are the gravitational force (G=mg), and the tension in the wire (S). There is considered the *Newton's second law*, which is projected onto the axes of an intrinsic system as:

$$m \cdot \bar{a} = m \cdot \bar{g} + \bar{S} \tag{1}$$

The scalar differential equations are:

$$\begin{cases} ma_\tau \ \ = ml\ddot{\varphi} = -mgsin\varphi \\ ma_\nu = ml\dot{\varphi}^2 = S - mgcos\varphi \end{cases} \tag{2}$$

The first equation from (2), in the case of ***small oscillations*** (α<=5°), when sin φ = φ and cos φ = 1, becomes:

$$\ddot{\varphi} + \frac{g}{l}\varphi = 0 \tag{3}$$

Relation (3), the differential equation of motion of the simple mathematical pendulum, is a homogeneous differential equation of second degree, from which, by integration, the law of motion is found as function of the angle φ, as follows:

$$\varphi(t) = C_1cos(pt) + C_2sin(pt) \tag{4}$$

where in the relation (3), there is noted $p = \sqrt{\frac{g}{l}}$.

From the initial conditions of the movement (at moment t = 0 $\varphi = \alpha$ and $\dot{\varphi} = 0$) the constants of integration are:

$$C_1 = \alpha, \ C_2 = 0 \tag{5}$$

Relations (5) replaced in (4) will give the law of motion of the pendulum in the case of ***small oscillations***: $\varphi = \alpha cos(pt)$ (6)

where α is angular amplitude, and p the pulsation.

The period of oscillation of a simple pendulum at small oscillations is given by relation:

$$T = \frac{2\pi}{p} = 2\pi\sqrt{\frac{l}{g}} \tag{7}$$

From relation (7) it can be noted that period (T) varies linearly with the radical of the length of the pendulum (l) and does not depend on its mass.

If it is considered the case of ***large oscillations*** (α>5°) in accordance with the specialized literature the period is:

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}sin^2\left(\frac{\alpha}{2}\right)\right) \tag{8}$$

Noting with A the distance (A = M_0M_1) and $A = 2l \cdot \sin\frac{\alpha}{2}$ (see Figure 1) relation (8) becomes:

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{16}\frac{A^2}{l^2}\right) \tag{9}$$

From relations (7) and (9) the gravitational accelerations g, in case of small and large oscillations, can be determined as:

$$g^m = \frac{4\pi^2 l}{T^2}; \qquad g^M = \frac{4\pi^2 l}{T^2}\left(1 + \frac{1}{16}\frac{A^2}{l^2}\right)^2 \tag{10}$$

3. Development of the laboratory work

There are mounted the pieces of the equipment on the frontal panel provided. According to **Table 1**, for different lengths of the wire (between 0.16-0.64 [m]), there will be determined the time for ten oscillations, making three measurements. After that will be established the time (**T_{medium}**) for one oscillation, the obtained values being registered in **Table 1**.

Table 1

No.	Length of the pendulum (l) [m]	$\sqrt{l}$	10 T [s]	T_{medium} [s]	α [^{0}]	g^* [m/s^2]	$\varepsilon_r = \frac{\lvert g^{Cluj} - g^* \rvert}{g^{Cluj}} \cdot 100$ [%]
1	0.16	0.4					
2	0.25	0.5					
3	0.36	0.6					
4	0.49	0.7					
5	0.64	0.8				-	

Note: We can measure the angle α (see the sixth column of the Table 1) using a protractor.

The results are used to represent the period variation referred to length of pendulum wire.(see Figure 2)

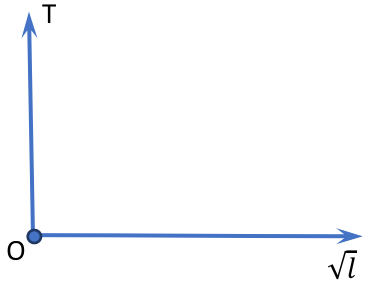


Figure 2

There are used relations (10) to calculate gravitational acceleration g^* in case of small/large oscillations.

Knowing the values of gravitational acceleration at pole and equator:

$$g^{pole} = 9.831 \text{ m/s}^2 \text{ and } g^{equator} = 9.781 \text{ m/s}^2$$

the geographical variation according with a degree of latitude can be determined as:

$$\Delta g_l = \frac{g^{pole} - g^{equator}}{90^\circ} = \frac{5}{9} \cdot 10^{-3} \quad m/s^2 grade$$

and knowing the northern latitude of Cluj-Napoca city $L^{CJ} = 46.77° \text{ N}$, results the value of gravitational acceleration in the town:

$$g^{Cluj} = g^{equator} + \Delta g_1 \cdot L^{CJ} = 9.781 + \frac{5}{9} \cdot 10^{-3} \cdot 46.77 = 9.807 \ m/s^2$$

with which can be calculated the relative error ε_r.

HIGHLIGHTING THE CORIOLIS INERTIAL FORCE

1. The purpose of the paper

The purpose of the paper is to highlight the Coriolis inertial force, by measuring the deformation (deflection/arrow) of an elastic bar, under the action of a force.

2 Experimental Stand Description

In Figure 1, is presented the scheme of the system used to determine the deformation of an elastic bar.

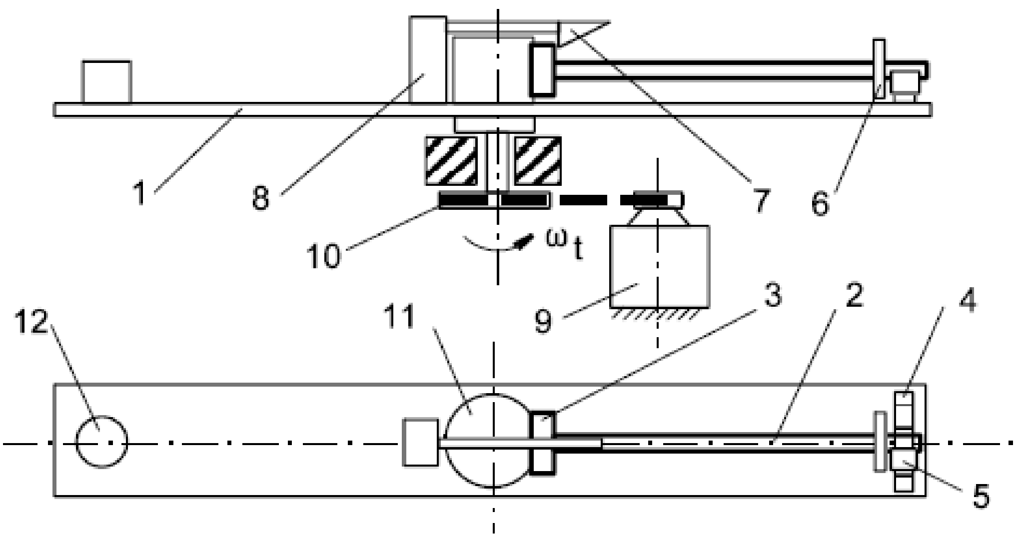


Figure 1

The components of the device are:

1 – plate; 2 – elastic bar; 3 – mobile body; 4 – graduated ruler; 5 – cursor;

6 – stopper; 7 – hindrance; 8 – electromagnet; 9 – electric motor;

10 – belt transmission; 11 – hub; 12 – counterweight.

The electric motor drives, through the transmission belt, the plate in the rotational motion with the angular velocity $\omega_t = \omega = cst.$ The elastic bar embedded in the hub rotates with the plate. The mobile body will move along the bar, from the initial position given by the hindrance put by the electromagnet, to the final position given by the stopper.

2. Theoretical considerations

The body (assimilated with a material point) moves with respect to the bar which is also in motion. The movement of the material point related to a mobile reference system (the bar) is called *relative motion*, and its movement in relation to a fixed reference system is called a*bsolute motion*. If is suppressed the relative motion (fix the body on the bar), the motion that remains is called *transport motion*. Each of the three motions have trajectory, velocity and acceleration corresponding to the motion. For the studied case, the relative motion is rectilinear, having the trajectory (Γr) being along the elastic bar. The transport motion is a uniform circular movement, with constant angular velocity, the trajectory being a circle (Γt).

In the relative motion of the material point, the dynamic equation has the form:

$$m \cdot \bar{a}_r = \bar{R} + \bar{R}_l + \bar{F}_{jt} + \bar{F}_{jc} \tag{1}$$

where $\bar{R}$ represents the resultant of active forces in the system, $\bar{R}_l$ represents the resultant of linkage forces, $\bar{F}_{jt}$ is the transport inertial force, and $\bar{F}_{jc}$ is the Coriolis inertial force. The two inertial forces have directions opposite to their corresponding accelerations.

According to Figure 2, there are considered two reference frames, $O_1x_1y_1z_1$-fixed, and $Oxyz$-mobile. The dynamic equation in relative motion (1), will be projected on the axes of the mobile system $Oxyz$.

The mobile system has the Ox axis along the bar, the axis Oy normal to the bar, and the axis Oz is identical with Oz_1 (axis of rotation) (see Figure 2).

There is known that in relative motion, are appearing the relative and transport trajectories. First there are established the relative trajectory (Γ_r a rectilinear straight line), and the transport trajectory (Γ_t a circle with radius x) as in Figure 2.

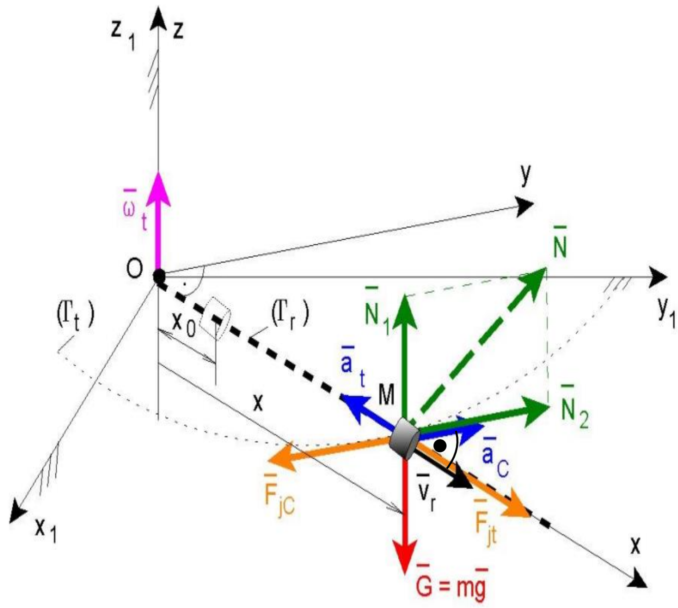


Figure 2 *(Spatial view)*

Further, there will be established the forces direction on the body, in relative movement, as presented in Figure 2:

- The **active force** is the gravitational force $\bar{G}$, opposite to O_z axis;
- The **linkage forces**, $\bar{N}_1$ (opposite to $\bar{G}$), and $\bar{N}_2$ parallel to O_y axis;
- The **transport inertial force** is defined as:

$$\bar{F}_{jt} = -m\bar{a}_t. \tag{3}$$

From the expression (3), there is observed that the transport force, and transport acceleration have opposite directions. The transport acceleration, in keeping with the fact that the angular velocity is constant, has only the normal component, oriented on the diameter of the trajectory Γ_t, established as:

$$\bar{a}_t = -\omega_t^2 \cdot \overline{OM}; \qquad a_t = \omega^2 \cdot OM = \omega^2 \cdot x \tag{4}$$

Hence, considering (3), the transport force is oriented along O_x axis.

