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Essential Physics for Engineers

U.T.PRESS
Cluj - Napoca, 2026
ISBN 978-606-737-831-3

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Editura U.T.PRESS
Str. Observatorului nr. 34
400775 Cluj-Napoca
Tel.: 0264-401.999
e-mail: utpress@biblio.utcluj.ro
http://biblioteca.utcluj.ro/editura

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Pregătire format electronic on-line: Gabriela Groza

ISBN 978-606-737-831-3

Foreword

Physics is the cornerstone of engineering, providing the essential framework for our technological world. While the academic landscape is rich with exhaustive treatises, the modern student requires a streamlined entry point. Essential Physics for Engineers was conceived for UTCN students as a clear, structured synthesis of core principles, distilling complex phenomena into a coherent, efficient guide. The material is organized into two fundamental pillars:

- Part I covers mechanics, rotations, and wave phenomena, extending into the study of energy and transport phenomena in fluids and porous materials.
- Part II explores electromagnetism—from electrostatics to electrodynamics—culminating in the foundations of quantum physics and the electronic properties of solids, including semiconductors and superconductors.

This edition embraces a contemporary vision by integrating various applications of the physical phenomena presented, ensuring that theoretical concepts are grounded in real-world engineering contexts. Rather than a final destination, this book is a dynamic starting point. Future editions will expand upon these foundations with applied problem sets, bridging the gap between theory and practice to provide the analytical depth indispensable for technological innovation.

Part I

I Kinematics and dynamics of material points

II Rotation of rigid bodies

III Oscillations

IV Elastic Waves

VI Elements of acoustics and ultra-acoustics

VII Thermal transport phenomena

VIII Fluid transport phenomena in porous materials

Part II

I Elements of electrostatics

II Electric current. Ohm's Law

III Elements of Magnetostatics

IV Elements of Electrodynamics

V Electromagnetic waves in vacuum

VI Materials in electric field

VII Materials in magnetic field

VIII Elements of quantum physics

IX Conductors, semi-conductors, insulators and superconductors

Part I

I

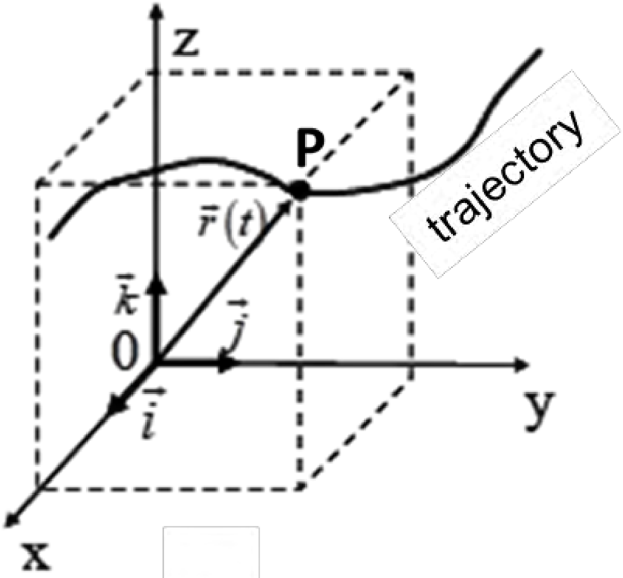
Elements of the kinematics and the dynamics of the material point

Contents:

1. Physical quantities in kinematics and dynamics

- **Kinematics** is that part of mechanics that studies the motion of objects without considering the causes producing it.
- **Dynamics** is that part of mechanics that studies the causes of motion (the effect of forces).
- **Material point** = physical object whose dimensions can be neglected in describing motion (e.g. motion of the Moon relative to the Earth)

1.1. The position vector ($\vec{r}\,(t)$)



- Describes the position of a material point in a tridimensional OXYZ coordinate system;
- Is characterized by the coordinates: x(t), y(t), z(t);
- Can be written in a vector form as:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$\vec{i}, \vec{j}, \vec{k}$ – the unit vectors of the three directions: OX, OY, OZ?

$$unit\ vector \Leftrightarrow \begin{cases} |\vec{i}| = |\vec{j}| = |\vec{k}| = 1 \\ \vec{i}\cdot\vec{j} = \vec{i}\cdot\vec{k} = \vec{j}\cdot\vec{k} = 1 \end{cases}$$

Observations:

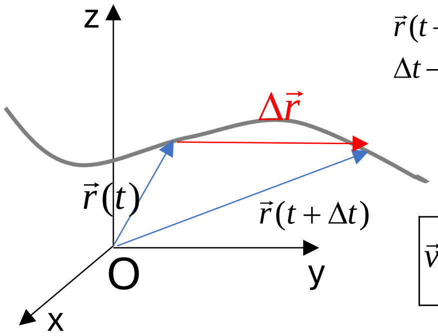
- The arrow of the position vector describes a **traiectory**
- The distance form the origin (O) to the P point can be calculated as the modulus of the position vector;

$$d_{OP} = |\vec{r}(t)| = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$$

1.2. The velocity vector ($\vec{v}(t)$)

- The velocity vector describes how quickly the position vector changes over time;
- We distinguish two types of velocity: **average velocity** and **instantaneous velocity**

The average velocity ($\vec{v}_m$)



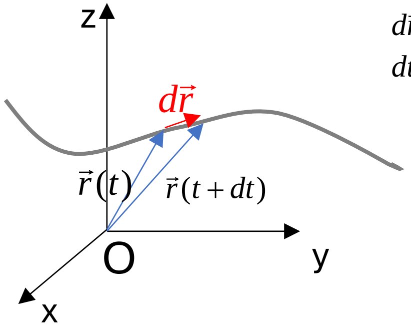
$\vec{r}(t+\Delta t)-\vec{r}(t)=\Delta\vec{r}$ – displacement vector

Δt – time interval of the displacement

$$\vec{v}_m=\frac{\vec{r}_2-\vec{r}_1}{t_2-t_1}=\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{t+\Delta t-t}=\frac{\Delta\vec{r}}{\Delta t}$$

- Measuring unit for velocity is **m/s**;
- Orientation of the average velocity is parallel to the displacement vector
- The "average <u>speed</u>" is not the magnitude of average velocity but the distance traveled divided by the travel time. It is a positive, scalar quantity.

The instantaneous velocity ($\vec{v}$)



$d\vec{r}$ – infinitezimal displacement during dt

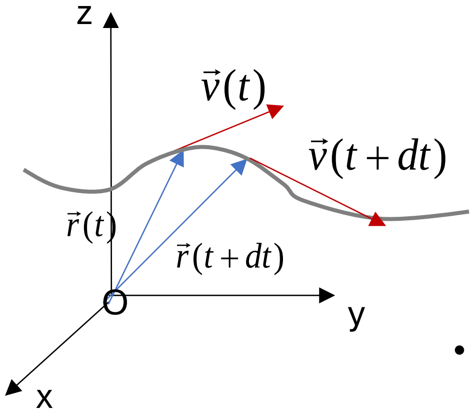
dt – infinitezimal time interval (*tinde spre* 0)

$$\vec{v}=\lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{t+\Delta t-t}=\frac{d\vec{r}}{dt}$$

- The Instantaneous velocity (at an instant t) is obtained as the **derivative of the position vector versus time**;
- Orientation of the instantaneous velocity is **tangent to the traiectory**
- The instantaneous <u>speed</u> is the magnitude of the instantaneous <u>velocity vector</u>, It cannot be negative.

1.3. The acceleration vector ($\vec{a}\,(t)$)

- The acceleration vector describes how fast the velocity vector changes over time;
- In the case of acceleration, only the **instantaneous acceleration** is defined



$$\boxed{\vec{a}=\frac{\vec{v}(t+dt)-\vec{v}(t)}{t+dt-t}=\frac{d\vec{v}}{dt}}$$

$$because: \quad v=\frac{d\vec{r}}{dt}$$

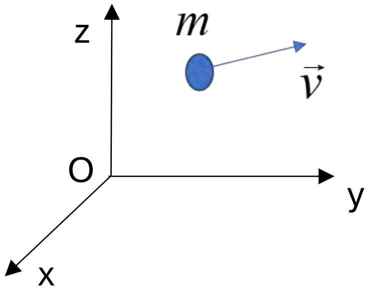
$$\Rightarrow \quad \vec{a}=\frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right)=\frac{d^2\vec{r}}{dt^2}$$

$d\vec{v}$ – infinitesimal valation of the velocity during dt interval

dt – infinitesimal time interval ($goes\ to$ 0)

- Instantaneous acceleration (at instant t) is obtained as the **derivative of the velocity vector versus time**;
- In SI, the measuring unit for acceleration is **m/s²**;

1.4. The momentum ($\vec{p}$)

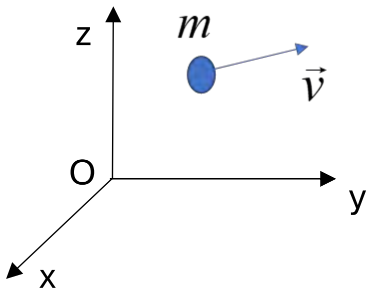


- If a material point having a mass *m* displaces with the velocity $\vec{v}$, it will have a momentum:

$$\boxed{\vec{p}=m\vec{v}}$$

- The SI unit for momentum is kg·m/s;

1.5. The translational kinetic energy (E_c)

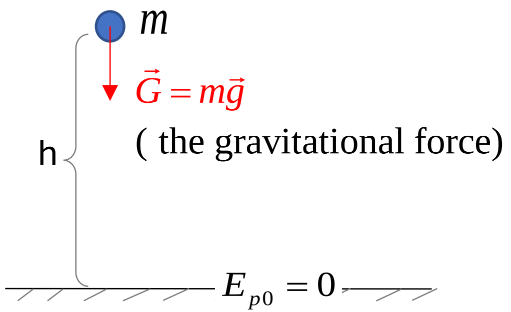


- If a material point with the mass *m* displaces with the speed *v* this will store a **kinetic energy**

$$E_c = \frac{mv^2}{2}$$

- The measuring unit for the kinetic energy (or any other form of energy) is **J=Joule**;

1.6. The potential gravitational energy (E_p)



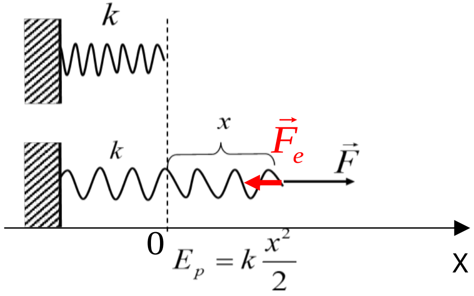
- If a material point having a mass *m* is placed in a gravitational field, at a height *h* relative to an arbitrary level chosen as a "zero" potential energy (E_{p0}=0), then the material point will store a **potential energy**

$$E_P = mgh$$

$g = 9.81\ m/s^2 -$ gravitational constant (acceleration);

- The measuring unit for the potential energy (or any other form of energy) is **J=Joule**;
- The measuring unit for gravitational force is N=Newton

1.7. Elastic potential energy (E_{pe})



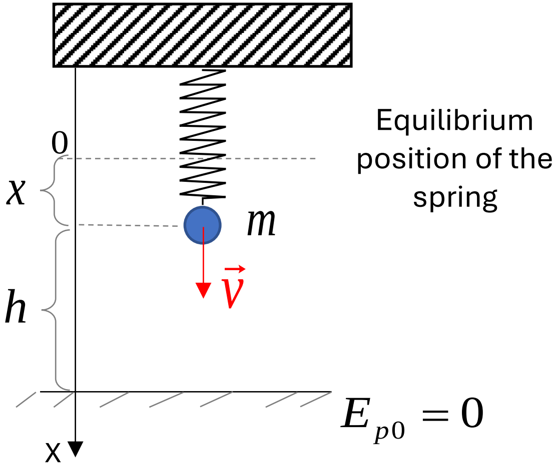
- If a spring of elastic constant *k* deforms over a distance *x* under the action of an external force *F*, then that spring will store an **elastic potential energy**

$$E_{pe} = \frac{kx^2}{2}$$

- The elastic force opposing the force F oriented along the OX is of the form:

$$\vec{F}_e = -kx\vec{i}$$

1.8. Total energy of a moving object hanging on a spring of negligible mass (E_{tot})



- If a material point of mass *m* is placed in the gravitational field, at a height *h* relative to the zero potential energy level (E_{p0}=0) and displaces with the velocity v, hanging on a spring with the elastic constant k, the system will store the **total energy**

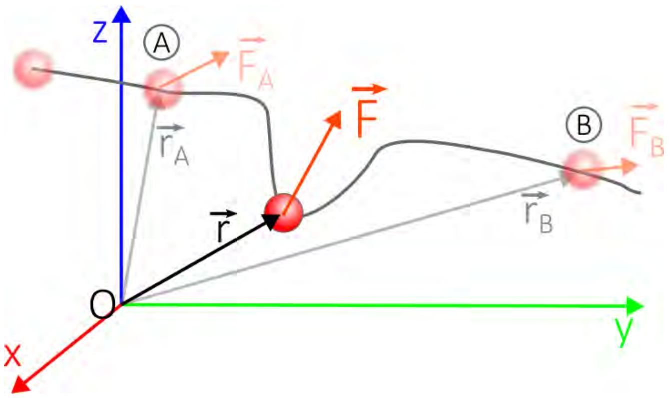
$$E_{tot} = E_c + E_P + E_{Pe} = m\frac{v^2}{2} + mgh + k\frac{x^2}{2}$$

- If no other external forces act on the system, the **total energy is conserved**;
- **Energy conservation** means that the sum of the three energies remains constant and only the energy forms are transformed.