- The **Coriolis inertial force** appears in the relative motion of the material point, due to the presence of the Coriolis acceleration. The Coriolis inertial force is defined:

$$\bar{F}_{jc} = -m\bar{a}_c = -2m\bar{\omega}_t \times \bar{v}_r \tag{5}$$

where:

m – mass of the material point [kg]

$\bar{a}_c$ - Coriolis acceleration [m/s²]

According to (5), there is observed that Coriolis inertial force, and correspondent acceleration are opposite. The Coriolis acceleration , determined with:

$$\bar{a}_c = 2\bar{\omega}_t \times \bar{v}_r \tag{5}$$

where:

$\bar{\omega}_t$ - angular transport velocity [rad/s]

$\bar{v}_r$ - relative velocity [m/s]

In keeping with the fact that the relative velocity $\bar{v}_r$ is oriented along O_x axis; there is established with (5) the orientation of Coriolis acceleration, parallel to O_y axis. According to (4), the Coriolis inertial force is oriented opposite to O_y axis direction.

Based on previous considerations, expression (1) becomes:

$$m \cdot \bar{\bar{a}}_r = \bar{G} + \bar{N}_1 + \bar{N}_2 + \bar{F}_{jt} + \bar{F}_{jc} \tag{5}$$

In keeping with the fact that the relative motion is along O_x axis, previous expression is projected on the axes of the mobile reference system *Oxyz* :

$$\begin{cases} m\ddot{x} = F_{jt} \\ 0 = N_2 - F_{jc} \\ 0 = N_1 - G \end{cases} \rightarrow \begin{cases} m\ddot{x} = m\omega^2 x \\ 0 = N_2 - 2m\omega\dot{x} \\ 0 = N_1 - mg \end{cases} \tag{6}$$

First equation from system (6) will be written as:

$$\ddot{x} - \omega^2 x = 0 \tag{7}$$

and its solution, considering the initial conditions of motion at time t=0,

$$x(0)=x0 \text{ and } \dot{x}(0) = v_0 = 0 \tag{8}$$

it will be exactly the law of relative motion:

$$x(t) = x_0 \mathrm{ch}(\omega\, t) \tag{9}$$

The relative velocity will be:

$$\dot{x} = \omega x_0 \mathrm{sh}(\omega\, t) = \omega\sqrt{x^2 - x_0^2} \tag{10}$$

The horizontal component of the normal reaction N_2 from second expression of the system (6) becomes:

$$N_2 = 2m\omega\dot{x} = 2m\omega^2\sqrt{x^2 - x_0^2} \tag{11}$$

The component that bends the bar in a horizontal plane determines a deflection, equal to:

$$f^{theor} = \frac{N_2 l_1^2\left(\frac{l_1}{3}+\frac{l_2}{2}\right)}{EI_z} \tag{13}$$

where: l_1, and l_2- are presented in Figure 3 ;

E – longitudinal modulus of elasticity [N/m²];

I_z – geometric inertia moment (for the bar $I_z = \frac{\pi \cdot d^4}{64}$) ;

$\omega = \frac{\pi n}{30}$ angular velocity (*rad/s*).

In keeping with previous consideration, the deflection is:

$$f^{theor} = \frac{16\pi}{675}\frac{n^2 m l_1^2\sqrt{x^2-x_0^2}(2l_1+3l_2)}{Ed^4} \tag{14}$$

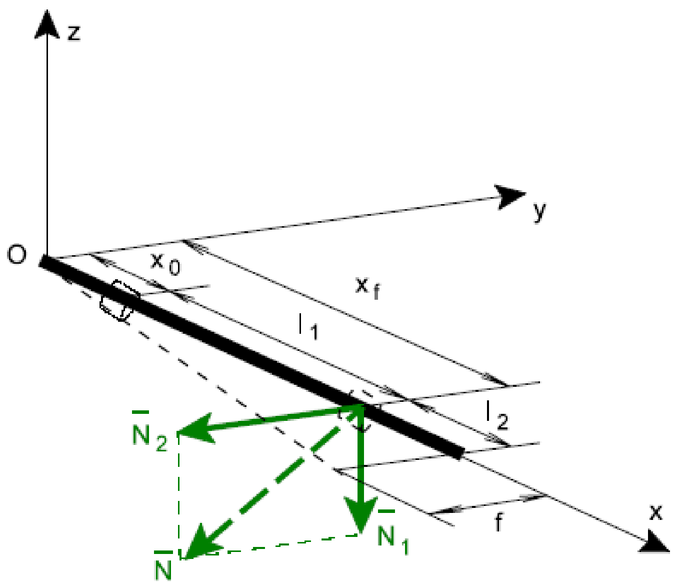


Figure 3

3. Development of the laboratory work

1. The body is positioned at x_0, initially stopped by the hindrance;

2. The cursor on the ruler is brought to the zero position, so that it touches the elastic bar;

3. The motor is started and is waiting for the velocity to stabilize;

4. The electromagnet is acted so it releases the body to move on the bar;

5. The motor is stopped and waits for the device to stop.

6. It is read on the ruler the displacement of the cursor (mm), which indicates the deformation of the elastic bar (arrow) under the action of the Coriolis force. Five values of these displacements will be measured, finally their arithmetic will be used, this being the experimental established deformation, which is compared with the theoretical one given by relation (14). The relative error is calculated by:

$$\varepsilon_r = \frac{\lfloor f^{theor} - f^{exp} \rfloor}{f^{theor}} \cdot 100[\%] \qquad (15)$$

The input data for theoretical establishing of deformation are:

$m = 0.27$ [kg] , $d = 0.011$ [m], $n = 350$ [rot/min], $l_1 = 0.315$ [m], $l_2 = 0.04$ [m], $x = 0.38$ [m], $x_0 = 0.065$ [m], $E = 2.1 \cdot 10^{11}$ [N/m^2].

All measurements are filled in the **Table 1** as below:

Table 1

No.	f^{theor} (mm)	f^{exp}(mm)	f_{med} (mm)	ε_r (%)
1				
2				
3				
4				
5				

DETERMINATION OF MECHANICAL MOMENTS OF INERTIA USING THE PHYSICAL PENDULUM METHOD

1. The purpose of the work

There will be applied the physical pendulum method to determine the axial mechanical moments of inertia in two situations:

a. For a body where the position of the center of gravity is known;

b. For a body where the position of the center of gravity is not known but admits an axis of symmetry.

2. Theoretical considerations

A ***physical pendulum*** consists of a body with mass M, suspended in a point O, taken out of equilibrium, and allowed to oscillate freely. As can be seen in Figure 1, from mechanical point of view, the movement of the physical pendulum is a rotational movement around the axis (Δ).

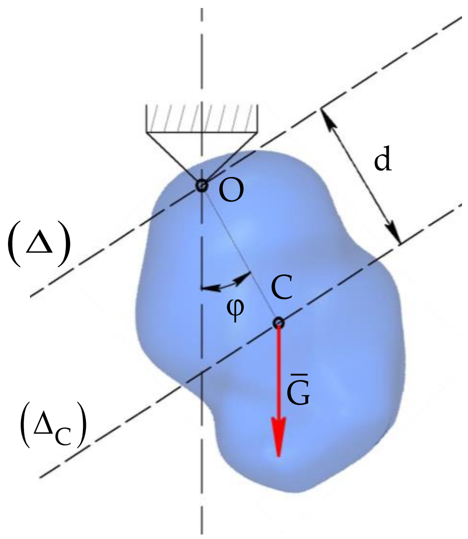


Figure 1

The axial mechanical moment of inertia (J_Δ) is a scalar that characterizes a body in rotational motion about an axis and is equal to sum the product of the elementary masses and their distances to the axis of rotation.

In Figure 1, there is considered a rigid body, having the mass m, being in rotational movement around axis (Δ). There must be determined, the value of inertia moment for the body, with respect to (Δ_C), which passes through the mass center.

There will be applied the kinetic moment theorem, applied in the case of rotation around a fixed axis for a rigid body:

$$\dot{K}_{\Delta} = M_{\Delta} \tag{1}$$

The angular momentum for the rotation around (Δ) axis is :

$$K_{\Delta} = J_{\Delta} \cdot \omega \tag{2}$$

There is known that ω is the angular velocity [rad/s] and ε the angular acceleration [rad/s²]. The correspondence between the angular displacement *φ*, and the above-mentioned cinematic parameters is:

$$\ddot{\varphi} = \dot{\omega} = \varepsilon \tag{3}$$

Applying the time derivative on (2), considering (3), the left member from (1) is calculated as:

$$\dot{K}_{\Delta} = J_{\Delta} \cdot \ddot{\varphi} \tag{4}$$

The moment of external forces is given by the moment of gravitational force, as:

$$M_{\Delta} = -m \cdot g \cdot d \cdot \sin\varphi \tag{5}$$

Substituting relations (4) and (5) in (1), passing all the terms in the left member, is obtained the equation:

$$J_{\Delta} \cdot \ddot{\varphi} + M \cdot g \cdot d \cdot \sin\varphi = 0 \tag{6}$$

representing the ***differential equation of the movement of the physical pendulum***.

There is considered that the pendulum performs small oscillations, $(\varphi \leq 5°)$, hence the following approximations can be made:

$$\sin\varphi \cong \varphi \text{ and } \cos\varphi \cong 1. \tag{7}$$

Based on (7) substituted in (6) is obtained a second order differential equation, homogeneous, which describes the small oscillations of the physical pendulum:

$$J_{\Delta} \cdot \ddot{\varphi} + M \cdot g \cdot d \cdot \varphi = 0 \tag{8}$$

There is introduced the notation:

$$\frac{M \cdot g \cdot d}{J_{\Delta}} = \omega^2 \tag{9}$$

and equation (8) takes the form: $\ddot{\varphi} + \omega^2 \cdot \varphi = 0$ (10)

Its solution or the motion law of the physical pendulum is a harmonic function:

$$\varphi = \alpha \cdot \cos(\omega t) \tag{11}$$

where α is the amplitude of motion.

Note: Expression (11) highlights the fact that the physical pendulum motion is an oscillation.

The period is determined as:
$$T = 2\pi \cdot \sqrt{\frac{M \cdot g \cdot d}{J_\Delta}} \tag{12}$$

The mathematical pendulum (see Figure 2)

A punctiform body (material point) of mass *m*, suspended in a point O by means of a flexible and inextensible wire of length l, taken out of equilibrium and allowed to oscillate freely, forms a *mathematical pendulum*.

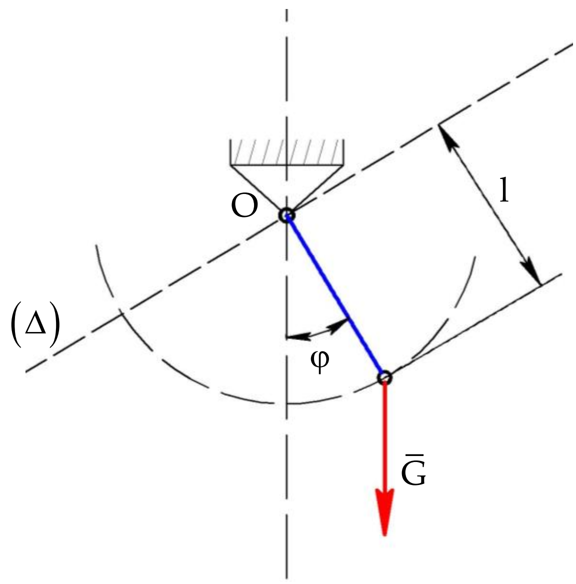


Figure 2

The movement of the mathematical pendulum from Figure 2, is also a rotation motion around the axis (Δ), so can be applied the **kinetic momentum theorem**, with respect to (Δ) axis described by expression (1). The kinetic momentum of the pendulum is:
$$K_\Delta = J_\Delta \cdot \omega = m \cdot l^2 \cdot \omega \tag{13}$$

The time derivative of (13), in keeping with (3) is:

$$\dot{K}_\Delta = m \cdot l^2 \cdot \ddot{\varphi} \tag{14}$$

The moment of external forces is given by the moment of gravitational force, as:

$$M_{\Delta} = -m \cdot g \cdot l \cdot \sin\varphi \tag{15}$$

There is considered that the pendulum is performing small oscillations, hence according to (7), there results:

$$M_{\Delta} = -m \cdot g \cdot l \cdot \varphi \tag{16}$$

Replacing relations (16), (14) in (1), the differential equation of small oscillations of the mathematical pendulum takes the form:

$$l \cdot \ddot{\varphi} + g \cdot \varphi = 0 \tag{17}$$

With the notation: $$\omega^2 = \frac{g}{l}, \tag{18}$$

the equation (17) becomes: $$\ddot{\varphi} + \frac{g}{l} \cdot \varphi = 0 \tag{19}$$

Its solution or the law of motion of the physical pendulum is a harmonic function, identical with(11), having the period: $$T = 2\pi\sqrt{\frac{l}{g}} \tag{20}$$

3. Development of the laboratory work

a. For bodies where the position of the center of gravity is known (regular shaped bodies)

From relation (12) it can be observed that if the position of the center of gravity (distance d) is known and the oscillation period of the physical pendulum T_f is timed, the axial moment of inertia J_{Δ} can be determined.

$$J_{\Delta} = M \cdot g \cdot d \cdot \frac{T_f^2}{4\pi^2} \tag{21}$$

Applying Steiner's theorem results: $$J_{\Delta} = J_{\Delta c} + M \cdot d^2 \tag{22}$$

The experimental value of the axial mechanical moment of inertia in relation to the axis Δ passing through the center of gravity of the disk (C) can be determined as:

$$J_{\Delta c}^{\exp} = J_{\Delta} - M \cdot d^2 = M \cdot g \cdot d \cdot \frac{T_f^2}{4\pi^2} - M \cdot d \tag{23}$$

For a disc of mass M, and diameter D (or radius R), the mechanical moment of axial inertia is:

$$J_{\Delta c}^{\text{theor}} = \frac{M \cdot R^2}{2} = \frac{M \cdot D^2}{8} \quad (24)$$

Having two values for the inertia moment, there will be determined the relative error according to:

$$\varepsilon_r = \frac{\left| J_{\Delta c}^{\text{theor}} - J_{\Delta c}^{\exp} \right|}{J_{\Delta c}^{\text{theor}}} \cdot 100 \quad [\%] \quad (25)$$

The disc will be sited, as presented in Figure 3. There will be timed the period for 10 oscillations of the physical pendulum to reduce the error due to human reaction time (one tenth of a second for each actuation of the timer).

Figure 3

There is established the average period for one oscillation T_{med}, then **Table 1** will be completed with the data and results obtained with (21), (23), (24), (25).

- The characteristics for the discs from laboratory are:
  - $M_{disc} = 6.7$ [kg], $d = 0.138$ [m], $R = 0.185$ [m];
  - $M_{disc} = 1.14$ [kg], $d = 0.094$ [m], $R = 0.124$ [m].

Table 1

No.	g [m/s²]	M_{disk} [kg]	d [m]	T [s] 10 T	T [s] T_{med}	J_Δ [$kg\ m^2$]	$J_{\Delta c}^{exp}$ [$kg\ m^2$]	$J_{\Delta c}^{theor}$ [$kg\ m^2$]	ε_r [%]
1									
2									
3									
4									
5									

b. The bodies whose center of gravity is not known, but admitting an axis of symmetry, will be suspended in two opposite points (see Figure 4), located on the axis of symmetry. Thus are created two physical pendulums, whose periods T_1 and T_2 are timed.

Figure 4

From the condition of synchronism (same period) of a physical pendulum with a mathematical one ($T_f = T_m$) based on relation (20), the lengths l_1 and l_2 of the mathematical pendulums synchronous with the physical ones are calculated as:

$$l_1 = \frac{g \cdot T_{1med}^2}{4 \cdot \pi^2}; \quad l_2 = \frac{g \cdot T_{2med}^2}{4 \cdot \pi^2} \tag{26}$$

Equating relations (12) and (20) yields:

$$J_\Delta = M \cdot d \cdot l \tag{27}$$

Relation (27) is replaced in (22) successively for the two positions of the physical pendulum, obtaining a system of three equations with the unknowns d_1, d_2 , and $J_{\Delta C}$ (see Figure 5), as follows:

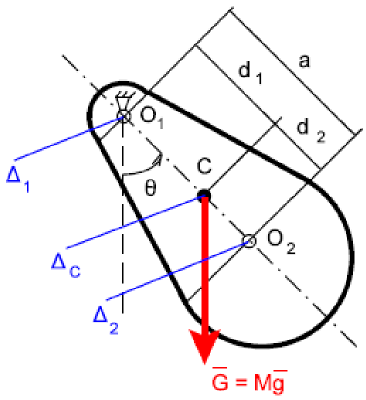


Figure 5

$$\begin{cases} M \cdot d_1 \cdot l_1 = J_{\Delta c} + M \cdot d_1^2 \\ M \cdot d_2 \cdot l_2 = J_{\Delta c} + M \cdot d_2^2 \\ \boldsymbol{d_1 + d_2 = a} \end{cases} \tag{28}$$

Solving the system (28) it results for the considered complex shaped body the following expression for calculation of the axial mechanical moment of inertia:

$$J_{\Delta c} = \frac{Ma(a-l_1)(a-l_2)(l_1+l_2-a)}{(2a-l_1-l_2)^2} \tag{29}$$

There will be timed 10 oscillation periods of the physical pendulum, for each of the five determinations in each position of the body (see Figure 4). There will be calculated the average periods, T_{1med} and T_{2med}. With (26) are determined the lengths l_1 and l_2 , which are included in (29), and conducts to the axial mechanical moment of inertia. All the obtained results are filled in **Table 2.**

Table 2

<table>
<tr><th rowspan="2">No.</th><th rowspan="2">g
[m/s²]</th><th rowspan="2">M
-[kg]</th><th rowspan="2">a
[m]</th><th colspan="3">T [s]</th><th rowspan="2">l_1
[m]</th><th rowspan="2">l_2
[m]</th><th rowspan="2">$J_{\Delta c}^{exp}$
$[kg\ m^2]$</th></tr>
<tr><th>10 T</th><th>T_{1med}</th><th>T_{2med}</th></tr>
<tr><td>1</td><td rowspan="10"></td><td rowspan="10"></td><td rowspan="10"></td><td></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="10"></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>6</td><td></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td></tr>
<tr><td>7</td><td></td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>9</td><td></td></tr>
<tr><td>10</td><td></td></tr>
</table>

Note: The characteristics for the connecting rods from laboratory are:

- $M = 2.9\ [kg],\ \ a = 0.323\ [m];$
- $M = 0.68\ [kg],\ \ a = 0.182\ [m].$

DETERMINATION OF MOMENTS OF MECHANICAL AXIAL INERTIA OF BODIES USING THE ROTATIONAL MOVEMENT

1. The purpose of the work

The aim of the work is to determine the moments of mechanical axial inertia for three types of objects (circular disk, tube and rod with two weights), using the rotational motion around a vertical axis as presented in Figures 1, 2, 3.

Figure 1

Figure 2

Figure 3

2. Experimental Stand Description

The experimental stand is presented in Figure 4, and the mechanical sketch in Figure 5.

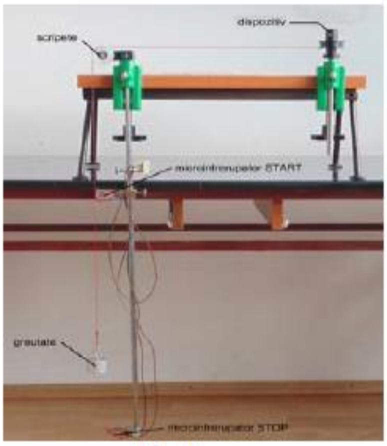


Figure 4

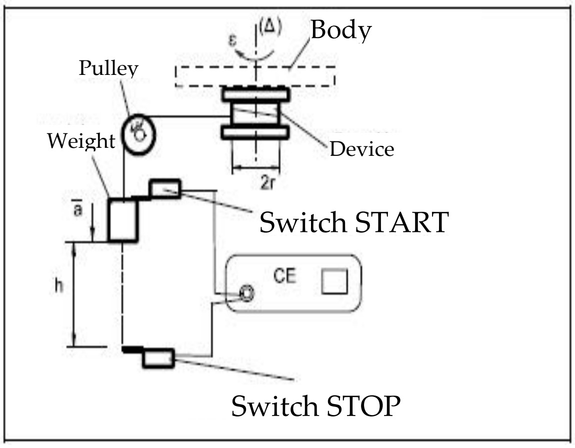


Figure 5

According to Figure 5, the body of which inertial moment must be determined, is fixed on the device, centrated with axis Δ. The object, and the device`s plate will rotate around the axis Δ with the angular acceleration ε. On the reel of the device, is wired a cable, which has at the other end attached a known weight. The cable is passed through a pulley, which is changing the wire direction.