1.9. The work (L)

The case of a variable force

Variable force ⬌ Magnitude or variable orientation



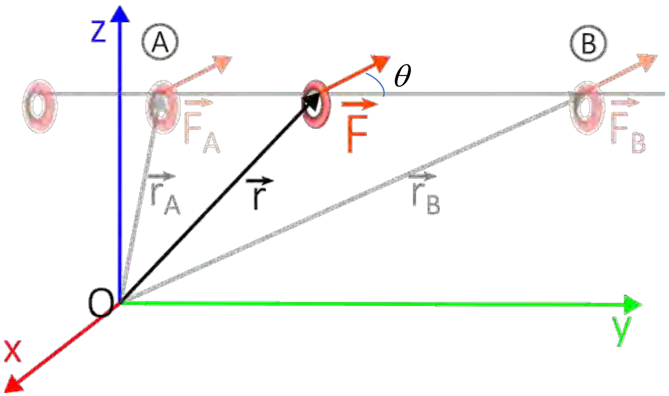
The total work produced by the force F in a displacement of its application point between the positions A and B can be calculated as an integral on the trajectory AB of the **elementary work** $dL = \vec{F} \cdot d\vec{r}$

$$L_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$$

The measuring unit for work is **Joule=J**

The case of a constant force applied to an axis constrained object

Constant force ⬌ Constant magnitude and orientation

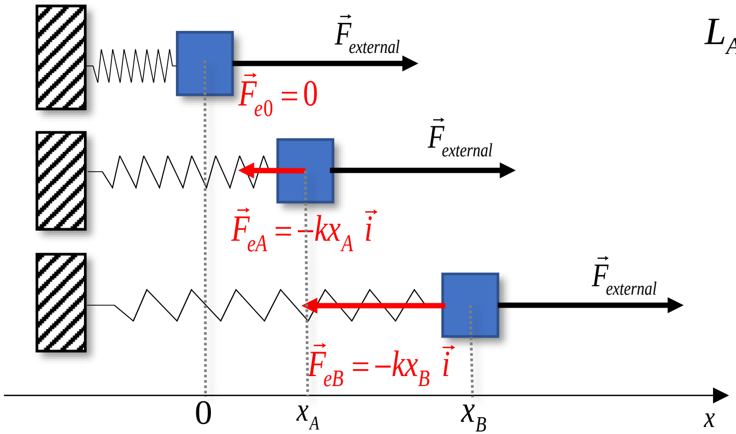


In the image above, the pebble can only move along the fixed string. Because the force F is constant on the trajectory constrained along Oy, the scalar product becomes $\vec{F} \cdot d\vec{r} = F dy \, cos\theta;$

$$L_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = F \cos\theta \int_{y_A}^{y_B} dy = Fd \cos\theta = \vec{F} \cdot \vec{d}$$

Applications:

The work of an elastic force



$$\vec{F}_e = -kx\vec{i} - \text{elastic force for a deformation } x \text{ of the spring}$$

$$L_{AB} = \int_A^B \vec{F} \cdot d\vec{r} - \text{the total work}$$

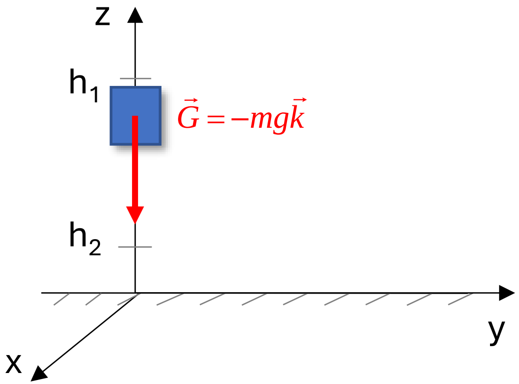
$$\Rightarrow \quad L_{AB} = \int_A^B -kx\vec{i} \cdot d\vec{r}$$

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k} = dx\vec{i} - \text{displacement only } \| \text{ with OX}$$

$$\Rightarrow \quad \boxed{L_{AB} = -\int_{x_A}^{x_B} kxdx = -\left(k\frac{x_B^2}{2} - k\frac{x_A^2}{2}\right) = -\left(E_{PB} - E_{PA}\right) = -\Delta E_P}$$

The work produced by the elastic force is **equal to the difference of the potential energies**

The work of the gravitational force



$$\vec{F} = \vec{G} = -mg\vec{k} - \text{gravitational force}$$

$$L_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$$

$$\Downarrow$$

$$L_{AB} = \int_A^B -mg\vec{i} \cdot d\vec{r}$$

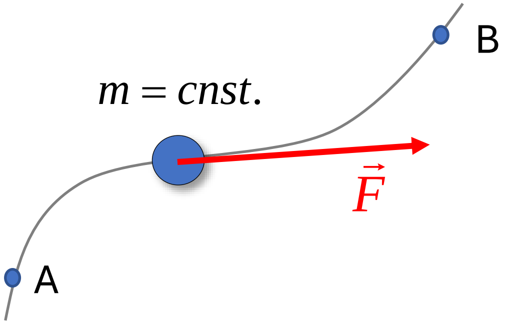
$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k} = dz\vec{k} - \text{displacement only } \| \text{ with OZ}$$

$$\boxed{L = \int_{h_1}^{h_2} -mg\ dz = mg(h_1 - h_2) = E_{p1} - E_{p2} = -\Delta E_p}$$

The work produced by the gravitational force is **equal to the difference of the potential energies**

1.10. Conservation of the total energy

The work and the variation of the kinetic energy



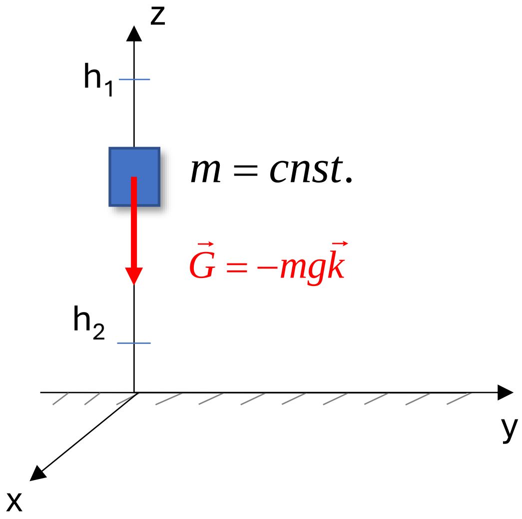
$$L_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$$

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$L_{AB} = m\int_A^B \frac{d\vec{v}}{dt} \cdot d\vec{r} = m\int_A^B \vec{v} \cdot d\vec{v} = m\frac{v_B^2}{2} - m\frac{v_A^2}{2}$$

The mechanical work produced by the force F on an object of constant mass m is equal to the increase in its kinetic energy

Conservation of the total energy



Combining the above result for mechanical work with that obtained in the case of gravitational force, we obtain:

$$L_{12} = m\frac{v_2^2}{2} - m\frac{v_1^2}{2} - \text{written as kinetic energy variation;}$$

$$L_{12} = mgh_1 - mgh_2 - \text{written as potential energy variation;}$$

$$mgh_1 + m\frac{v_1^2}{2} = mgh_2 + m\frac{v_2^2}{2}$$

The total energy in free fall is conserved

1.11. The Power (P)

- Power is the physical quantity that shows the mechanical work produced by a force in a unit of time. There are two types of power: instantaneous power and average power.

- **Instantaneous power** produced by the force F during the elementary time interval dt is defined as:

$$P = \frac{dL}{dt}$$

dL= $\vec{F} \cdot d\vec{r}$-the elementary work during dt

- **The average power** produced by the force F during the finite time interval Δt is given as:

$$P_{avg} = \frac{L}{\Delta t}$$

L= $\int_A^B \vec{F} \cdot d\vec{r}$-the total work during the interval Δt

The measuring unit for power is Watt=W

$$1W = \frac{1J}{1s}$$

- Another unit also used for power is the Horsepower (hp): 1 hp = 736W (the power needed to lift a 75kg body to 1m in 1s - the European definition). This unit of measurement was originally established by James Watt and is close to the power exerted by an average horsepower. However, the British system uses 1HP=746W

2. The principles of Newtonian mechanics

- All classical mechanics is based on three principles, formulated by Isaac Newton in 1687:

P1 (inertia): *A body maintains its state of rest or uniform rectilinear motion as long as no resultant external forces act on it.*

P2 (force principle, fundamental law): *The rate of change of the momentum of a body is equal to the resultant force acting on that body*

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$\vec{F}$ – resultant force

$\vec{p} = m\vec{v}$ – momentum of the body

Only in the case *m=cnst.,* the 2-nd principle can be rewritten:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d\left(m\vec{v}\right)}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Attention!

$$\vec{F} = m\vec{a}$$

can be applied only to objects having **constant mass**

P3 (action and reaction):

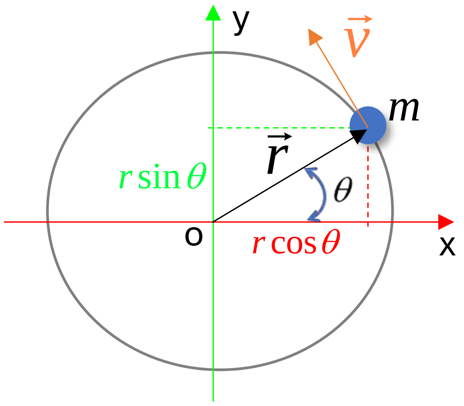
If a body A acts on the body B with a force $\boldsymbol{F_{AB}}$ *(action force) then the body B will react on the body A with a force* $\boldsymbol{F_{BA}}$ *(reaction force)**, with the same modulus but opposed sense.***

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Applications: the circular motion of the material point

- It is the movement where the magnitude r of the position vector stays constant in time and only the angle θ with the OX axis changes;
- The movement of the material point can be described here using both **cartesian** (OXY) and **polar** (r, θ) coordinates

The relation between cartesian and polar coordinates



$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$$
$$x(t) = r\cos\theta$$
$$y(t) = r\sin\theta$$

$$\begin{cases} \vec{v}(t) = \dfrac{dx(t)}{dt}\vec{i} + \dfrac{dy(t)}{dt}\vec{j} = v_x\vec{i} + v_y\vec{j} \\ v_x(t) = -r\dfrac{d\theta}{dt}\sin\theta = -r\omega\sin\theta \\ v_y(t) = r\dfrac{d\theta}{dt}\cos\theta = r\omega\cos\theta \end{cases}$$

The angular velocity:

$\omega = \dfrac{d\theta}{dt}$ – angular velocity

$[\omega]_{SI} = rad/s$

The relation between angular and tangential velocity:

$$v = \sqrt{v_x^2 + v_x^2} = r\omega$$

The kinetic energy in circular motion can be expressed in polar coordinates:

$$E_c = \frac{mv^2}{2} = \frac{mr^2\omega^2}{2} = \frac{I\omega^2}{2}$$

$I = mr^2$ – inertia of the material point around rotation axis ($kg \cdot m^2$)

II

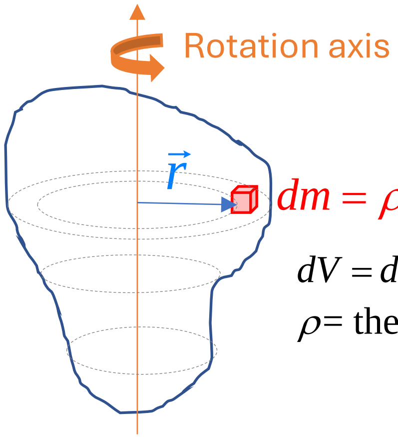
Elements of the kinematics and dynamics of a rigid body

Contents:

1. The rigid body

- All bodies around us have a finite, non-zero geometric dimension;
- Often the shape of the body also influences its motion, not just its mass. For example, a cylinder rolling freely on an inclined plane from height H will have a velocity at the bottom of the plane that depends on its mass distribution (empty or full cylinder);
- To describe the motion of physical bodies, having a finite shape, the notion of a **rigid body** is introduced

The rigid body *consists of a system of material points (volume elements) whose distance from each other remains invariable (does not change during the motion of the rigid solid).*



$dm = \rho dV$ - mass element

$dV = dxdydz$ - volume element

ρ= the mass density of the body

Obs.:

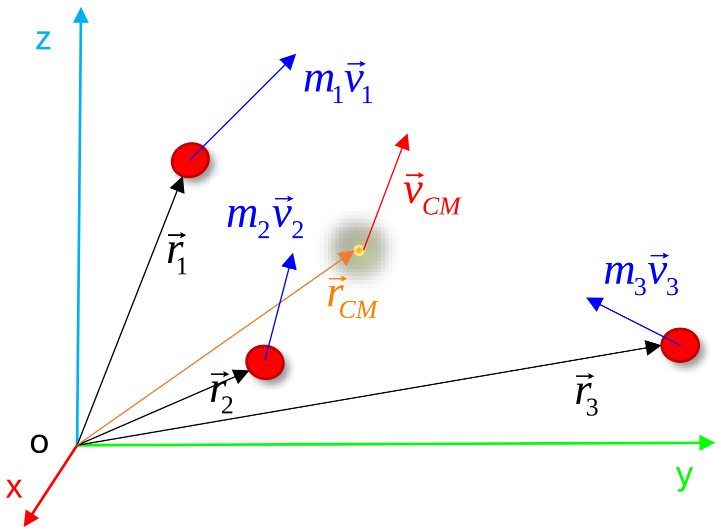
- The rotational motion of the rigid solid about an axis is equivalent to the rotational motion of the volume elements forming the rigid body;
- These elements are at different distances *r* but rotate at the same angular velocity ω

- To understand the rotational motion of the rigid solid we will first define the **center of mass** of a system of material points and describe the circular motion of a point in the system around the center of mass

2. Center of mass of a system of material points

- Consider in the following a system of material points consisting of bodies of masses $m_1 \ldots m_N$ at the positions described by the position vectors $r_1 \ldots r_N$ and moving with velocities $v_1 \ldots v_n$ (as shown in the figure).