3. Theoretical considerations

Since it is a matter of a material system in motion under the action of some forces, one of the principles of **Analytical Mechanics** can be applied, namely the **D'Alembert's principle**, which is stated as follows: *The mechanical system is in fictitious dynamic equilibrium under the action of the given (active) forces, linkage forces and of inertia forces.*

In case of a *material point*, the inertia force has an opposite sense to the acceleration, and D'Alembert's principle can be expressed by:

$$\bar{R} + \bar{R}_l + \bar{F}_j = 0 \tag{1}$$

In case of a *rigid body* in general motion, each elementary mass has acceleration, the elementary inertia force having an opposite sense. Reducing the system of elementary inertia forces in the center of gravity C, the *force - couple system* of the inertia forces (R_j, M_{jc}) is obtained, and D'Alembert's principle is expressed by two vectorial equations as:

$$\bar{R} + \bar{R}_l + \bar{F}_j = 0 \qquad \bar{M}_c + \bar{M}_{lc} + \bar{M}_{jc} = 0 \tag{2}$$

Note:

1. in translational motion – the force - couple system of inertia forces is reduced to a single resultant R_j;
2. in rotational motion around the fixed axis that passes through C, the force - couple system of inertia forces is reduced to a single torque M_{jc}.

Appling the D'Alembert's principle to solve dynamic problems, there must be considered the ***kineto-static method*** (equilibrium equations are written as in statics to which kinematic relations are added).

For the considered case, the friction will be neglected, the pulley will be considered ideal (with the role to change the direction of the wire). The bodies will be separated and introduced the active forces **G** (the gravitational force of the body of mass m); the connecting forces **S** (the tension in the wire), **H** and **V** (the reactions in the cylindrical joint of the device, located in the horizontal plane) and the inertia force $\mathbf{F_j}$ (related to the translational movement of the mass) and torque $\mathbf{M_j}$ (related to the rotational movement of the device) (see Figure 6). The mass *m* descends with the acceleration *a*, and the device rotates with the angular acceleration ε.

The force and moment of inertia are:

$$\bar{F}_j = -m\bar{a} \qquad \bar{M}_j = -J_\Delta \bar{\varepsilon} \tag{3}$$

with scalar quantities: $F_j = ma; \qquad M_j = J_\Delta \varepsilon$ (4)

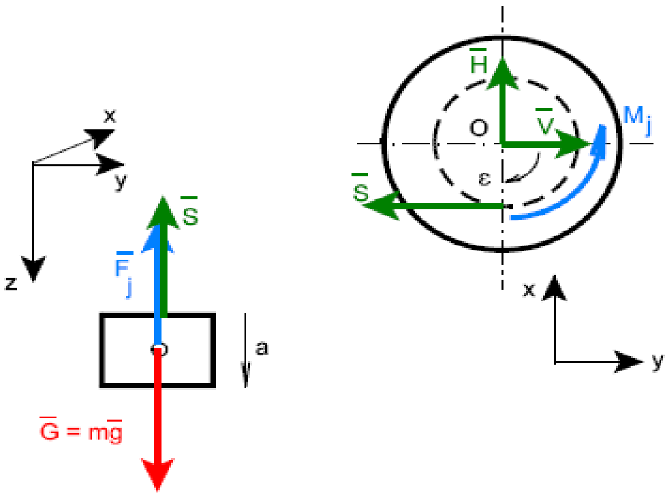


Figure 6

For the body of mass *(m)* situated at the end of wire (which performs translational motion) is written an equation of vertical force projection. For the device (which performs rotational motion), an equation of moment in relation to point O. To the motion expressions, there is added the kinematic relation between *a* and *ε* (due to the wire):

$$\begin{gathered} G - S - F_j = 0 \\ S\,r - M_j = 0 \\ a = \varepsilon\, r \end{gathered} \tag{5}$$

Considering the relations (4) and eliminating ε, system (5) takes the form:

$$\begin{gathered} mg - S - ma = 0 \\ S\,r - J_\Delta \frac{a}{r} = 0 \end{gathered} \tag{6}$$

From the second equation it results the tension in the wire: $S = \frac{J_\Delta}{r^2} a$ (7)

which, if replaced in the first equation, allows the calculation of the acceleration:

$$a = \frac{mgr^2}{J_\Delta + mr^2} \tag{8}$$

It can be seen that the acceleration is constant, so the weight descends *uniformly accelerated.*

The movement of the gravitational force $\bar{G}$ is a rectilinear vertical translational movement with constant acceleration a. If is denoted by z the displacement related to the starting position, the following kinematic relationships are valid:

$$v = \dot{z}; \qquad a = \dot{v} = \ddot{z} \tag{9}$$

the last representing a 2nd grade differential equation related to time, that can be integrated as follows:

$$\dot{v} = a = \frac{dv}{dt} \quad \rightarrow \quad dv = adt \quad \rightarrow v = at + C_1 \tag{10}$$

$$v = \dot{z} = a \cdot t + C_1 \ \rightarrow \ \frac{dz}{dt} = at + C_1 \ \rightarrow \ dz = (at + C_1)dt$$

$$z = \frac{a \cdot t^2}{2} + C_1 t + C_2 \tag{11}$$

Constants of integration C_1 and C_2 are determined from *the initial condition of motion* at moment t = 0, v = 0, a = 0 from which substituting in relations (7) and (8) results that:

$$C_1 = C_2 = 0 \tag{12}$$

Thereby, relations (9) and (11) becomes:

$$v = \dot{z} = a \cdot t\,; \qquad z(t) = \frac{a \cdot t^2}{2} \tag{13}$$

When the weight reaches the bottom, the condition $t = t_c$ and $z(t_c) = h$ are considered, as a result the acceleration is:

$$a = \frac{2h}{t_c^2} \tag{14}$$

By comparing relation (8) with (14) the axial mechanical moment of axial inertia can be determined:

$$J_\Delta = \frac{mgr^2 t_c^2}{2h} - mr^2 \tag{15}$$

4. Development of laboratory work

a. A blank measurement (without a body on the device) will be performed finding for the mass $m=m_1$, a fall time t_1, determining the mechanical moment of axial inertia of the device, as:

$$J_{\Delta 0} = \frac{m_1 g r^2 t_1^2}{2h} - m_1 r^2 \tag{16}$$

b. There is placed the body (disk) on the device plate and is changed the mass m = m_2. There will be measured the fall time t_2. The mechanical moment of axial inertia given by relation (16) is a sum of moments of inertia of the disk and the device:

$$J_{\Delta c} + J_{\Delta o} = \frac{m_2 g r^2 t_2^2}{2h} - m_2 r^2 \tag{17}$$

From (17) it results that the value of the experimental moment of mechanical axial inertia of the body:

$$J_{\Delta c}^{exp} = \frac{m_2 g r^2 t_2^2}{2h} - m_2 r^2 - J_{\Delta 0} \tag{18}$$

c. Repeat point b) for the tube and the rod with two bodies. A set of measurements is performed in each case, and the results are filled in **Table 1**. For the rod, step c) is repeated, considering three different distances (d) for each value of weight $M_2 = M_3$ (Figure 7), and the results are filled in **Table 2**.

d. It is known that a disk of mass M and radius R, or diameter D has the moment of inertia:

$$J_{\Delta c}^{theor} = \frac{MR^2}{2} = \frac{MD^2}{8} \tag{19}$$

e. For the body presented in Figure 2, that can be approximated with a tube of mass (*M*) and radius R, r, the moment of inertia in relation to the axis of symmetry is:

$$J_{\Delta c}^{theor} = \frac{M(R^2+r^2)}{2} = \frac{M(D^2+d^2)}{8} \tag{20}$$

f. The body from Figure 3 is considered composed of three parts (see Figure 7): bar/tube - 1 and two identical hollow cylinders 2 and 3. The moments of inertia of each part will be calculated in relation to its center of gravity and then with Steiner's theorem, the moments of inertia of these parts 2 and 3 are brought to the axis of rotation of the device; in the end all these will add up as follows:

$$J_{\Delta c}^{theor} = \frac{M_1}{12}\left(3r_1^2 + 3r_2^2 + L^2\right) + 2\left[\frac{M_2}{12}\left(3r_2^2 + 3r_3^2 + L_1^2\right) + M_2 d^2\right] \tag{21}$$

g. With relations (19), (20) and (21) the theoretical mechanical moments of inertia are determined, which will be filled in Table **1** and **Table** 2.

h. Finally, the relative error between the theoretical and the experimental axial mechanical moment of axial inertia can be calculated, for each individual experiment, as follows:

$$\varepsilon_r = \frac{\left|J_{\Delta c}^{theor} - J_{\Delta c}^{exp}\right|}{J_{\Delta c}^{theor}} \cdot 100\ [\%] \tag{22}$$

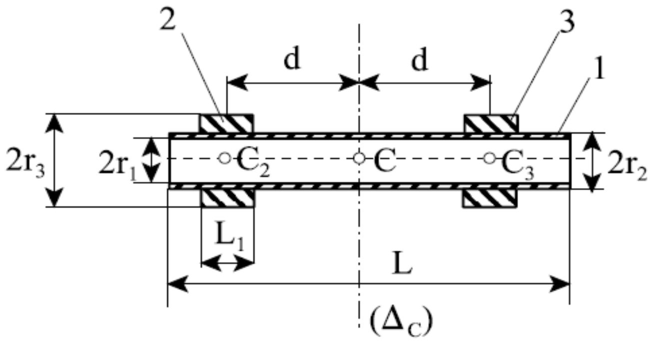


Figure 7

i. The input data for the three bodies are:

- **Body 1 (disk)** – M = 0.943 [kg], D = 0.12 [m];
- **Body 2 (tube)** – M = 0.9 [kg], D = 2R = 0.119 [m], d = 2r = 0.109 [m];
- **Body 3** (tubular bar with two masses)

  M_1 = 0.125 [kg], $M_2 = M_3$ = 0.1/0.2 /0.4

  L = 0.55 [m] , L_1 = 0.014/0.029/0.058 [m];

$d_1 = 2r_1$=0.007 [m], $d_2 = 2r_2$ = 0.01 [m], $d_3 = 2r_3$ =0.035 [m]

r = 0.02 [m], h = 0.4 [m].

Table 1

Body	**h**	**m_1**	**m_2**	**t_{fall} [s]**				$J_{\Delta O}$	$J_{\Delta c}^{exp}$	$J_{\Delta c}^{theor}$	**ε_r**
	[m]	**[kg]**	**[kg]**	**t_1**	**$t_1$med**	**t_2**	**$t_2$med**	**[N m]**	**[N m]**	**[N m]**	**[%]**
Disk											
Tube											

Table 2

Body	M_2	L_1	d	t_{fall} [s]		$J_{\Delta O}$	$J_{\Delta c}^{exp}$	$J_{\Delta c}^{theor}$	ε_r
	[kg]	[m]	[m]	t_2	t_{2med}	[kg m]	[kg m]	[kg m]	[%]
Tubular bar with two masses	0.1	0.014							
	0.2	0.029							
	0.4	0.058							

DETERMINATION OF THE COEFFICIENT OF DYNAMIC FRICTION ON THE INCLINED PLANE

1. The purpose of the paper

The aim of the work is to determine the coefficient of dynamic friction on the inclined plane. The method used consists in applying the ***Kinetic Energy Theorem*** in differential form to the mechanical system presented in the Figure 1, or Figure 2.