***The center of mass** of the system is the geometric place where the entire mass of the system can be concentrated. It is indicated by the position vector:*



$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{m_{tot}}$$

N – number of material points in the system (here N=3)

The velocity of the center of mass of the system can be calculated as the time derivative of the position vector of the center of mass:

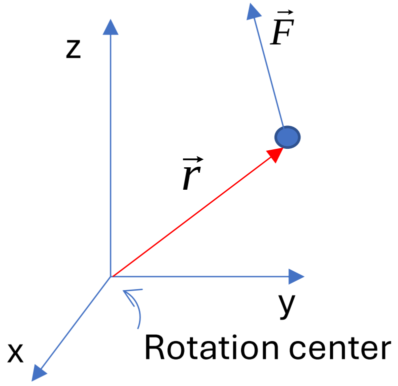
$$\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{1}{m_1 + m_2 + m_3}\frac{d}{dt}\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3\right) = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{m_{tot}} = \frac{\vec{P}_{tot}}{m_{tot}}$$

$\vec{P}_{tot} = \sum_{i=1}^{N} m_i\vec{v}_i$ -total momentum of the system

3. Rotational dynamics of the rigid solid

- Since the rigid solid consists of a system of material points, we first refer to the material point and introduce the main physical quantities involved in the dynamics of rotation of the material point around a center of rotation

3.1. The torque of a force about a rotation center

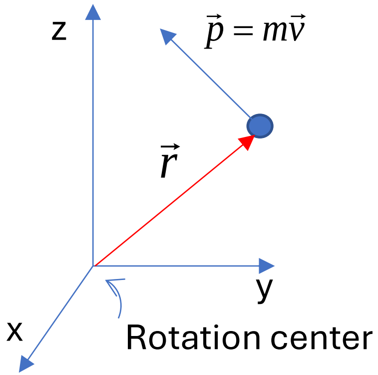


If a force **F** acts on a body at a distance **r** from a center of rotation, then that force exerts a torque

$$\vec{M} = \vec{r} \times \vec{F}$$ - The torque

The SI unit of torque is Nm

3.2. The angular momentum of a material point



If a body at a distance **r** from a rotating center is moving with velocity **v**, then it will have **angular momentum**

$$\vec{L} = \vec{r} \times \vec{p}$$ - The angular momentum

The SI unit of angular momentum is kgm^2/s

3.3. The second law of dynamics for the rotational motion of a material point

- Consider a material point at a distance **r** from a rotation center, displacing with the velocity **v.**
- The angular momentum of the material point is:

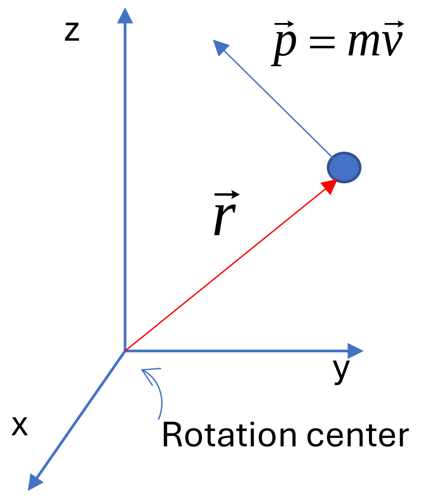
$$\vec{L} = \vec{r} \times \vec{p}$$

- The time derivative of the angular momentum is:

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \underbrace{\vec{v} \times m\vec{v}}_{\substack{=0 \text{ because} \\ \text{the vectors} \\ \textit{are parallel}}} + \vec{r} \times \vec{F} = 0 + \vec{r} \times \vec{F} = \vec{M}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{M}}$$

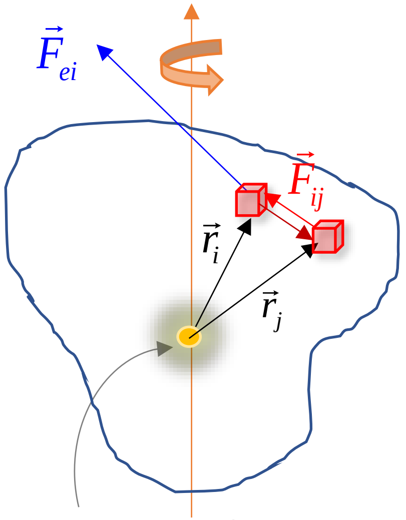
-the second law of dynamics for the material point found in rotation



Notes:

- This law is analogous Newton's 2nd law of translation;
- If **M**=0 then **L**=cnst., i.e. angular momentum is conserved.

3.4. The 2nd law of dynamics for the rigid body



Center of rotation (does not have to be the center of mass)

The rigid body is made of a system of material points with the masses m_i displacing with the velocities $\mathbf{v}_i$

$$\vec{L}=\sum_{i=1}^{N}\vec{r}_i\times\vec{p}_i$$

The total angular momentum of the rigid body is the sum of the individual angular momenta:

By differentiating the total kinetic momentum versus time, one gets:

$$\frac{d\vec{L}}{dt}=\frac{d}{dt}\left(\sum_{i=1}^{N}\vec{r}_i\times\vec{p}_i\right)=\sum_{i=1}^{N}\frac{d\vec{r}_i}{dt}\times\vec{p}_i+\sum_{i=1}^{N}\vec{r}_i\times\frac{d\vec{p}_i}{dt}=\sum_{i=1}^{N}\underbrace{\vec{v}_i\times m\vec{v}_i}_{=0}+\sum_{i=1}^{N}\vec{r}_i\times\left(\vec{F}_{ei}+\sum_{j=1}^{N}\vec{F}_{ji}\right)=\sum_{i=1}^{N}\vec{r}_i\times\vec{F}_{ei}+\underbrace{\sum_{i,j=1}^{N}\vec{r}_i\times\vec{F}_{ji}}_{=0}=\vec{M}_e$$

because:

$$\sum_{i,j=1}^{N}\vec{r}_i\times\vec{F}_{ji}\underset{\text{we note j instead of i}}{=}\sum_{i,j=1}^{N}r_j\times F_{ij}\Rightarrow\sum_{i,j=1}^{N}r_i\times F_{ji}=\frac{1}{2}\sum_{i,j=1}^{N}\left(r_i\times F_{ji}+r_j\times F_{ij}\right)\underset{F_{ij}=-F_{ji}\text{ principe 3}}{=}\frac{1}{2}\sum_{i,j=1}^{N}\left(r_i-r_j\right)\times F_{ji}\underset{\substack{\text{internal forces are parallel}\\\text{with the position vector connecting}\\\text{the 2 particles}}}{=}0$$

$\vec{F}_{ei}=$ external force acting on particle "i";

$\sum_{j=1}^{N}\vec{F}_{ji}=$ sum of forces acting on particle "i";

$\vec{M}_e=\sum_{i=1}^{N}\vec{r}_i\times\vec{F}_{ei}=$ sum of torques of the external forces acting on the rigid body

$$\frac{d\vec{L}}{dt}=\vec{M}_e$$

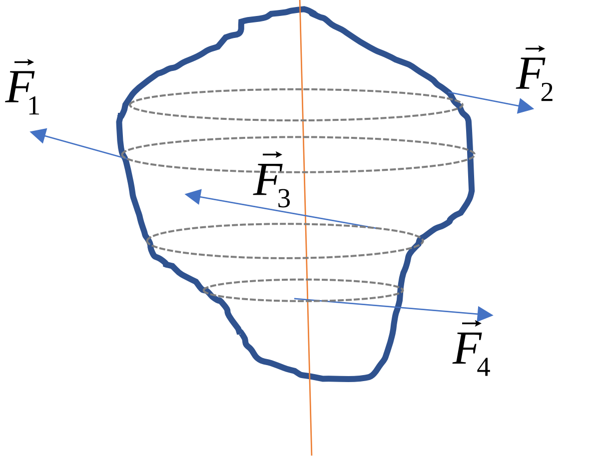
The 2nd law of dynamics for the rigid body

3.5. Equilibrium conditions of the rigid body

- 2nd law of the rigid body dynamics, together with the 2nd law of material point dynamics (here the center of mass is assimilated to a material point) allow to establish equilibrium conditions for the rigid body.

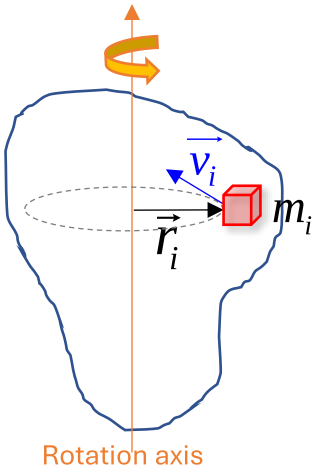
A rigid body is found ***in equilibrium*** *(uniform translation or rest, uniform rotation or rest) if two conditions are satisfied simultaneously:*

- *The resultant of the external forces is zero:* $\boxed{\sum \vec{F} = 0} \longrightarrow \frac{d\vec{P}}{dt} = 0 \Rightarrow \boxed{\vec{P} = cnst.} \Leftrightarrow m\vec{v} = cnst.$
- *The resultant of the external torque is zero:* $\boxed{\sum \vec{M} = 0} \longrightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \boxed{\vec{L} = cnst.} \Leftrightarrow I\omega = cnst.$



Both forces and torques compensate each other, so it follows that the rigid body is at equilibrium

3.6. Rotational kinetic energy of the rigid solid



We can consider the rotational kinetic energy as the sum of the energies of the volume elements forming the rigid solid

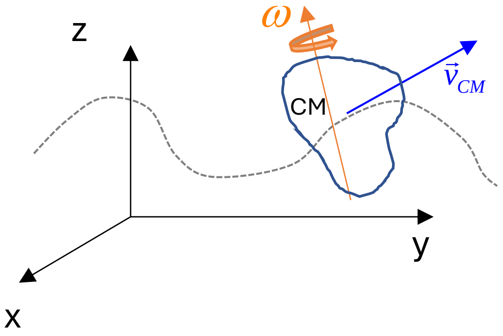
$$E_{crot} = \sum_{i=1}^{N} m_i \frac{v_i^2}{2} = \sum_{i=1}^{N} m_i \frac{\omega^2 r_i^2}{2} = \sum_{i=1}^{N} m_i r_i^2 \frac{\omega^2}{2} = I\frac{\omega^2}{2} \Leftrightarrow \boxed{E_{crot} = I\frac{\omega^2}{2}} \text{ – rotational kinetic energy}$$

$$\boxed{I = \sum_{i=1}^{N} m_i r_i^2} \text{ – moment of inertia of the rigid body}$$

Comments:

- The moment of inertia I plays in the rotational motion of the rigid body the role played by the mass m in the translational motion of the material point;
- The moment of inertia I is a measure of the rotational inertia of the rigid body (that is, a body in rotational motion is harder to stop the higher its moment of inertia is);

3.7. Total kinetic energy of a rigid body



For a rigid body rotating with angular velocity **ω** and whose center of mass moves with velocity $\boldsymbol{v_{CM}}$, the total kinetic energy is the sum of the translational and rotational kinetic energies:

$$\boxed{E_{ctot} = E_{crot} + E_{ctrans} = I\frac{\omega^2}{2} + m\frac{v_{CM}^{\ 2}}{2}}$$

$\vec{v}_{CM}$ – velocity of the center of mass

ω – angular velocity

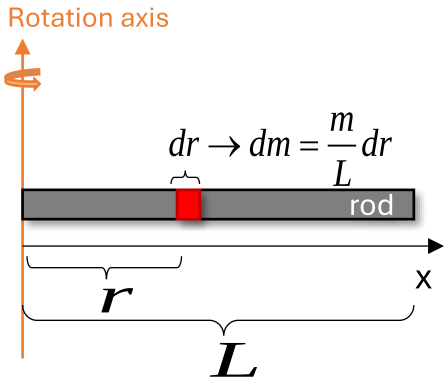
I - moment of inertia

m- mass of the rigid body

3.8. Calculation of the moment of inertia of a bar rotating about one end

- Moment of inertia I plays an essential role in rotational motion. We previously observed that: $I = \sum_{i=1}^{N} m_i r_i^2$

- To illustrate the calculation of the moment of inertia I, we consider a rod having length L and mass m that rotates around one end. The element m_i of the rigid solid at distance r_i from the axis of rotation is replaced by an *infinitesimal element dm* at variable distance *r*.



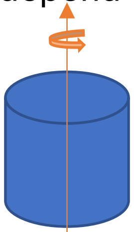
The infinitesimal moment of inertia dI of a length element dr containing a mass dm is given by:

$$dI = r^2 dm \qquad \text{where } dm = \frac{m}{L} dr \text{ - the mass element} \quad \Rightarrow \quad dI = \frac{m}{L} r^2 dr$$

The total moment of inertia is calculated as the **infinite sum** of such elementary moments of inertia, i.e. **as an integral:**

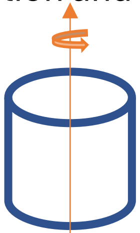
$$I = \sum_{\infty} dI = \int_0^M r^2 dm \quad \to \quad \mathrm{I} = \frac{m}{L} \int_0^L r^2 dr = m \frac{L^2}{3}$$

- Other examples of moments of inertia (they can be calculated in a similar way; one can observe that they depend on the mass distribution and the position of the rotation axis relative to the body)



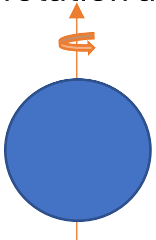
$I = \frac{mr^2}{2}$

Full cylinder



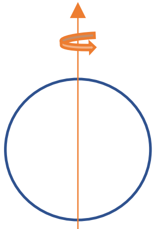
$I = mr^2$

Empty cylinder



$I = \frac{2}{5} mr^2$

Full sphere



$I = \frac{2}{3} mr^2$

Empty sphere

4. Translation-rotation analogy

There is a perfect analogy between rotation and translation. Thus, the laws governing rotational motion have the same form as those governing translational motion, except that instead of the physical quantities of translation, equivalent quantities are used.

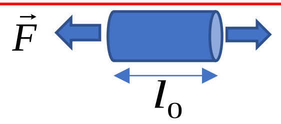
Translation	Rotation
$\vec{r}(t)$ – position vector	$\theta(t)$ - angular coordinate
$\vec{v}(t)=\frac{d\vec{r}(t)}{dt}$ – velocity vector	$\omega=\frac{d\theta(t)}{dt}$ - angular velocity
$\vec{a}(t)=\frac{d\vec{v}(t)}{dt}$ – acceleration vector	$\varepsilon=\frac{d\omega(t)}{dt}$ – angular acceleration
m – mass	I - momentum of inertia
$\vec{p}=m\vec{v}$ -momentum	$\vec{L}=I\vec{\omega}$ - angular momentum
$\vec{F}$ - force	$\vec{\mathrm{M}}$ - torque
$\frac{d\vec{p}}{dt}=\vec{F}$ - 2nd law	$\frac{d\vec{L}}{dt}=\vec{M}$ - 2nd law
$E_c=m\frac{v^2}{2}$ - kinetic energy	$E_{crot}=I\frac{\omega^2}{2}$ - kinetic energy
$P=Fv$ - power	$P=M\omega$ - power

Note: This analogy allows one to quickly deduce a formula characterizing the rigid body if one knows the formula for the translational motion.

5. Introduction to non-rigid solids. Deformations and elasticity

- In many situations, the assumption of rigidity is not valid. Different types of mechanical stress (from external forces) **can modify a material's shape and dimensions** through reversible or irreversible deformation.
- *Elastic deformation is reversible:* the material returns to its original shape once the load is removed.
- *Plastic (permanent) deformation is irreversible:* the material does not fully recover after the applied stress is released.

Hooke's law and the tensional stress of a uniform bar



$$\frac{F}{S} = E\frac{\Delta l}{l_0}$$ – Hooke's law

F – applied force; S – section of the element

Δl – element deformation; l_0 – initial length

we further denote $\sigma = \frac{F}{S}$ and $\varepsilon = \frac{\Delta l}{l} = \frac{l - l_0}{l}$

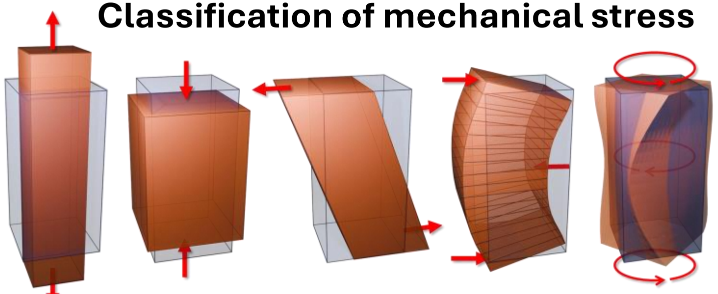
$$\sigma = E\varepsilon$$

σ = (tensile/compressive) stress (Pa)

ε = the relative (tensile/compressive) strain

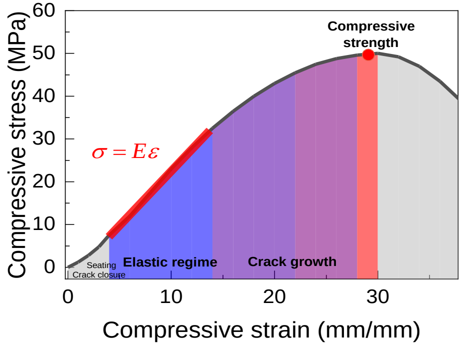
E = *Young's* modulus

Classification of mechanical stress



- For many materials, in the elastic domain, **Hooke's law** is valid both for tension and compression. For composite materials, they are not equivalent. *Such materials resist well to compressive stress but fail easier under tensile stress.*

Stress vs strain for concrete



Compressive strength

- The stress required to induce fractures into the material is known as: *breaking stress* or for compressive stress, the term used is **compressive strength** and is measured in Pa.

III

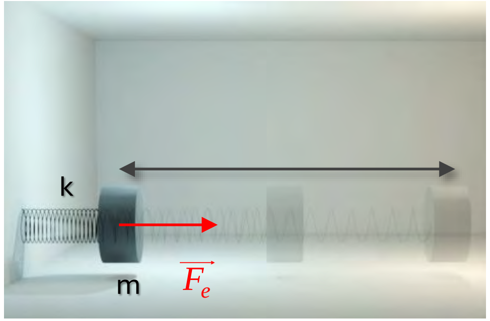
Oscillatory motion

Contents:

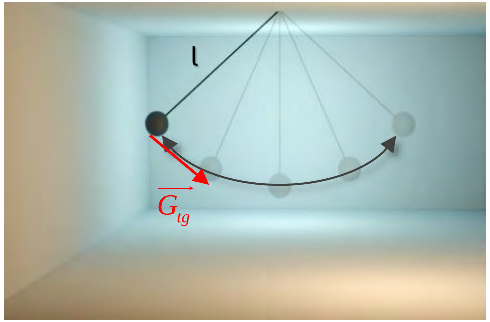
1. The harmonic oscillator
2. The damped oscillator
3. The forced oscillator. Resonance

The oscillatory motion *is a repetitive motion, produced under the action of an elastic force (restoring force), and characterized by a certain periodicity in time*

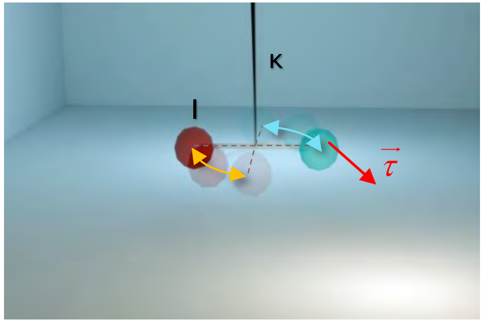
Examples of oscillations



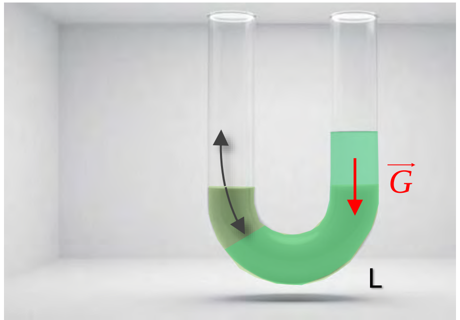
Mass-spring system



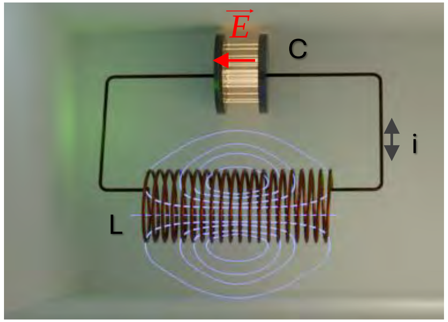
Gravitational pendulum



Torsional Pendulum

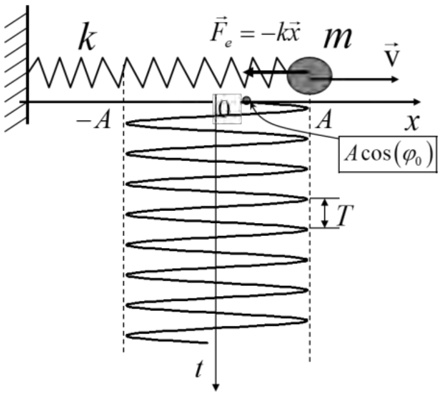


Liquid Column in “U” shaped tube



LC oscillators

1. The harmonic oscillator



- Oscillations take place **under the action of an elastic force alone**:

$$\vec{F}_e = -kx\vec{i}$$

- From Newton's 2nd Law: $\vec{F} = m\vec{a} = ma_x\vec{i}$ – onedimensional motion

$$a_x = \frac{d^2x}{dt^2}$$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

- The characteristic angular frequency is further introduced as: $\omega_0 = \sqrt{\frac{k}{m}}$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \omega_0^2 x = 0}$$

The differential equation of the harmonic oscillator

Note: The solution of the differential equation of the harmonic oscillator is obtained using the method of the characteristic equation (see math courses).

solution

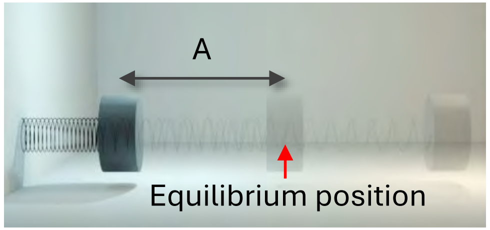
$$\boxed{x(t) = A\cos(\omega_0 t + \varphi)}$$

(the elongation of oscillations)

A – amplitude of oscilation (m);
ω_0 – angular frequency (rad/s);
φ – initial phase (rad).

1.2. The parameters of the oscillatory motion

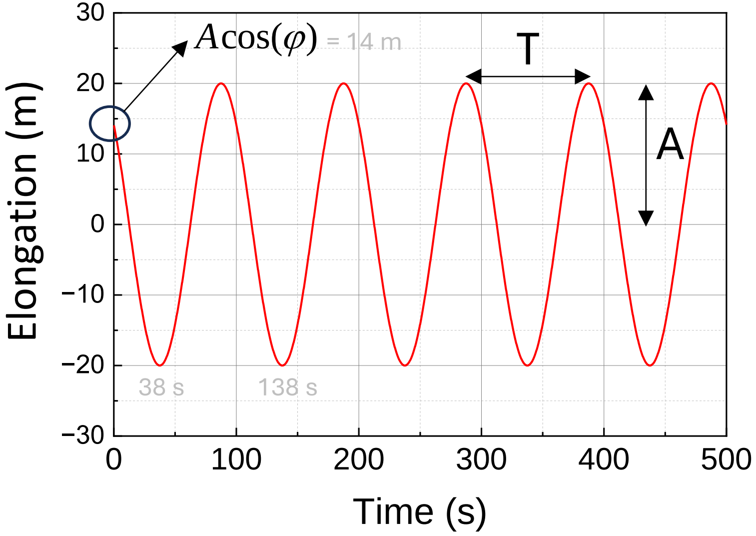
The mass-spring system:



The position of the object:

$$x(t) = A\cos(\omega_0 t + \varphi)$$

A – amplitude of oscilation (m);
ω_0 – angular frequency (rad/s);
φ – initial phase (rad).



The period of oscillation (T)

Definition: *The period represents the time needed for one complete oscillation*

$$\Leftrightarrow x(t) = x(t+T)$$
$$x(t) = A\cos(\omega_0 t + \varphi)$$

$$A\cos(\omega_0 t + \varphi) = A\cos[\omega_0 (t+T) + \varphi]$$
$$\Leftrightarrow \omega_0 (t+T) + \varphi = \omega_0 t + \varphi + 2\pi$$
$$\Leftrightarrow \omega_0 T = 2\pi$$

$$\Rightarrow T = \frac{2\pi}{\omega_0}$$

The measuring unit for period is **s**=second

The frequency of oscillation (ν)

Definition: *Frequency gives the number of complete oscillations in a second:*

$$\nu = \frac{1}{T}$$

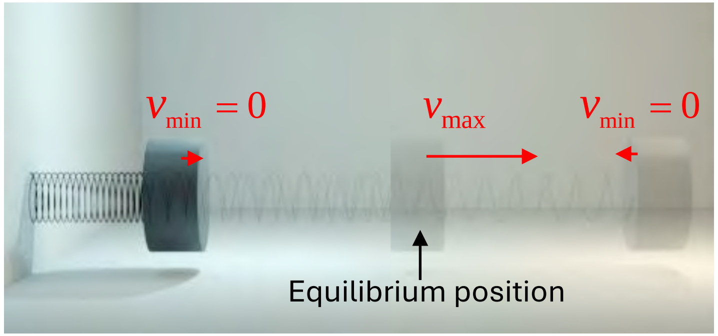
The measuring unit for frequency is **Hz**=Hertz 1Hz = 1 oscillation/s

The velocity of harmonic oscillator(v)

The instantaneous velocity of the harmonic oscillator is calculated as the derivative of the elongation with respect to time

$$v = \frac{dx(t)}{dt} = -\omega_0 A\sin\left(\omega_0 t + \varphi\right) = -v_{\max} \sin\left(\omega_0 t + \varphi\right)$$

$$where: \ v_{\max} = \omega_0 A$$



The total energy of the harmonic oscillator (E_{tot})

$$E_{tot} = E_c + E_{pe} = m\frac{v^2}{2} + k\frac{x^2}{2}$$

$$v = -\omega_0 A\sin\left(\omega_0 t + \varphi\right)$$

$$x = A\cos\left(\omega_0 t + \varphi\right)$$

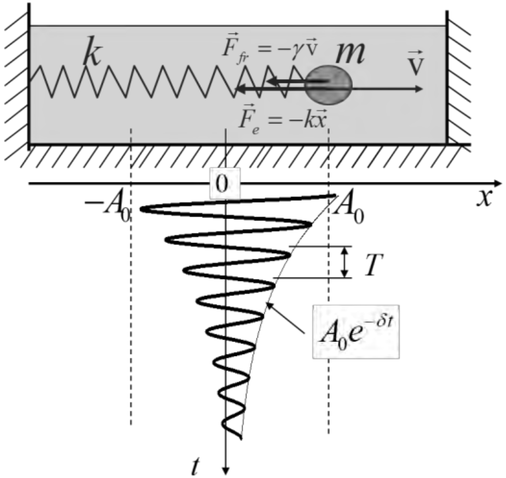
$$E_{tot} = m\frac{\omega_0^{\,2} A^2}{2}\sin^2\left(\omega_0 t + \varphi\right) + k\frac{A^2}{2}\cos^2\left(\omega_0 t + \varphi\right)$$

$$k = m\omega_0^{\,2} - \textit{the}\ \text{defining equation for the characteristic frequency}$$

$$\boxed{E_{tot} = m\frac{\omega_0^{\,2} A^2}{2} = k\frac{A^2}{2}}$$

Note: The total energy of the harmonic oscillator remains constant over time and is divided into kinetic and elastic potential energies.