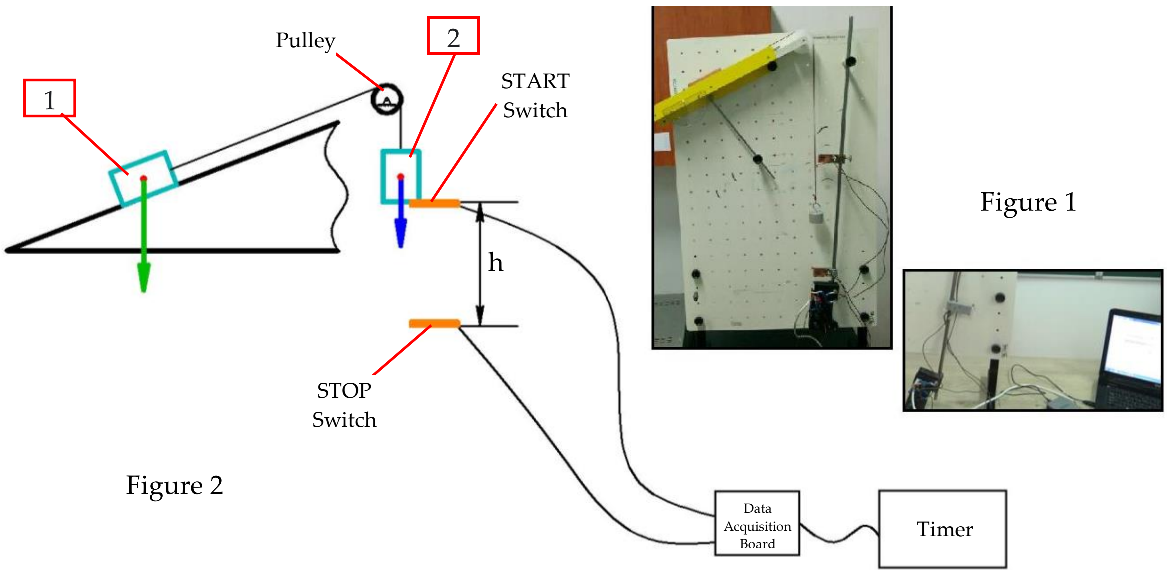


Figure 1

Figure 2

2. Experimental Stand Description

There is considered the laboratory stand presented in Figure 1, whose sketch is presented in Figure 2. The mechanical system is composed of two bodies, 1- situated on an inclined plane; 2 – is tied by 1, through a wire, passed over a pulley. Knowing that body 2, has a greater weight, it will vertically descend. At each of the two positions of the body 2, there are switches, which are connected to a data acquisition system, linked to a timer, which is registering the moving time on displacement h.

3. Theoretical considerations

For the study, according to the Figure 3, there is considered a rigid body $Q = m_1 \cdot g$, situated on a rough inclined plane at angle α. The rigid Q, is tied through an ideal wire, which is passed on a pulley. At the other end of the wire, there is tied another rigid body $P = m_2 \cdot g$. When body P descending with the acceleration a, the body Q, is ascending on the inclined plane, with the same acceleration.

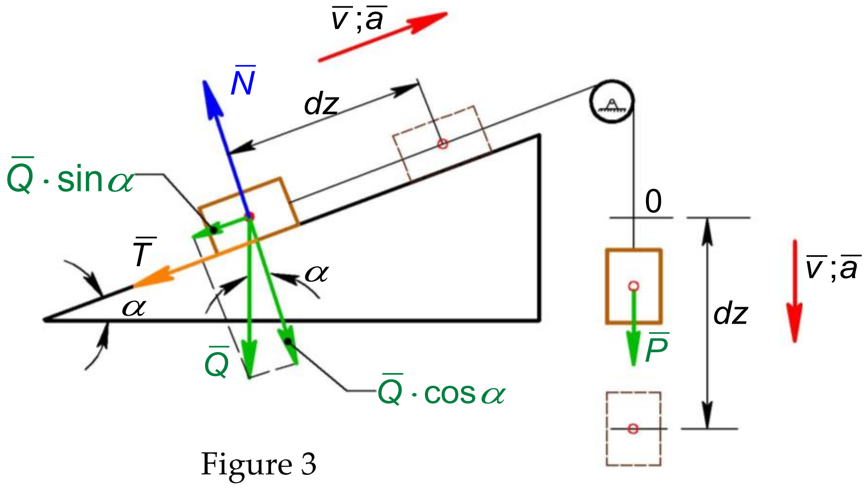


Figure 3

The ***Theorem of Kinetic Energy*** will be applied in differential form as:

$$dE_c = dL \tag{1}$$

For the above considered mechanical system, the kinetic energy is:

$$E_c = \frac{1}{2}\frac{P}{g}v^2 + \frac{1}{2}m_1 v^2 = \frac{1}{2}\left(m_1 + \frac{P}{g}\right)\cdot v^2 \tag{2}$$

The time derivative of kinetic energy determined in (2) is:

$$dE_c = \left(m_1 + \frac{P}{g}\right)\cdot \mathrm{v}d\mathrm{v} \tag{3}$$

The elementary mechanical work, corresponding to an elementary displacement dz is:

$$dL = P\cdot dz + Q\cdot sin\alpha\cdot dz - \mu\cdot Q\cdot cos\alpha\cdot dz = [P - Q\cdot(sin\alpha + \mu cos\alpha)]\cdot dz \tag{4}$$

There are equalized relations (3) with (4) and it results:

$$(m_1 + m_2)\cdot \mathrm{v}d\mathrm{v} = [m_2\cdot g - m_1\cdot g\cdot(sin\alpha + \mu cos\alpha)]\cdot dz \tag{5}$$

The previous expression is divided with dt, resulting:

$$(m_1 + m_2)\cdot \mathrm{v}\cdot \mathrm{a} = \left[m_2 g - m_1 g\cdot(sin\alpha + \mu cos\alpha)\right]\cdot \mathrm{v} \tag{6}$$

Simplifying with v, expression (6) becomes:

$$(m_1 + m_2)\cdot \mathrm{a} = \left[m_2 - m_1(sin\alpha + \mu cos\alpha)\right]\cdot g \tag{7}$$

From (7) results that the acceleration of the system is:

$$a = \frac{m_2 - m_1\cdot(sin\alpha + \mu cos\alpha)}{m_2 + m_1}\cdot g \tag{9}$$

4. Development of the laboratory work

The motion of weight P is a vertical rectilinear translational movement with constant acceleration a. If is denoted by z the motion from the initial starting position, the following kinematic relationships are valid:

$$v = \dot{z}; \qquad a = \dot{v} = \ddot{z} \tag{10}$$

The last expression representing a second order differential equation in relation to time, which can be integrated as it follows:

$$\dot{v} = a = \frac{dv}{dt} \quad \rightarrow \quad dv = adt \quad \rightarrow v = at + C_1 \tag{11}$$

$$v = \dot{z} = a \cdot t + C_1 \ \rightarrow \ \frac{dz}{dt} = at + C_1 \ \rightarrow \ dz = (at + C_1)dt$$

$$z = \frac{a \cdot t^2}{2} + C_1 t + C_2 \tag{12}$$

The *constants of integration* C_1 şi C_2 are determined from *initial conditions of the movement* at time ***t = 0***: ***v = 0, a = 0***. Hence, replacing the *initial conditions* in relations (11) and (12), results:

$$C_1 = C_2 = 0 \tag{13}$$

Thus, the relations (11) and (12) are becoming:

$$v = \dot{z} = a \cdot t\,; \qquad z(t) = \frac{a \cdot t^2}{2} \tag{14}$$

When the body P reaches the final position (down), there are set the following conditions in previous relation: $t = t_c$ and $z(t_c) = h$. So the acceleration is:

$$a = \frac{2h}{t_c^2} \tag{15}$$

Considering expressions (9) and (15), results:

$$\frac{m_2 - m_1 \cdot (\sin\alpha + \mu cos\alpha)}{m_2 + m_1} \cdot g = \frac{2h}{t_c^2} \tag{16}$$

or:

$$m_2 - m_1 \cdot (sin\alpha + \mu cos\alpha) = \frac{2h \cdot (m_1 + m_2)}{t_c^2 \cdot g} \tag{17}$$

From (13) results that the friction coefficient when sliding μ:

$$\mu = \frac{1}{cos\alpha}\left[\frac{m_2}{m_1} - sin\alpha - \frac{2h}{t_c^2 \cdot g}\left(1 + \frac{m_2}{m_1}\right)\right] \tag{18}$$

respectively:

$$\mu = \frac{1}{cos\alpha}\left[\left(1 - \frac{2h}{t_c^2 \cdot g}\right) \cdot \frac{m_2}{m_1} - sin\alpha - \frac{2h}{t_c^2 \cdot g}\right] \quad (15)$$

To measure de sliding friction coefficient (μ) the next steps must be followed:

a. For the pair of materials (rigid body and inclined plane) and an initially established angle of inclination (α), the time and height of fall (t_c and h) are measured.

b. There are made three measurements and calculate the average fall time, filling the values in **Table 1.**

c. With relation (15), is calculated the sliding friction coefficient (μ).

d. Repeat steps **a-c** for each pair of materials.

Table 1

Materials in contact	**α[°]**	**h[m]**	**t_c [s]**	**t_c^{medium} [s]**	**μ**
Al-Steel					
Al – Al					
Wood – Steel					
Wood – Al					
Wood – Wood					
Al – Rubber					
Wood - Rubber					

In the specialized literature, the sliding friction coefficients have the values indicated in **Table 2.**

Table 2

Surface of contact	**Sliding (μ)**
Metal - Metal	0.09-0.15
Metal - Wood	0.3-0.4
Wood - Wood	0.48
Rubber – Bitumen	0.19-0.34 Smooth wheel 0.28 Tooted wheel 0.55

THE VARIATION OF THE COEFFICIENT OF DYNAMIC FRICTION
WITH THE LINEAR SPEED OF A BAR

1. The purpose of the paper

In the paper, there will be highlighted the variation of the dynamic coefficient of friction related to the speed. The method used consists in the analysis of the oscillatory motion according to the mass center motion theorem. There is reminded that the dynamic friction is defined as the frictional force which appears between two surfaces when they are in a moving position.

2. Experimental Stand Description

There is considered a *bar* having the length l and gravitational force $\overline{G}$, which rests freely on *rollers 2 and 3*, located at a distance 2a one from the other (see Figure 1).