2. The damped oscillator



- Damped oscillations occur in a system if a **frictional force** (resistance) acts on the oscillating body **in addition to the elastic force**:

$\vec{F}_e = -kx\vec{i}$ – elastic force

$\vec{F}_R = -\gamma\vec{v} = -\gamma v_x\vec{i} = -\gamma\dfrac{dx}{dt}\vec{i}$ – friction force proportional with speed

$\vec{F} = \vec{F}_e + \vec{F}_R$ – total force acting on the body

- By applying Newton's 2nd law: $\vec{F} = m\vec{a} = ma_x\vec{i} = m\dfrac{d^2x}{dt^2}\vec{i}$

(one dimensional motion)

$$\Rightarrow \frac{d^2x}{dt^2} + 2\frac{\gamma}{2m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$\omega_0 = \sqrt{\dfrac{k}{m}}$ – *the natural*, characteristic angular frequency

$\delta = \dfrac{\gamma}{2m}$ – the damping coefficient

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = 0}$$

The differential equation of a damped oscillator

solution

$$\boxed{x(t) = A_0e^{-\delta t}\cos(\omega t + \varphi)}$$

The elongation of the damped oscillator

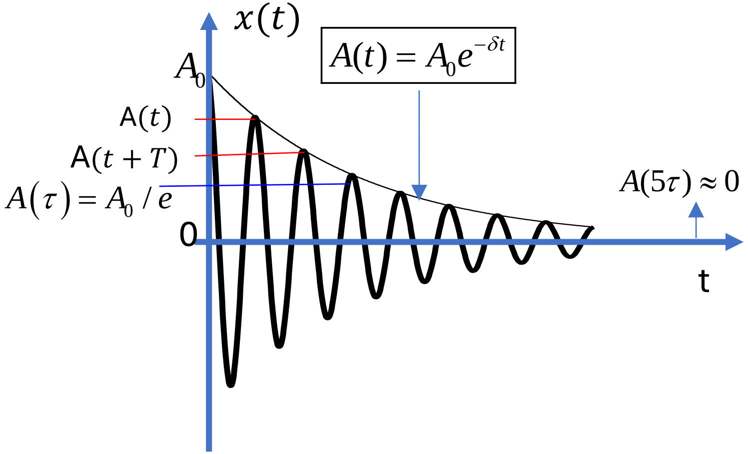
$A_0e^{-\delta t}$ – amplitude of oscillation;

$\omega = \sqrt{\omega_0^2 - \delta^2}$ – angular frequency of oscillation;

φ – initial phase.

Note: The solution of the differential equation of the damped oscillator is obtained using the method of the characteristic equation (see math courses).

2.1. The parameters of the damped oscillator



The period of the damped oscillator (T)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \delta^2}} > T_0 = \frac{2\pi}{\omega_0}$$

(the period is bigger than the characteristic period)

The frequency (ν)

$$\nu = \frac{1}{T} = \frac{\sqrt{\omega_0^2 - \delta^2}}{2\pi}$$

(smaller than the own frequency: $\nu_0 = \frac{1}{T_0}$)

The Logarithmic decrement of the attenuation (Δ)

- Indicates the degree of attenuation of the amplitude of oscillations over a period.

$$\Delta = \ln \frac{A(t)}{A(t+T)}$$

- It is defined by the relation:

$$A(t) = A_0 e^{-\delta t}$$

$$A(t+T) = A_0 e^{-\delta(t+T)}$$

$$\Rightarrow \Delta = \delta T$$

The relaxation time (τ)

- The time after which the amplitude of the oscillations is **reduced by e=2.718** times

$$A(\tau) = \frac{A_0}{e}$$

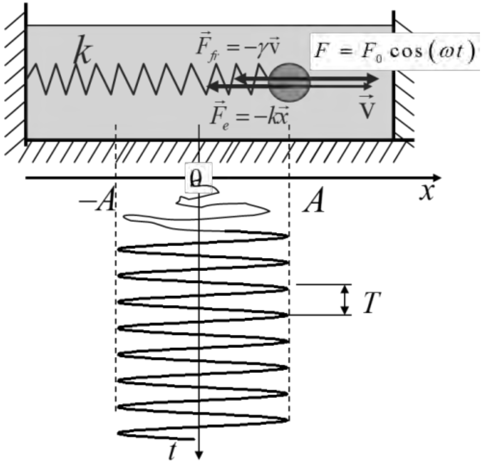
$$A(\tau) = A_0 e^{-\delta\tau}$$

$$\Rightarrow \tau = \frac{1}{\delta}$$

- Using relaxation time, the elongation of a damped oscillator can be written as:

$$x(t) = A_0 e^{-\frac{t}{\tau}} \cos(\omega t + \varphi)$$

3. The forced oscillator



- **Forced oscillations** occur in a system if an **external, periodic force** acts on the oscillating body in addition to the **elastic** and **frictional** (resistance) forces:

$\vec{F}_e = -kx\vec{i}$ – elastic force

$\vec{F}_R = -\gamma\vec{v} = -\gamma v_x\vec{i} = -\gamma\frac{dx}{dt}\vec{i}$ – resistance force, proportional with the velocity

$\vec{F}_{ext} = F_0\cos(\omega t)\vec{i}$ – periodical external force, with angular frequency ω and amplitude F_0

$\vec{F} = \vec{F}_e + \vec{F}_R + \vec{F}_{ext}$ – total force on the oscillating body

- By applying **Newton's 2nd law:** $\vec{F} = m\vec{a} = ma_x\vec{i} = m\frac{d^2x}{dt^2}\vec{i}$ – 1D motion

$$m\frac{d^2x}{dt^2} + 2\frac{\gamma}{2m}\frac{dx}{dt} + kx = F_0\cos\omega t$$

$\omega_0 = \sqrt{\frac{k}{m}}$ – characteristic angular frequency

$\delta = \frac{\gamma}{2m}$ – damping coefficient

$$\frac{d^2x}{dt^2} + 2\delta\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t)$$

The differential equation of the forced oscillator

solution

$$x(t) = A(\omega)\cos(\omega t + \varphi)$$

The elongation of oscillation

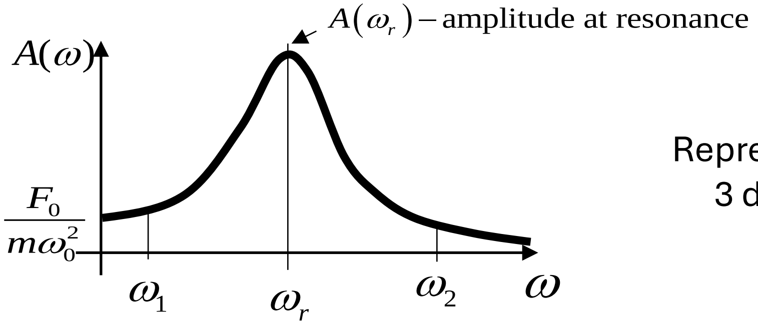
$A(\omega) = \frac{F_0}{m}\frac{1}{\sqrt{(\omega^2 - \omega_0^2)^2 + 4\delta^2\omega^2}}$ – the amplitude of forced oscillations

$\tan\varphi = \frac{2\delta\omega}{\omega^2 - \omega_0^2}$ – *the* initial phase

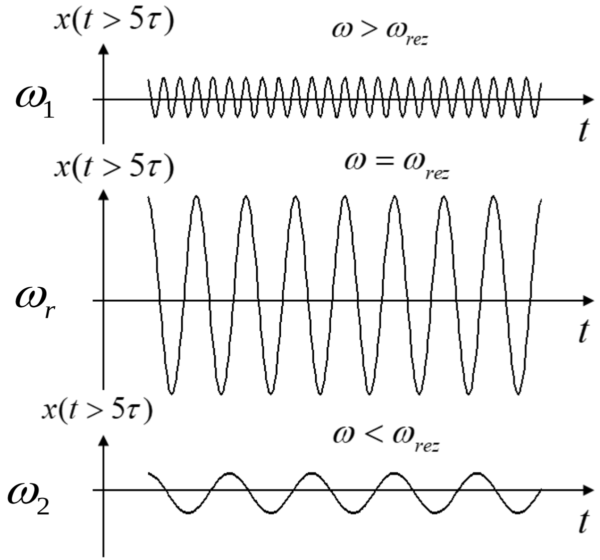
Notes: The solution of the differential equation of the damped oscillator is obtained as *the sum of the solution of the homogeneous equation and a particular solution* (see the math courses). The solution of the homogeneous equation smooths out over time, canceling itself out. So *only the particular solution remains.*

3.1. Resonance

- The **resonance phenomenon** consists of **maximizing the amplitude** of the forced oscillations for an **external force frequency close to the oscillator's natural frequency**. The frequency at which the amplitude of the oscillations becomes **maximum** is called the **resonance frequency**.



Representing oscillations at 3 different frequencies



- **The resonance frequency** is obtained from the condition of maximizing the amplitude

$$\left.\frac{dA(\omega)}{d\omega}\right|_{\omega=\omega_r} = 0$$

$$A(\omega) = \frac{F_0}{m}\frac{1}{\sqrt{\left(\omega^2-\omega_0^2\right)^2 + 4\delta^2\omega^2}}$$

$$\Rightarrow \boxed{\omega_r = \sqrt{\omega_0^2 - 2\delta^2}} \text{ – resonance angular frequency}$$

$$A(\omega_r) = \frac{F_0}{m}\frac{1}{2\delta\sqrt{\omega_0^2-\delta^2}} \text{ – amplitude of oscillations at resonance}$$

IV

Elastic waves

Contents:

1. Classifications of waves
2. The equation of plane harmonic waves
3. The energy carried by elastic waves
4. Wave interference
5. Wave diffraction
6. Reflection and refraction of waves
7. Mechanisms of attenuation
8. The Doppler effect and applications

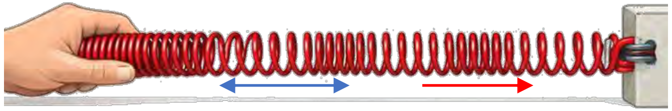
1. Classifications of waves

- *Waves are oscillations that propagate in space from one point to another via a force field*

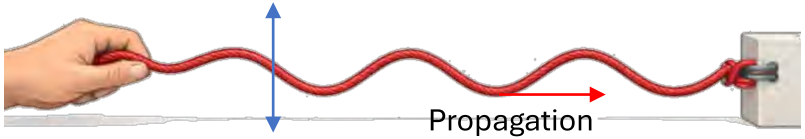
Elastic force field	→	Elastic waves
Pressure forces	→	Sound waves in fluids
Electric and magnetic force field	→	Electromagnetic waves

- Waves can also be classified by the direction of wave propagation relative to the direction of particle oscillation

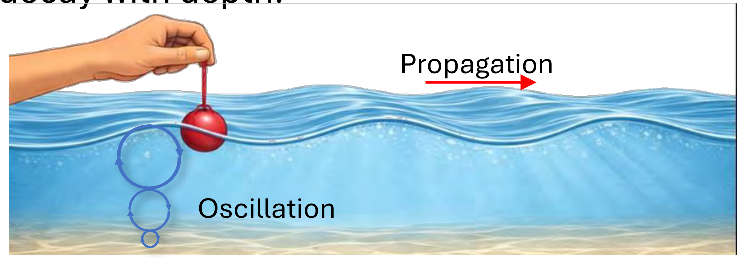
✓ **Longitudinal waves** = direction of oscillation parallel to direction of propagation:



✓ **Transverse waves** = direction of oscillation perpendicular to the direction of propagation:



✓ **Surface waves** = particle motion is elliptical. They occur at the boundary between two media and decay with depth:



	Solids	Liquids	Gases
Transverse	Shear waves	-	-
Longitudinal	Compressional waves	Sound	Sound
Surface waves	Love, (solid/solid) Rayleigh (solid/gas)	Sea waves, Capillary waves, (water/air)	Kelvin–Helmholtz (gas/gas)

- **Definition:** *The wavelength is the distance travelled by the wave in a period T*

$$\boxed{\lambda = cT}$$

λ – wavelength

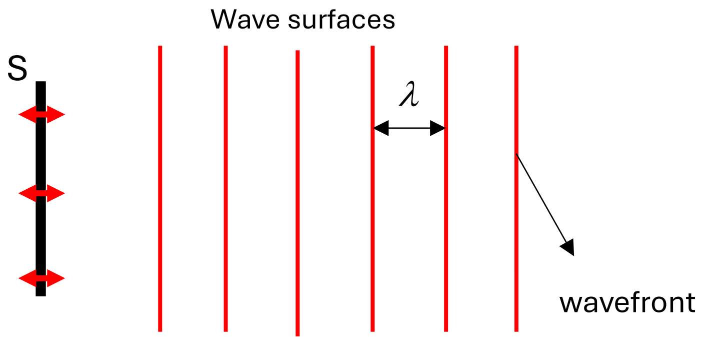
c – *propagation* speed

T – *period*

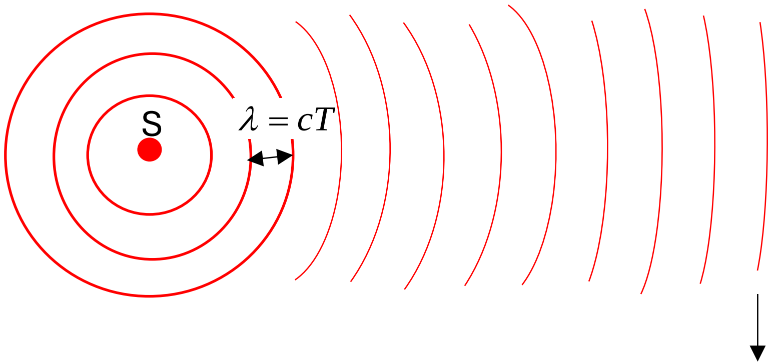
- **The wavefront** = *the geometric location of points in space oscillating in phase at a given time instant*

- The waves can be classified by considering the shape of the wavefront (i.e. plane waves, spherical waves, etc.)

Plane waves = wave surfaces are parallel planes



Spherical waves = wave surfaces are concentric spheres

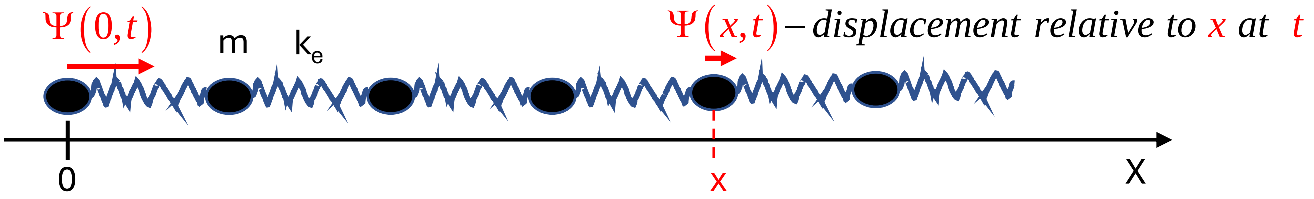


At a great distance from the source, spherical waves turn into plane waves

Simmulations:
https://phet.colorado.edu/en/simulations/category/physics

2. The equation of plane harmonic waves

- Consider a harmonic perturbation propagating in a system of springs and bodies that approximates a one-dimensional medium (body = molecule or particle; spring = interaction forces):



- The displacement from the equilibrium position of the body located in the position 0: (harmonic oscillations of pulsation ω and amplitude A are assumed)

$$\Psi(0,t) = A\cos\omega t$$

- The displacement from the equilibrium position of the body located in position x is delayed by Δt:

$$\Psi(x,t) = \Psi(0,t-\Delta t)$$

- This delay is the time the wave travels from 0 to x:

$$\Delta t = \frac{x}{c}$$

where c = the propagation speed of the wave

Wave function for plane harmonic waves:

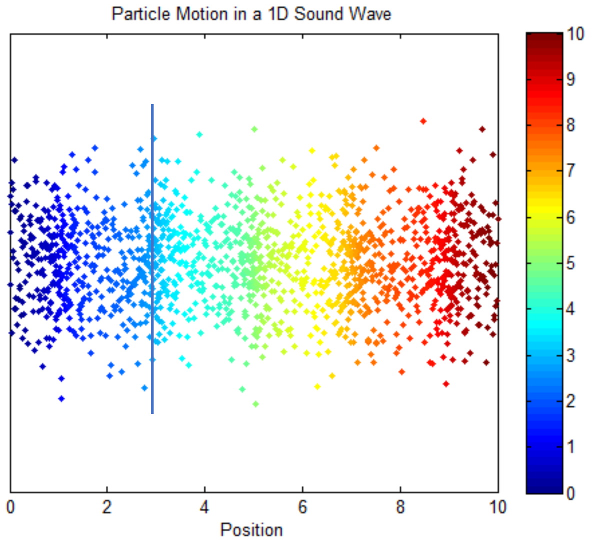
$$\Psi(x,t) = A\cos\omega\left(t - \frac{x}{c}\right)$$

Note: The wave function Ψ(x,t) tells us how much a particle, initially found at x, displaces relative to its equilibrium position (along a specific axis, here OX) at time t. The equilibrium position refers to the location of the particle at x when the medium is unperturbed by the wave.

2.1. The Differential equation of plane harmonic waves

- The wave function $\Psi(x,t) = A\cos\omega\left(t - \frac{x}{c}\right)$ is the solution to a second order differential equation with partial derivatives.

- To find the differential equation for waves we will partially derivate the wave function with respect to both t and x



$$\frac{\partial\Psi(x,t)}{\partial t} = -A\omega\ \sin\omega\left(t - \frac{x}{c}\right) \qquad \frac{\partial\Psi(x,t)}{\partial x} = A\frac{\omega}{c}\ \sin\omega\left(t - \frac{x}{c}\right)$$

$$\frac{\partial^2\Psi(x,t)}{\partial t^2} = -A\omega^2\ \cos\omega\left(t - \frac{x}{c}\right) \qquad \frac{\partial^2\Psi(x,t)}{\partial x^2} = -A\frac{\omega^2}{c^2}\ \cos\omega\left(t - \frac{x}{c}\right)$$

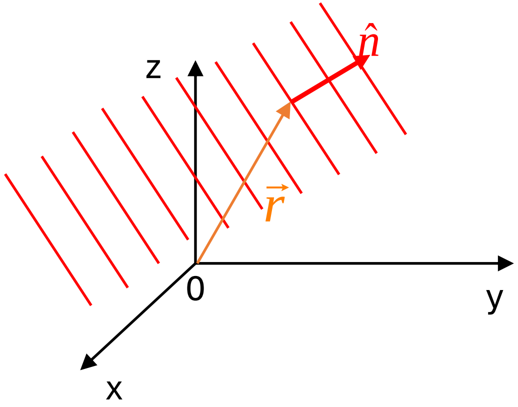
$$\frac{\dfrac{\partial^2\Psi(x,t)}{\partial t^2}}{\dfrac{\partial^2\Psi(x,t)}{\partial x^2}} = c^2 \Leftrightarrow \boxed{\frac{\partial^2\Psi(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\Psi(x,t)}{\partial t^2} = 0}$$

The differential equation of plane harmonic waves with propagation direction II OX (longitudinal waves)

- The solution to this equation can be expressed in several forms considering that:

$$\left.\begin{aligned} &\omega = \frac{2\pi}{T} - \text{the angular frequency} \\ &\lambda = cT - \text{the wavelength} \\ &k = \frac{2\pi}{\lambda} - \text{the wavenumber} \end{aligned}\right\} \quad \Psi(x,t) = A\cos\omega\left(t - \frac{x}{c}\right) \Leftrightarrow \left\{\begin{aligned} &\boxed{\Psi(x,t) = A\cos(\omega t - kx)} \\ &\boxed{\Psi(x,t) = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)} \end{aligned}\right.$$

2.2. The case of plane harmonic wave propagation in an arbitrary direction



$$\vec{k}=\frac{2\pi}{\lambda}\hat{n}-\textit{the}\ \text{wavevector}$$

$\hat{n}-\textit{the}$ unit vector indicating propagation direction

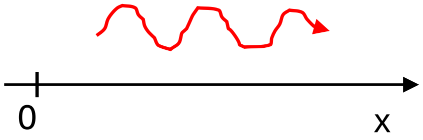
The differential equation of the wave propagating in an arbitrary direction indicated by the wave vector **k**

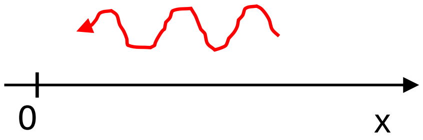
$$\boxed{\nabla^2\Psi\left(\vec{r},t\right)-\frac{1}{c^2}\frac{\partial^2\Psi\left(\vec{r},t\right)}{\partial t^2}=0} \Rightarrow \Psi\left(\vec{r},t\right)=A\cos\left(\omega t-\vec{k}\cdot\vec{r}\right)$$

The solution of the differential equation

$$\textit{where}\quad \nabla^2 \underset{noted}{=} \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\ \text{(The Laplace operator)}\quad \textit{or}\quad \frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta \underset{noted}{=} \square\ \text{(The D'Alambert operator)}$$

2.3. Progressive waves and regressive waves





progressive=displacement parallel with OX

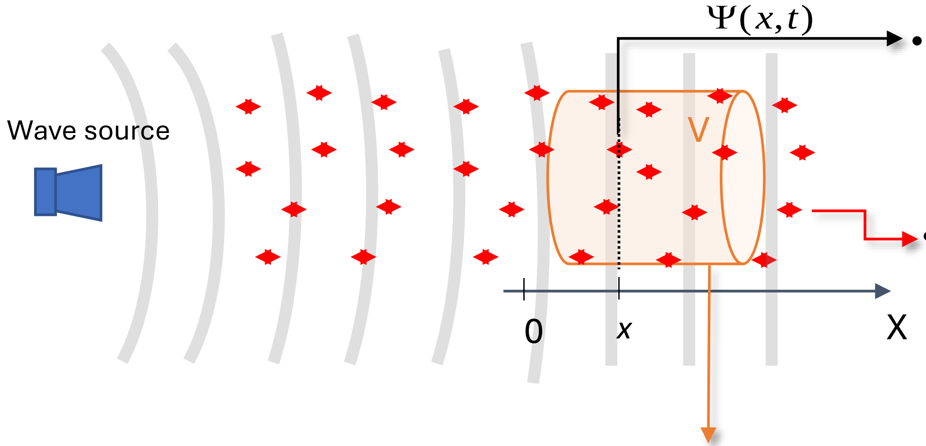
$$\Psi_p\left(x,t\right)=A\cos\left(\omega t-kx\right)$$

regressive=displacement antiparallel with OX

$$\Psi_r\left(x,t\right)=A\cos\left(\omega t+kx\right)$$

3. The energy carried by elastic waves

- Since the elastic wave moves the particles of the medium through which it propagates, it follows that it carries energy. The energy carried is not accompanied by mass transport because particles only oscillate around their equilibrium positions and do not travel long distances.



- The waves become plane waves at a distance from the source and the particles of the medium move relative to the equilibrium position according to the relation: $\Psi(x,t) = A\cos(\omega t - kx)$
- The particles of the medium are like harmonic oscillators oscillating with angular frequency ω and amplitude A. Each particle of the medium stores energy in the presence of a wave:

$$E_c = \frac{m\omega^2 A^2}{2}$$

-The maximum kinetic energy of a harmonic oscillator

- The energy stored by the wave in volume V of the cylinder is the sum of the energies of all particles in that volume:

$$E_{ctot} = NE_c = N\frac{m\omega^2 A^2}{2}$$

$N = nV$ – nr. of particles inside V

n – particle density [particles/m^3]

$n \cdot m = \rho$ – mass density of the medium

$$\Rightarrow E_{ctot} = nm\frac{\omega^2 A^2}{2}V = \rho\frac{\omega^2 A^2}{2}V$$

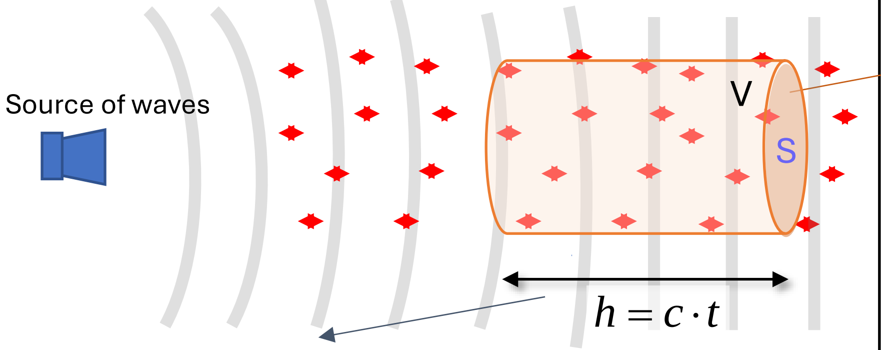
by denoting $w = \frac{E_{ctot}}{V}$

The energy density of the wave

$$\Rightarrow w = \rho\frac{\omega^2 A^2}{2} \qquad [w]_{SI} = \frac{J}{m^3}$$

3.1. The energy flux and the wave intensity

- ***The energy flux (ϕ)*** *represents the energy transported by waves through a given surface area S, in the unit time:* $\Phi = \frac{E_S}{t}$ The measuring unit for flux is: Watt (W)



Observation: only the waves found at a distance **< ct** will be able to pass through the surface S in the given time t.