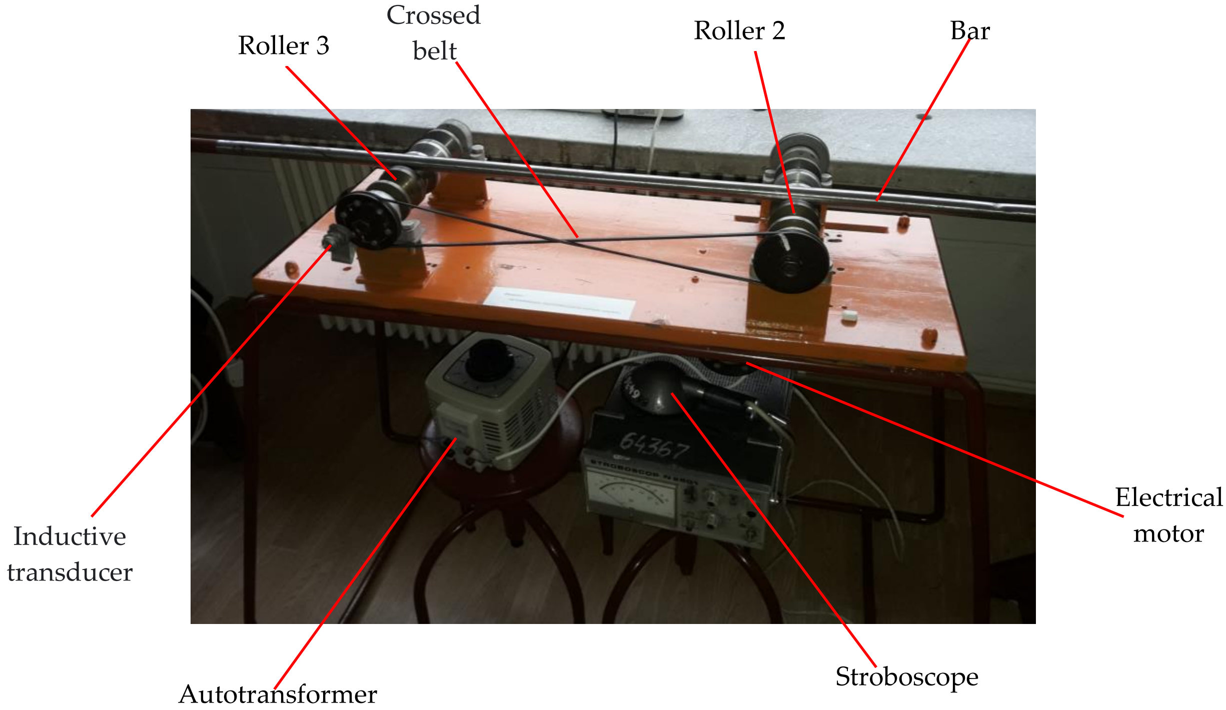


Figure 1

Roller 2 rotates counterclockwise, being driven by an *electric motor* through a belt drive. *Roller 3* rotates clockwise being driven by roller 2 through a *crossed belt*. The two rollers, having the same diameter, will have the same revolution (RPM). Their revolution can be changed with the help of an *autotransformer*, that controls the supply voltage of the electric motor, and it is measured with an *inductive transducer* connected to a *stroboscope* with the role of a frequency meter.

3. Development of the laboratory work

According to Figure 2, a bar is placed on the rollers, in an off-center position; when the rollers are rotating with n [RPM], the bar will begin to perform alternative motion related to the stable equilibrium position (*point 0*, where the center of mass is in the middle of the *distance 2a* between the axes of the two rollers).

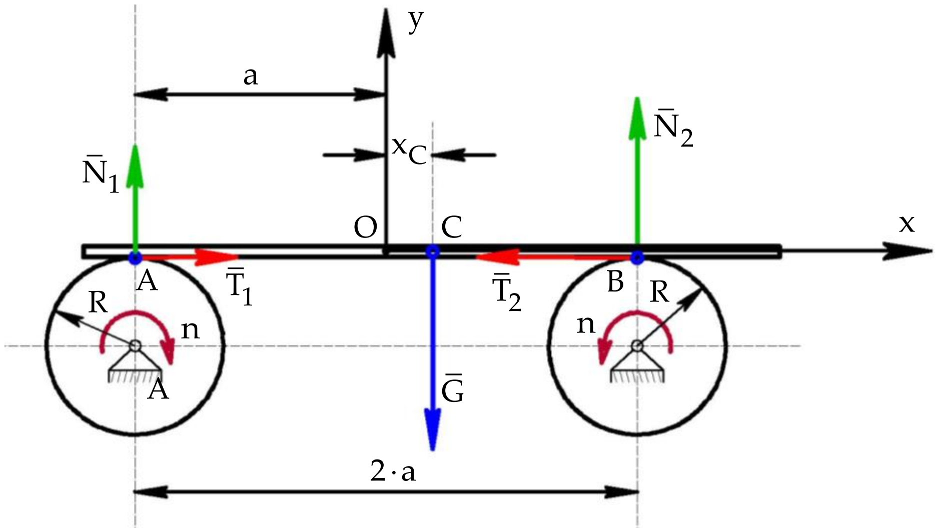


Figure 2

According to Figure 2, with the change of the mass center position, the values of the normal reactions $\bar{N}_1$ and $\bar{N}_2$, and the corresponding frictional forces $\bar{T}_1$ and $\bar{T}_2$ are modifying their values.

For example, for the position shown in Figure 2, N_1 and T_1 are decreasing and N_2 and T_2 are increasing. When the frictional force T_1 decreases low enough, the sliding of the bar on the rollers occurs, and with this, the direction of motion of the bar changes also (from right to left). For a certain position of the bar, characterized by the *displacement x of the center of mass C related to point 0*, the equation of motion of the center of mass is:

$$m\ddot{x} = T_1 - T_2 \tag{1}$$

where: $$T_1 = \mu \cdot N_1; \quad T_2 = \mu \cdot N_2 \tag{2}$$

By writing two moment equilibrium equations for points A and B, $\sum M_A = 0, \sum M_B = 0$, the reactions N_1 and N_2 are found as:

$$\begin{aligned} 2a \cdot N_2 - G \cdot (a + x) = 0 \Rightarrow N_2 = \frac{a + x}{2a} \cdot G; \\ 2a \cdot N_1 - G \cdot (a - x) = 0 \Rightarrow N_1 = \frac{a - x}{2a} \cdot G \end{aligned} \tag{3}$$

where, ***a*** is the distance from the equilibrium point *0* of the mass center *C*, to a one of the roller\`s axes.

On the basis of Coulomb\`s law, results:

$$T_1 = \mu \cdot G \cdot \frac{a - x}{2a}; \quad T_2 = \mu \cdot G \cdot \frac{a + x}{2a} \tag{4}$$

Replacing (4) in (1), it is obtained:

$$m \cdot \ddot{x} + \mu \cdot G \cdot \frac{x}{a} = 0 \tag{5}$$

The solution of the differential equation (5) is:

$$x = \alpha \sin(\omega t + \varphi) \tag{6}$$

Note: $\omega^2 = \mu \cdot \frac{g}{a}$, ***and at*** $t = 0; \dot{x}(0) = 0; x(0) = x_0$.

It results that the motion of bar\`s mass center is a harmonic oscillatory motion with the center in O.

The period of this motion is: $$T = \frac{2\pi}{\omega} \Rightarrow \frac{4 \cdot \pi^2 \cdot a}{\mu \cdot g} = T^2 \tag{7}$$

From (7), it results:

$$\mu = \frac{4 \cdot \pi^2 \cdot a}{g} \cdot T^{-2} = K \cdot T^{-2} = \frac{K}{T^2} \tag{8}$$

where $K \approx 1{,}2$ is a constant value.

By measuring the period *T* of the oscillatory motion, the coefficient of friction is calculated with relation (8). To increase the accuracy of the calculations, the duration required for three complete oscillations of the bar is timed and are filled the values in **Table 1**.

To establish a link between the value of the sliding friction coefficient and the corresponding speed, the number of pulses per second emitted by the *inductive transducer* is read on the *stroboscope indicator* (on a scale of 0 ÷ 100). Knowing that six pulses are emitted at one rotation (because of the six screws assembled on the roller) by multiplying by 10 it is obtained the revolution (RPM) and then the speed is calculated as:

$$v_i = \frac{\pi \cdot n_i}{30} \cdot R \tag{9}$$

The bar is placed on a pair of rollers and three periods of oscillation at different revolutions are timed, filling the measured values in **Table 1**. There are repeated the operations described above for other rollers made of different materials.

Table 1

a = 302 [mm]		R = 50 [mm]		g = 9.81 [m/s²]	
No of set:		**Materials:**			
No.	**3T [s]**	*Period T [s]*	*Revolution n [rot/min]*	*Coefficient of friction μ*	*Velocity v [m/s]*
1					
2					
3					
4					
5					

Using data from **Table 1**, on a system of axis, having on the abscissa *speed v* and on the ordinate the *coefficient of friction on sliding* μ, the curve μ = f(v) for different combinations of materials will be drawn (ex. Figure 3 - Steel-Aluminum or Figure 4 Steel-Bronze).

Figure 3

Figure 4

DETERMINATION OF KINETIC ENERGY IN THE CASE OF A PLANE MECHANISM

1. The purpose of the work

Determination of the ***kinetic energy***, the ***equivalent mechanical moment of inertia reduced to the axis of the crank*** and the ***angular momentum*** (***kinetic moment)*** in relation to the axis of the crank. For the eccentric crank mechanism, supposing that the connecting rod being as an homogeneous bar, presented in Figure 1, in the position given by angle φ, there will be determined analytically and graph-analytically, the following :

a) kinetic energy of the mechanism;

b) the equivalent mechanical moment of inertia reduced to point O of the mechanism;

c) the angular momentum of the mechanism in relation to point O;

d) relative errors between analytical and graphic values in the case of kinetic energy and angular momentum.

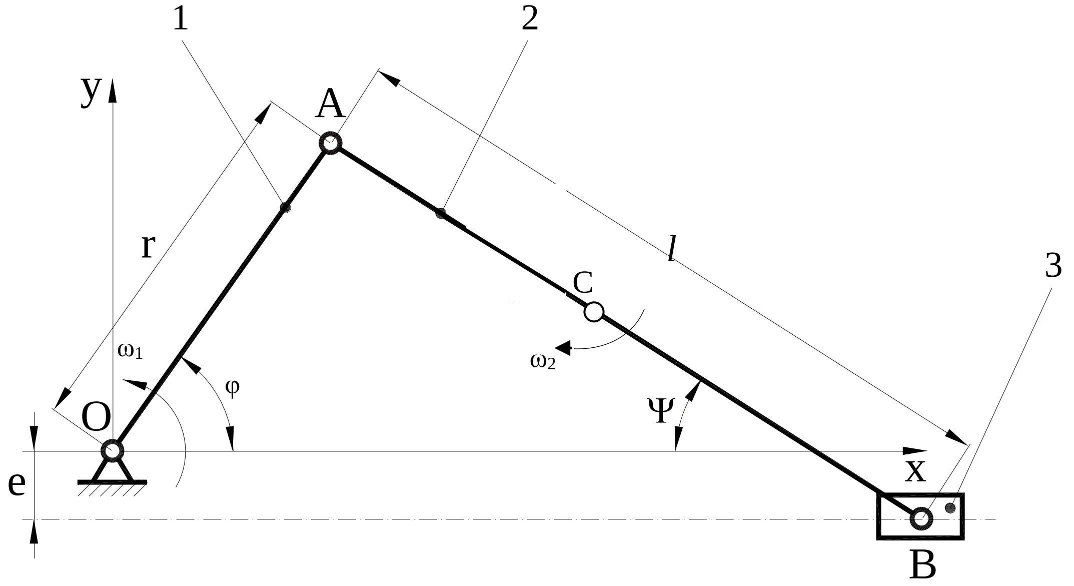


Figure 1

According to Figure 1, the elements of the mechanical system, which will be analyzed, are: **1** – crank (m); **2** – connecting rod (b); **3** – piston (p).

There will be considered as input data the following:

$n_1 = 1000 + 10 \cdot j$ [rot/min] (*j – semigrup number*);

$\varphi = 10 + 5 \cdot i$ [°] (*i – serial number of the student in the semigrup*);

$r = 40$ [mm]; $\ell = 80$ [mm]; $e = 5$ [mm];

$M_m = 0.25$ [kg]; $M_b = 0.5$ [kg]; $M_p = 0.125$ [kg].

2. Development of laboratory work

2.1. Analytical method

Step 1. Depending on the number of the semigroup and the serial number in the semigroup, values of speed n_1 and of angle φ are determined and filled in **Table 1**:

Table 1

i	j	n_1	φ

Step 2. The connecting rod and the crank will be assimilated with **homogeneous bars**. The kinetic energy of the mechanism is equal to the sum of the kinetic energies of the component elements *(crank, connecting rod and piston)*:

$$E_C = E_{C1} + E_{C2} + E_{C3} \tag{1}$$

where:

$E_{C1} = \frac{1}{2} \cdot J_0 \cdot \omega_1^2$ - the kinetic energy of the crank

$E_{C2} = \frac{1}{2} \cdot M_b \cdot v_C^2 + \frac{1}{2} \cdot J_C \cdot \omega_2^2$ - the kinetic energy of the connecting rod (2)

$E_{C3} = \frac{1}{2} \cdot M_p \cdot v_B^2$ - the kinetic energy of the piston

Step 3. The mechanical moments of inertia of the crank and connecting rod are calculated;

a) Calculation of the mechanical moment of inertia of the crank OA:

$$J_0 = \frac{M_m \cdot r^2}{3} \tag{3}$$

b) Calculation of the mechanical moment of inertia of connecting rod AB:

$$J_C = \frac{M_b \cdot \ell^2}{12} \tag{4}$$

Step 4. Calculation of the angular velocities of the connecting rod and the crank:

a) angular velocity ω_1:

$$\omega_1 = \frac{\pi \cdot n_1}{30} \tag{5}$$

b) the angular velocity of connecting rod AB is:

$$\omega_2 = \frac{r}{\ell} \cdot \frac{\cos\varphi}{\cos\psi} \cdot \omega_1 \tag{6}$$

where:

$$\sin\psi = \frac{r \cdot \sin\varphi + e}{\ell}, \qquad \cos\psi = \sqrt{1 - \sin^2\psi} \tag{7}$$

Step 5. The value of the crank's kinetic energy will be calculated with:

$$E_{C1} = \frac{1}{2} \cdot J_0 \cdot \omega_1^2 \tag{9}$$

Step 6. Are determined the coordinates of points A, B and C in plane, used to determine the velocities of points A, B and C needed to calculate the kinetic energies of the connecting rod and piston.

a) *The coordinates of point A* are:

$$\begin{cases} x_A = r \cdot \cos\varphi \\ y_A = r \cdot \sin\varphi \end{cases} \tag{10}$$

b) *The coordinates of point B* are:

$$\begin{cases} x_B = r \cdot \cos\varphi + \ell \cdot \cos\psi \\ y_B = -e \end{cases} \tag{11}$$

c) *The coordinates of point C* are:

$$\begin{cases} x_C = r \cdot \cos\varphi + \dfrac{\ell}{2} \cdot \cos\psi \\ y_C = \dfrac{\ell}{2} \cdot \sin\psi - e \end{cases} \tag{12}$$

Step 7. Determination of the projections of the linear velocities of points B and C on the axes Ox and Oy, by using the expressions:

$$\begin{cases} v_{Bx} = v_{Ax} + \omega_2(y_B - y_A) \\ v_{By} = v_{Ay} - \omega_2(x_B - x_A) \end{cases} \tag{13}$$

$$\begin{cases} v_{Cx} = v_{Ax} + \omega_2(y_C - y_A) \\ v_{Cy} = v_{Ay} - \omega_2(x_C - x_A) \end{cases} \tag{14}$$

where: $\begin{cases} v_{Ax} = -y_A \cdot \omega_1 \\ v_{Ay} = x_A \cdot \omega_1 \end{cases}$

Step 8. The modulus of the velocities of points B and C are:

$$v_B = \sqrt{v_{Bx}^2 + v_{By}^2}; \qquad v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} \tag{15}$$

Step 9. The kinetic energies of the connecting rod and piston are calculated:

a) Kinetic energy of the connecting rod:

$$E_{C2} = \frac{1}{2} \cdot M_b \cdot v_C^2 + \frac{1}{2} \cdot J_C \cdot \omega_2^2 \tag{16}$$

b) Kinetic energy of the piston:

$$E_{C3} = \frac{1}{2} \cdot M_p \cdot v_B^2 \tag{17}$$

Step 10. The analytical **total kinetic energy** of the mechanism will be:

$$E_c^{an} = E_{C1} + E_{C2} + E_{C3} \tag{18}$$

Step 11. The equivalent (*reduce*) mechanical moment of inertia reduced to point O is:

$$J_0^{red} = \frac{2 \cdot E_c}{\omega_1^2} \text{ [kg m}^2\text{]} \tag{19}$$

Note - ***The reduced equivalent moment of inertia*** *is the moment of inertia of a fictitious body that has the same state of motion (kinetic energy) as the entire mechanism.*

Step 12. The kinetic moment relative to point O of each element of the mechanism (crank, connecting rod and piston) is calculated using the relations:

$$K_0^{(1)} = J_0 \cdot \omega_1$$

$$K_0^{(2)} = M_b(x_C v_{Cy} - y_C v_{Cx}) - J_C \omega_2 \tag{20}$$

$$K_0^{(3)} = M_p\left(x_B v_{By} - y_B v_{Bx}\right)$$

Step 13. **The total (analytical) kinetic** moment relative to point O of the mechanism will be calculated as a sum of the component kinetic moments:

$$K_O^{an} = K_0^{(1)} + K_0^{(1)} + K_0^{(3)} \quad [\text{kg m}^2\ \text{s}^{-1}] \tag{21}$$

2.2. The graph - analytical method (see Figure 2)

The mechanism is represented on the length scale, chosen suitable for the input data, in the configuration given by the angle φ. The angular velocity ω_1 of the crank, the mechanical moment of inertia of the crank related to O, the mechanical moment of inertia of the connecting rod related to C are already calculated analytically (see relations (3)-(5)), and the velocity of point A is:

$$v_A = \omega_1 \cdot r \tag{22}$$

Point A of the connecting rod moves on a circle of radius r, and point B on a line parallel to the Ox axis. The instantaneous center of velocity of the connecting rod I_2 is found at the intersection of the normal at velocities taken to points A and B (the crank *OA* is extended and a perpendicular line to B on the Ox axis is drawn).

In planar motion, the velocities distribution related to the instantaneous center of velocity I_2 is specific to a rotational movement relative to this center. Hence:

$$v_A = \omega_2 \cdot I_2A, \qquad \bar{v}_A \perp I_2A \tag{23}$$

$$v_B = \omega_2 \cdot I_2B, \qquad \bar{v}_B \perp I_2B \tag{24}$$

$$v_C = \omega_2 \cdot I_2C, \qquad \bar{v}_C \perp I_2C \tag{25}$$

The distances I_2A, I_2B, I_2C are measured on the drawing and multiplied by the length scale.

From (23) results:

$$\omega_2 = \frac{v_A}{I_2A} \tag{26}$$

With relations (24) and (25) velocities of points B and C are graphically determined.

a) The t**otal kinetic energy** of the mechanism by graph - analytical method is determined with the relation:

$$E_C^{gr} = \frac{1}{2} \cdot J_0 \cdot \omega_1^2 + \frac{1}{2} \cdot M_b \cdot v_C^2 + \frac{1}{2} \cdot J_C \cdot \omega_2^2 + \frac{1}{2} \cdot M_p \cdot v_B^2 \tag{27}$$

b) The graph-analytical kinetic moment relative to point *O* will be calculated with the relation:

$$K_O^{gr} = K_0^{(1)} + K_0^{(1)} + K_0^{(3)} \tag{28}$$

where:

$$K_0^{(1)} = J_0 \cdot \omega_1 \tag{29}$$

$$K_0^{(2)} = M_b \cdot v_C \cdot d \mp J_C \cdot \omega_2 \tag{30}$$

$$K_0^{(3)} = M_p \cdot v_B \cdot e \tag{31}$$

The distance ***d*** that will be submitted in relation (30), is obtained by measuring the distance from point *O* to the support of the vector $\overline{v}_C$ on the drawing and multiplied by the scale. Also in relation (30), the sign (-) is taken when the direction of the velocity of point *B* is to the left, and the sign (+) when the direction of the velocity of point *B* is to the right.

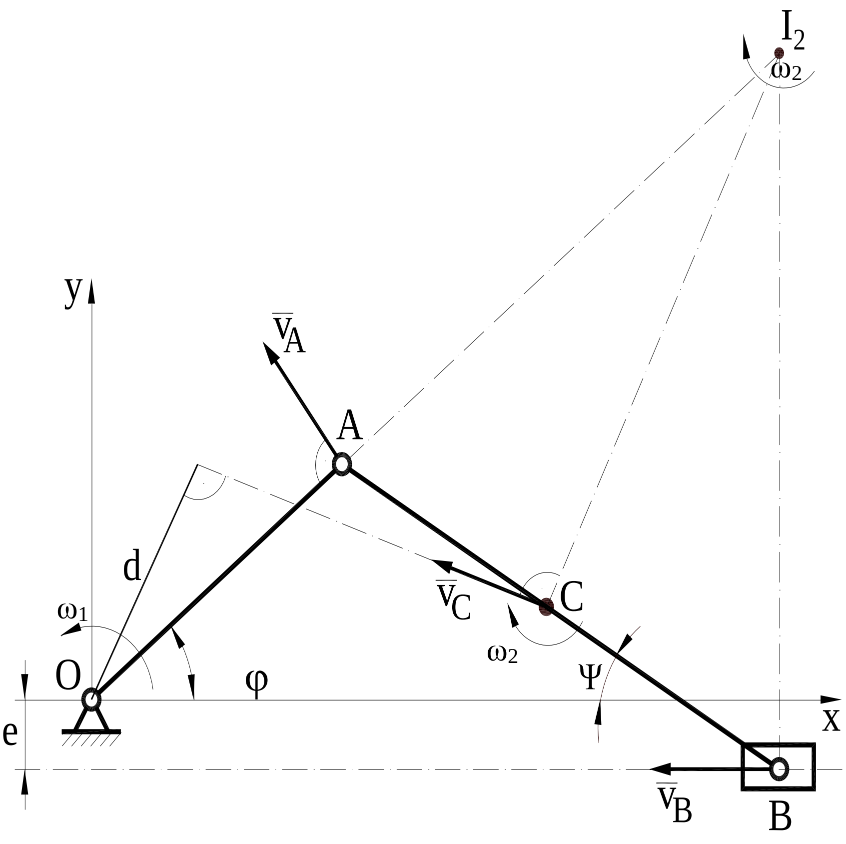


Figure 2

The relative errors, in general, are evaluated with the relation:

$$\varepsilon_r = \frac{\left|V^{an} - V^{gr}\right|}{V^{an}} \cdot 100 \quad [\%] \tag{32}$$

where: V – is the analised component;

Table 2

E_C^{an} [J]	E_C^{gr} [J]	ε_{rEc} [%]	K_O^{an} [kgm^{2}s^{-1}]	K_O^{gr} [kgm^{2}s^{-1}]	ε_{rKo} [%]

DETERMINATION OF THE RESTITUTION COEFFICIENT AT COLLISION

1. The purpose of the work

The aim of the laboratory work is the experimental determination of the coefficient of restitution due to the collision of two bodies of the same material or of different materials.