- The energy passing through surface S during t is contained in the cylinder of base area S and height h=ct:

$$E_S = wV = wSct = \rho c \frac{\omega^2 A^2}{2} St$$

-consider the energy flux: $\Phi = \frac{E_S}{t}$

$$\Rightarrow \Phi = \rho c \frac{\omega^2 A^2}{2} S$$

Note: $Z = \rho \cdot c$ **– the acoustic impedance** of the medium, $[Z] \overset{SI}{=} \frac{kg}{m^2 s} = Ry$ (from "Rayl")

$$\Phi = Z \frac{\omega^2 A^2}{2} S$$ -Energy flux of elastic waves

- ***The intensity (I)*** *of waves represents the energy carried by waves per unit area in the unit time (1s):*

$$I = \frac{E_S}{St} = \frac{\Phi}{S}$$

$$I = \rho c \frac{\omega^2 A^2}{2} = Z \frac{\omega^2 A^2}{2}$$ - The intensity of elastic waves

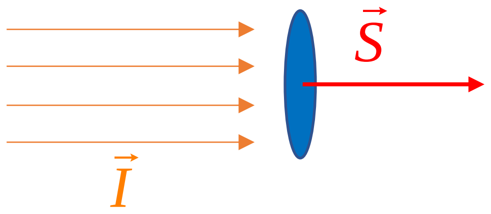
The measuring unit for intensity is: Watt/m^2 (W/m^2)

3.2. The relation between flux and intensity

Observations:

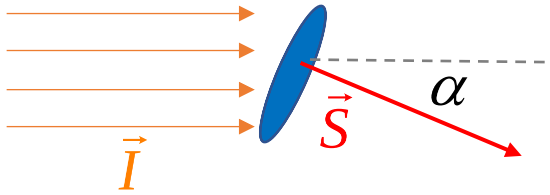
If the wave energy is constant on the surface S and it is placed perpendicular to the wave path, then we have:

$$\Phi = IS$$



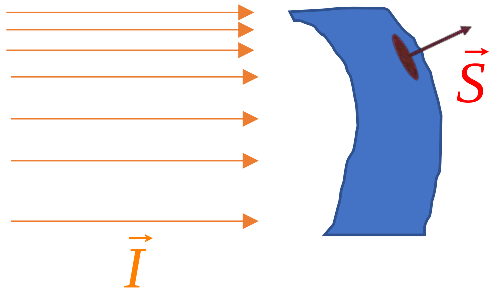
If the energy is constant on the surface but the surface is laid at a certain angle, then we have:

$$\Phi = \vec{I} \cdot \vec{S} = IS\cos\alpha$$



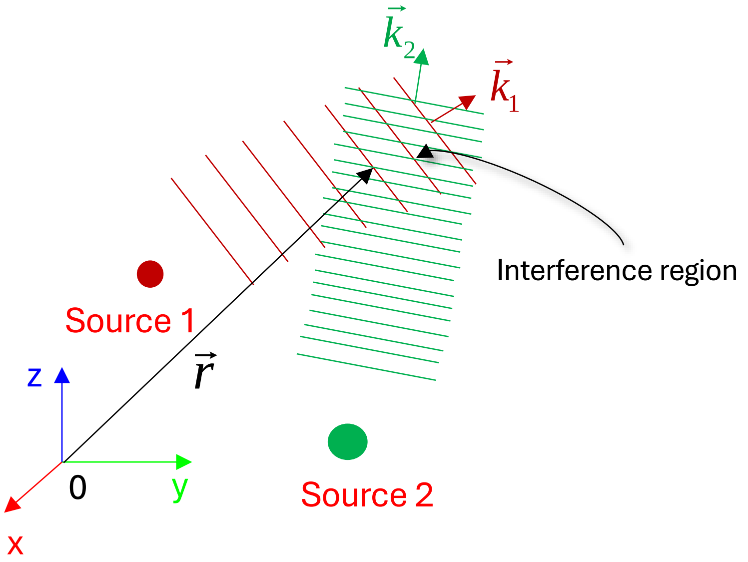
If energy falls inhomogeneously on the surface, then the flux is calculated as an integral on the surface:

$$\Phi = \iint_S \vec{I} \cdot d\vec{s}$$



4. Wave interference

- ***Interference** is the phenomenon of superposition in space of two or more waves.* The result of interference may be an increased amplitude (constructive interference) or a reduced amplitude (destructive interference).



Wave functions of the two waves in the interference region:

$$\Psi_1(\vec{r},t) = A_1 \cos(\omega_1 t - \vec{k}_1 \cdot \vec{r})$$

$$\Psi_2(\vec{r},t) = A_2 \cos(\omega_2 t - \vec{k}_2 \cdot \vec{r})$$

$A_1, A_2 - amplitudes;$
$\omega_1, \omega_2 - angular\ frequencies;$
$k_1, k_2 - wavenumbers$

The medium does not change in the presence of waves

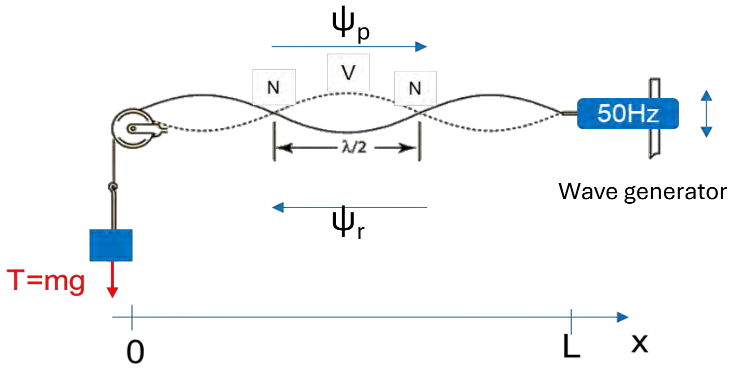
The wave function resulting from the interference of the two waves if they do not change the medium through which they propagate:

$$\Psi(r,t) = \Psi_1(\vec{r},t) + \Psi_2(\vec{r},t) = A_1 \cos(\omega_1 t - \vec{k}_1 \cdot \vec{r}) + A_2 \cos(\omega_2 t - \vec{k}_2 \cdot \vec{r})$$

Instructive simulation of interference phenomenon:
https://phet.colorado.edu/en/simulation/wave-interference

4.1. The interference of a progressive wave with a regressive wave

- Consider a vibrating string passed over a pulley and able to be moved to the other end with a certain frequency.



T-tension in the rope;
L-length of the rope;
m-mass of the body;
M-mass of the rope;
μ=M/L-linear density of the string

$c = \sqrt{\frac{T}{\mu}}$ – propagation speed of transverse waves in rope

Experimental illustration of the superposition of a progressive wave with a regressive wave:
https://www.youtube.com/watch?v=-gr7KmTOrx0

Simulation of the experiment of superimposing a progressive wave with a regressive wave:
http://www.phy.hk/wiki/j/Eng/resonanceString/resonanceString_js.htm

- The superposition between a progressive and a regressive wave passed through the string gives rise to interference phenomena:

$\Psi_p(x,t) = A\cos(\omega t - kx + \pi)$ – wave function of progressive wave;

$\Psi_r(x,t) = A\cos(\omega t + kx)$ – wavefunction of regressive wave;

Noting that: $\cos\alpha + \cos\beta = 2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2})$

- The wave resulting from interference becomes:

$$\Psi(x,t) = \Psi_p(x,t) + \Psi_r(x,t) = A\cos(\omega t - kx + \pi) + A\cos(\omega t + kx) =$$

$$= 2A\cos\left(\omega t + \frac{\pi}{2}\right)\cos\left(kx - \frac{\pi}{2}\right) \Leftrightarrow$$

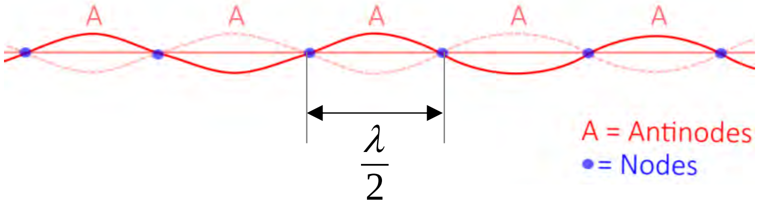
$$\Leftrightarrow \boxed{\Psi(x,t) = 2A\cos\left(kx - \frac{\pi}{2}\right)\cos\left(\omega t + \frac{\pi}{2}\right)}$$

The amplitude of the resulting wave depends on the position passing through maxima (**antinodes**) and minima (**nodes**);

The phase of the resulting wave is independent of position = **standing wave**;

Position of wave function maxima and minima

$$\Psi(x,t) = \underbrace{\boxed{2A\cos\left(kx - \frac{\pi}{2}\right)}}_{\text{amplitude of the resulting wave}} \cdot \cos\left(\omega t + \frac{\pi}{2}\right)$$



Position of maxima (antinodes)

$$\cos\left(kx - \frac{\pi}{2}\right) = \pm 1 \quad \Leftrightarrow kx - \frac{\pi}{2} = n\pi \quad (n = 0,1,2,\ldots) \Leftrightarrow$$

$$\Leftrightarrow \left.\begin{aligned} kx &= (2n+1)\frac{\pi}{2} \\ k &= \frac{2\pi}{\lambda} - wavenumber \end{aligned}\right\} \Leftrightarrow \boxed{x_n = (2n+1)\frac{\lambda}{4}}$$

Distance between two consecutive antinodes

$$x_{n+1} - x_n = [2(n+1)+1]\frac{\lambda}{4} - (2n+1)\frac{\lambda}{4} = \frac{\lambda}{2}$$

Position of minima (Nodes)

$$\cos\left(kx - \frac{\pi}{2}\right) = 0 \quad \Leftrightarrow kx - \frac{\pi}{2} = (2n+1)\frac{\pi}{2} \quad (n = 0,1,2,\ldots) \Leftrightarrow$$

$$\Leftrightarrow \left.\begin{aligned} kx &= (2n+2)\frac{\pi}{2} = (n+1)\pi \\ k &= \frac{2\pi}{\lambda} - wavenumber \end{aligned}\right\} \Leftrightarrow \boxed{x_n = (n+1)\frac{\lambda}{2}}$$

Distance between two consecutive nodes

$$x_{n+1} - x_n = (n+1+1)\frac{\lambda}{2} - (n+1)\frac{\lambda}{2} = \frac{\lambda}{2}$$

The distance between two consecutive antinodes or two consecutive nodes is λ/2. This allows an easy determination of the wavelength by directly measuring the distance between nodes or antinodes

4.2. Interference of two harmonic waves of the same amplitude but different frequency

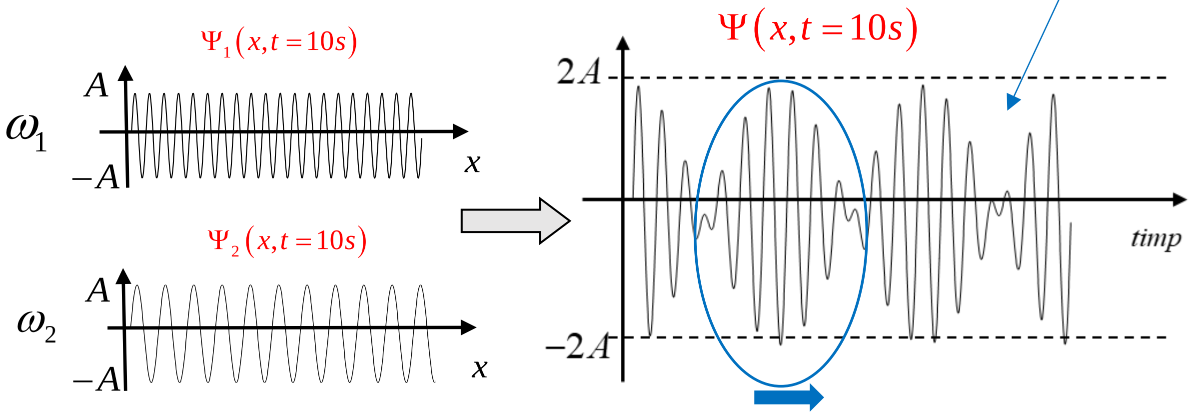
Consider below two harmonic waves of the same amplitude but different frequency moving in the same direction. The result of the superposition of the two waves is calculated as described below.

Interfering waves

Interference result

$$\left.\begin{aligned}\Psi_1(x,t) &= A\cos(\omega_1 t - k_1 x);\\ \Psi_2(x,t) &= A\cos(\omega_2 t - k_2 x);\end{aligned}\right\} \Rightarrow \Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t) = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t - \frac{k_1 - k_2}{2}x\right)\cos\left(\frac{\omega_1 + \omega_2}{2}t - \frac{k_1 + k_2}{2}x\right)$$

amplitude depends on position and time



A **wave packet** traveling at c-wave speed

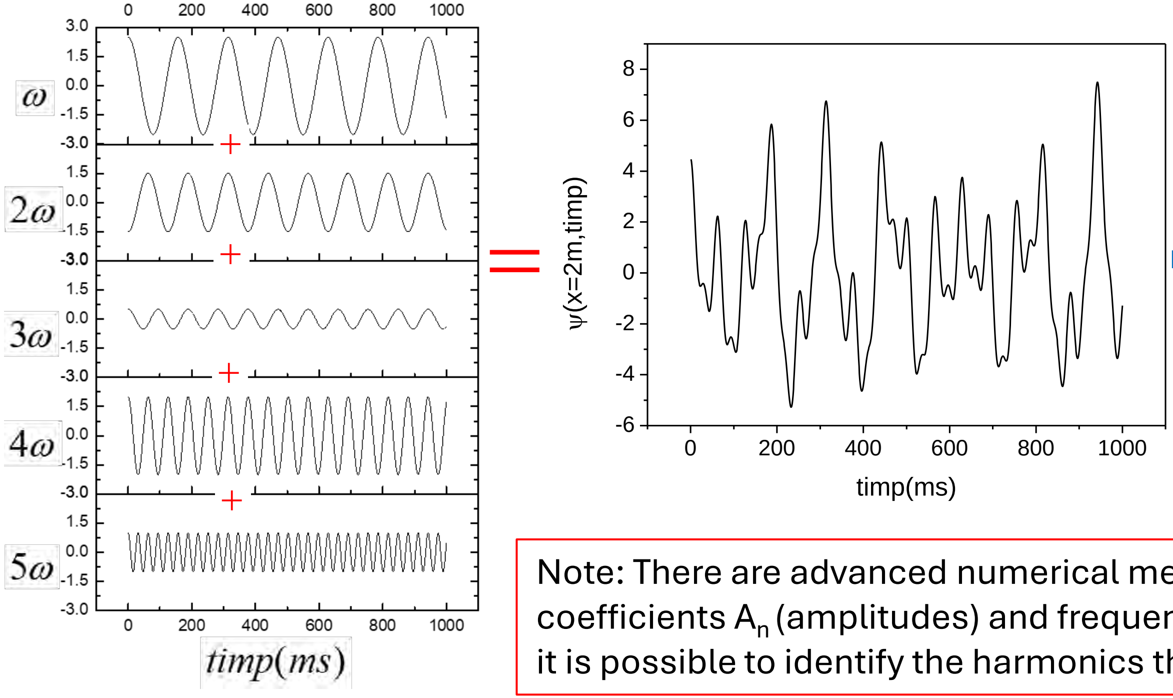
- The amplitude of the resulting wave depends on position and is in some positions it can become greater than the waves that compose it;
- So-called **groups of waves** are formed that travel through space

Note: the superposition of two harmonic waves of constant amplitude results in a non-harmonic wave whose amplitude varies in space and time and which travels in space. This wave "resembles" a real wave produced, for example, by our voice.