2. Theoretical considerations

In Figure 1 is presented the model of a mechanical system consisting of two bodies, *body 1* of mass m_1 moving on a circular trajectory in the vertical plane and *body 2* of mass m_2 fixed in a rigid mechanical structure of mass $m_2 >> m_1$, at rest .

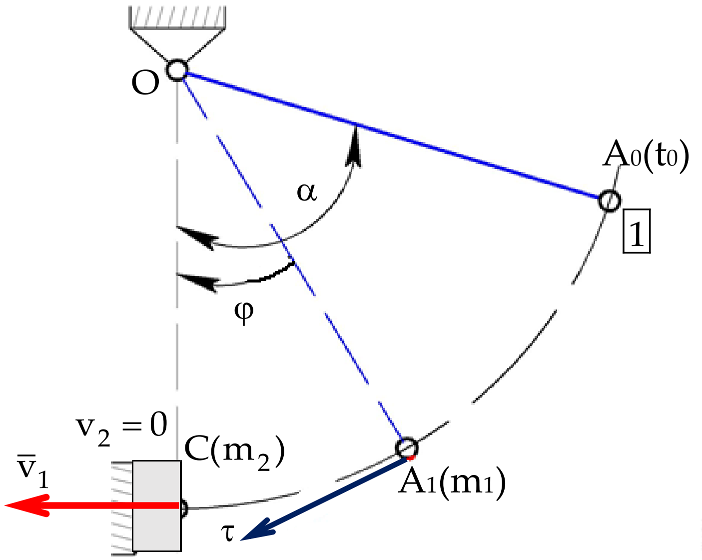


Figure 1

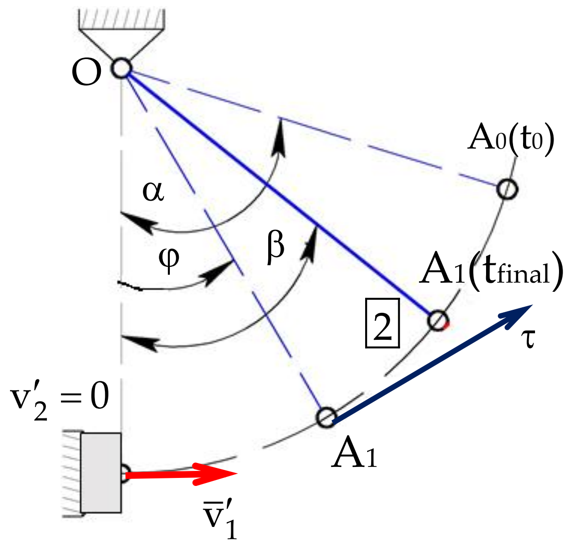


Figure 2

Body 1, is fixed on the end A of a rod *OA* of length *l* and negligible mass, articulated to a fixed horizontal axis at point *O*, thus forming a **physical pendulum**. *To produce the collision*, the mass *m1* is released from the rest position OA_0, performing a circular movement with the velocity v_1, and after the collision with mass m_2 performing a motion in the opposite direction to the velocity v_1 as in Figure 2.

The collision is a mechanical phenomenon that occurs in a short period, with sudden variations of velocity and that generates very large forces called *impacts* expressed by relation (1).

$$\int_{t_0}^{t_1} \bar{F} dt = \bar{P} \tag{1}$$

Collision events are characterized by the collision *coefficient of restitution* (velocity of the same direction) and expressed as:

$$k = \frac{P_2}{P_1} = \frac{v_2' - v_1'}{v_1 - v_2} \tag{2}$$

where: P_1 represents the normal percussion before collision, and P_2 the normal percussion after collision.

Figure 1 shows the movement of mass m_1 before collision, and Figure 2 shows the movement of mass m_1 after the collision. It is considered that *the velocities of body 2 before and after the collision are zero*, respectively: $v_2 = v_2' = 0$, which conducts to:

$$k = -\frac{v_1'}{v_1} \tag{3}$$

To determine the velocity v_1, the physical pendulum equation is solved according to Figure 1, on descendant motion:

$$\ddot{\varphi} + \frac{Mgd}{J} sin\varphi = 0 \tag{4}$$

where M is the mass of pendulum, d is the distance from point O to the mass center, and J is the axial inertia moment with respect to horizontal axis which passes through point O. Changing the variable time with φ, and separating the variables, the differential equation becomes:

$$d\left(\frac{\dot{\varphi}^2}{2}\right) = -\frac{Mgd}{J} sin\varphi \mathrm{d}\varphi = 0 \tag{5}$$

by integrating relation (5) it follows:

$$\frac{\dot{\varphi}^2}{2} = \frac{Mgd}{J} cos\varphi + C_1 \tag{6}$$

If the initial conditions of the movement are imposed, at the moment $t_1 = 0$, namely $\varphi(0) = \alpha$ and and $\dot{\varphi}(0) = 0$, in equation (6) the integration constant C_1 will be obtained as:

$$C_1 = -\frac{Mgd}{J} cos\alpha \tag{7}$$

By replacing relation (7) in (6) it will result the expression of velocity on descending zone, as:

$$\frac{\dot{\varphi}^2}{2} = \frac{Mgd}{J}(cos\varphi - \cos\alpha) = 0 \tag{8}$$

The expression of the velocity of *body 1* before the collision is obtained by imposing the condition $\varphi = 0$ in relation (8):

$$\mathrm{v}_1 = l\dot{\varphi} = l\sqrt{2\frac{Mgd}{J}(1 - cos\alpha)} = 2l\sqrt{\frac{Mgd}{J}}\sin\frac{\alpha}{2} \tag{9}$$

To determine the velocity $\mathrm{v}_1^{'}$, after the collision, the equation of the physical pendulum (see Figure 2) will be solved on ascending zone, according to relations (4)-(6) resulting the constant of integration C_2.

$$\frac{\dot{\varphi}^2}{2} = \frac{Mgd}{J} cos\varphi + C_2 \tag{10}$$

If the initial conditions of the movement are imposed, at the moment $t = f_{inal} = 0$, namely $\varphi = \varphi(t_{final}) = \beta$ and $\dot{\varphi}(t_{final}) = 0$, in equation (10) the constant of integration C_2 will result as:

$$C_2 = -\frac{Mgd}{J} cos\beta \tag{11}$$

By replacing relation (11) in (10) it will result:

$$\frac{\dot{\varphi}^2}{2} = \frac{Mgd}{J}(cos\varphi - \cos\beta) \tag{12}$$

The expression of the velocity $\mathrm{v}_1^{'} = l\dot{\varphi}$ that *body 1 has it after collision* is obtained by imposing the condition $\varphi = 0$ in relation (12):

$$\mathrm{v}_1^{'} = l\sqrt{2\frac{Mgd}{J}(1 - cos\beta)} = 2l\sqrt{\frac{Mgd}{J}}\sin\frac{\beta}{2} \tag{13}$$

Substituting relations (9) and (13) into relation (2) results the *coefficient of restitution k*:

$$k = \frac{\sin\frac{\beta}{2}}{\sin\frac{\alpha}{2}} \tag{14}$$

3. Experimental Stand Description

Figure 3 shows, the laboratory stand for the study of collisions, which is used to determine the measurements of angles α and β from expression (14) to determine the coefficient of restitution due to collision k. The bodies to collide are interchangeable (having mass m_1 and m_2) and are used to analyze the collisions between the same material bodies or different materials bodies.

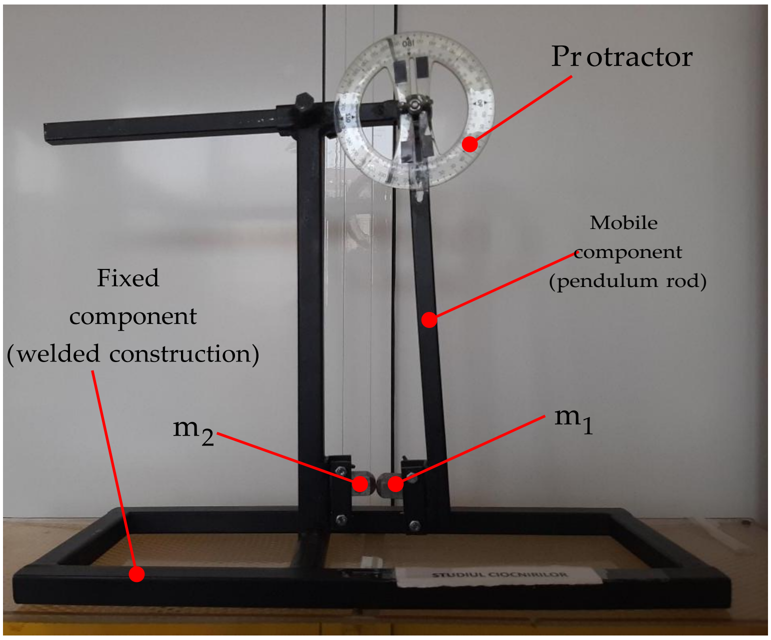


Figure 3

4. Development of the laboratory work

A set of three measurements is performed for collisions between the same material (see Figure 4), and for different materials, measuring on the protractor the angles before and after the collision. Using expression (14), there is calculated the coefficient of restitution for them, and the obtained values are filled in **Table 1.**

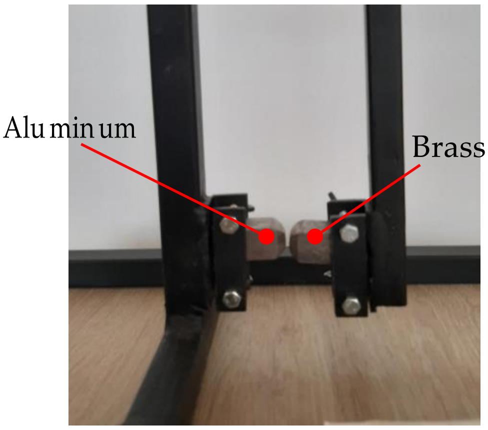


Figure 4

Table 1

No.	Materials	α	β	k_i	k_{medium}
1	Al-Al				
2	Steel-Steel				
3	Brass-Brass				
4	Steel-Al				
5	Steel-Brass				
6	Brass-Al				

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