4.3. Decomposition of a wave into a sum of harmonic waves (Fourier analysis)

Above we considered only the case of two harmonic waves overlapping in different positions at the same instant in time. Let's see what happens if we take several harmonic waves and superimpose them in the same position (x=2m)

Overlapping harmonic waves **The wave obtained by superposition**



Any wave can be decomposed as a sum of harmonic waves of precisely determined amplitudes and frequencies, i.e.:

$$\Psi(x,t) = \sum_{n=1}^{N} A_n \cos(\omega_n t - k_n x)$$

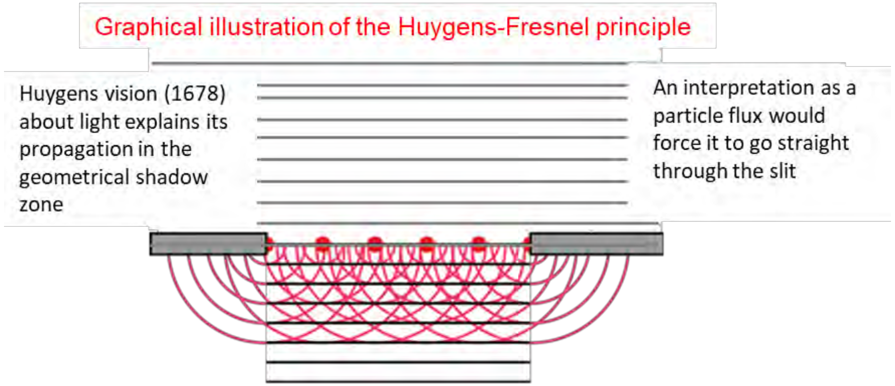
This decomposition of a signal into its harmonics is called **Fourier analysis**.

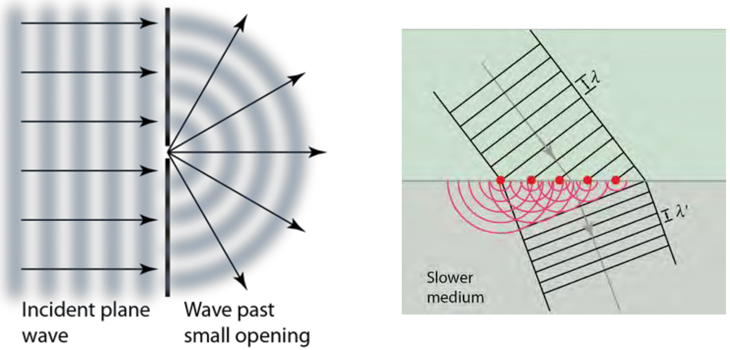
Note: There are advanced numerical methods that allow extracting the coefficients A_n (amplitudes) and frequencies from a recorded signal and thus it is possible to identify the harmonics that compose the signal.

5. Wave diffraction

- Wave diffraction is the phenomenon of waves bypassing obstacles and their penetration into the geometric shadow region (complicated to treatise mathematically, so here we deal only with physical interpretation).
- The phenomenon of diffraction becomes more important if the wavelength of the waves is greater or of the same order of magnitude as the size of the objects ($\lambda \gtrsim d$).
- The phenomenon of diffraction is explained based on the Huygens–Fresnel principle: Each point on a wavefront can be considered as the starting point of an elementary wave propagating with the same frequency and phase as the wavefront. These elementary waves overlap (interfere) and the new wavefront is created by summing up all these elementary waves.

The Huygens–Fresnel principle explains the passage of sound through small holes (e.g., the slightly opened window) as well as the refractive phenomenon of waves



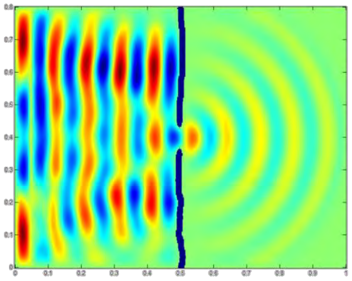


Diffraction simulation:
https://phet.colorado.edu/en/simulation/wave-interference

http://hyperphysics.phy-astr.gsu.edu/

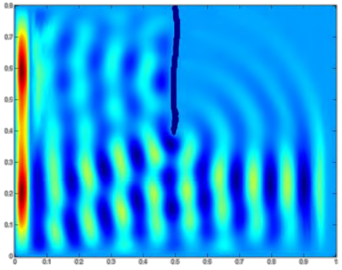
Examples of diffraction and the role of wavelength

Small opening

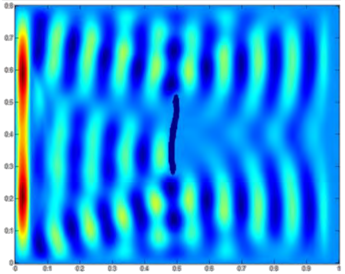


Slit opening: 0.1

Diffraction at the corner

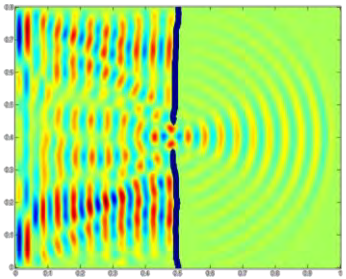


Diffraction around obstacles



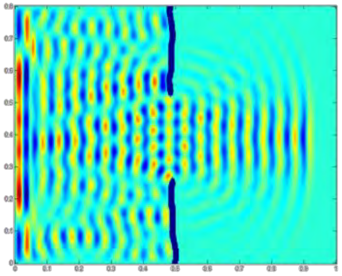
$\lambda = 0.1$ m; $\nu = 100$ Hz; $A = 2$;

Small wavelength



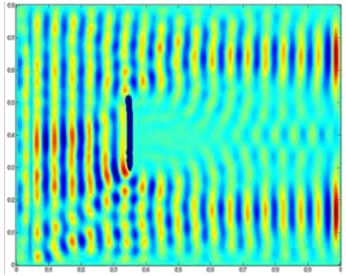
Slit opening: 0.1

Large opening



Slit opening: 0.2

Diffraction around obstacles



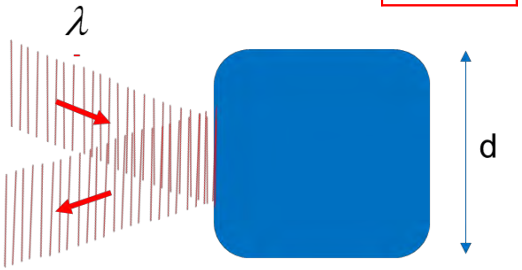
$\lambda = 0.05$ m; $\nu = 100$ Hz; $A = 2$;

Color code: **Red/Yellow** = crest (+), **Blue** = trough (-), **Green/Cyan** = equilibrium point, **Black** = obstacle

Effect of diffraction on reflection

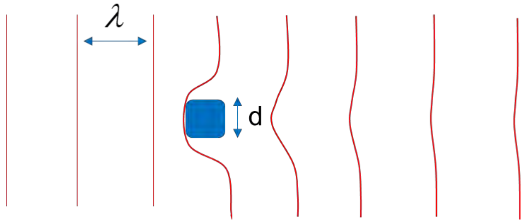
Reflection dominates

$\lambda \ll d$



Negligible reflection

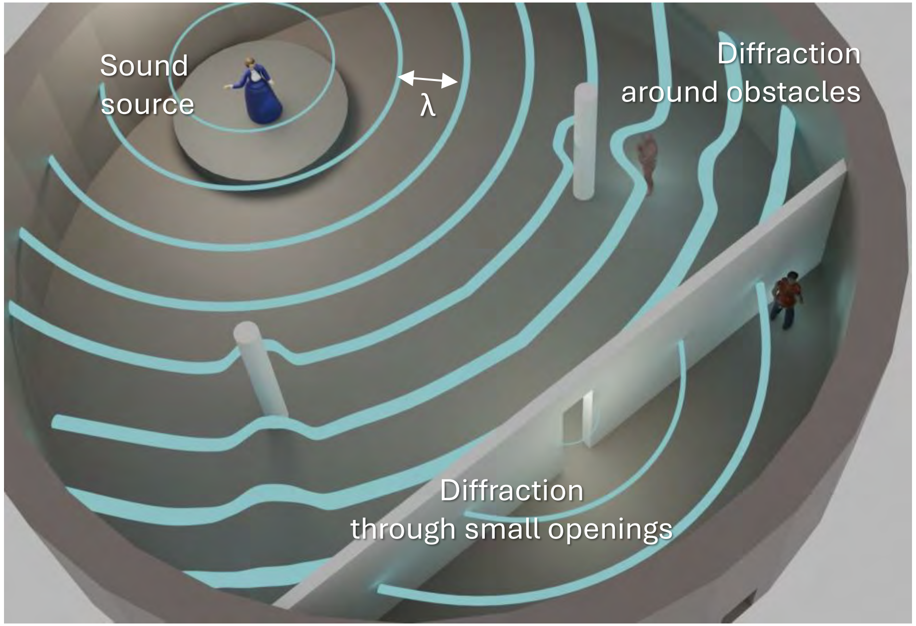
$\lambda \geq d$



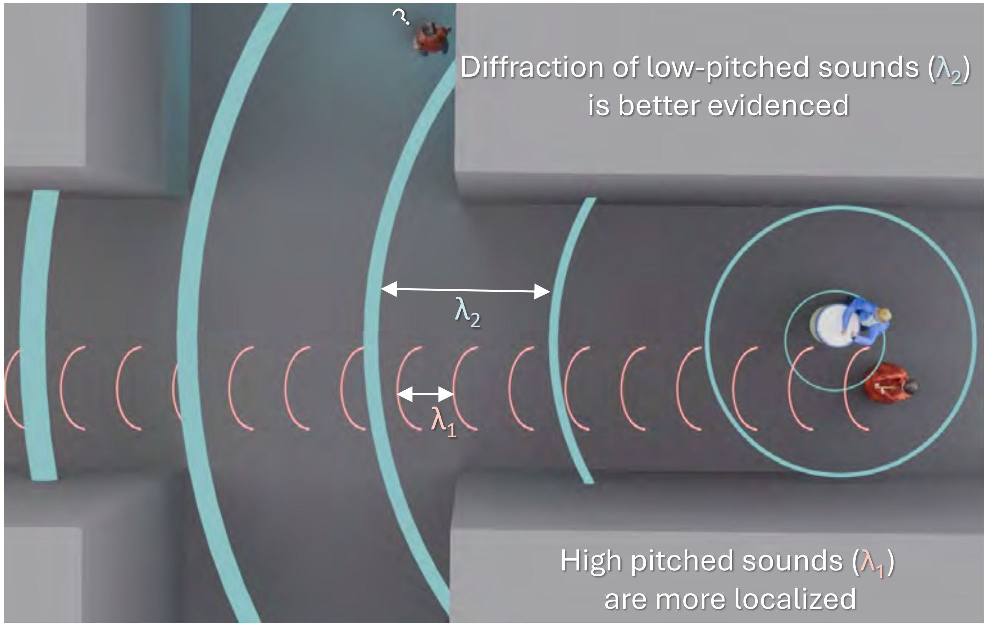
Note: To obtain optical or ultra-acoustic images of objects it is necessary to have reflection on them. Therefore, the wavelength must be shorter than the studied objects. Due to diffraction phenomena, we cannot conventionally observe objects smaller than 1 μm under an optical microscope and it is necessary to use an electron microscope.

Examples of diffraction in the case of sound

Diffraction on small objects and through halls



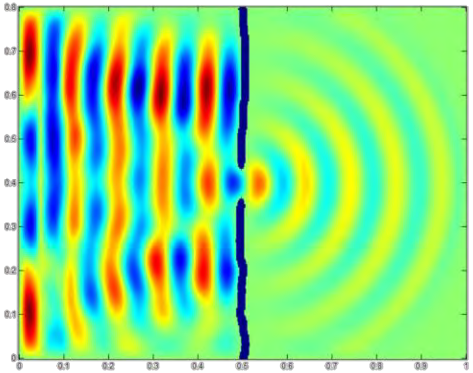
Diffraction when hearing a street band



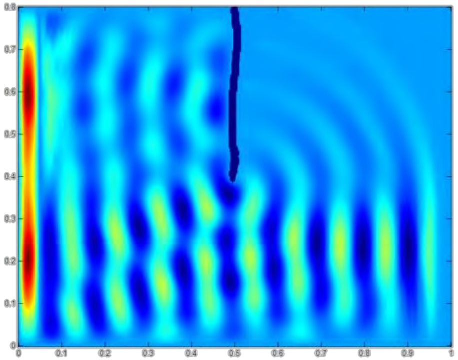
http://hyperphysics.phy-astr.gsu.edu/

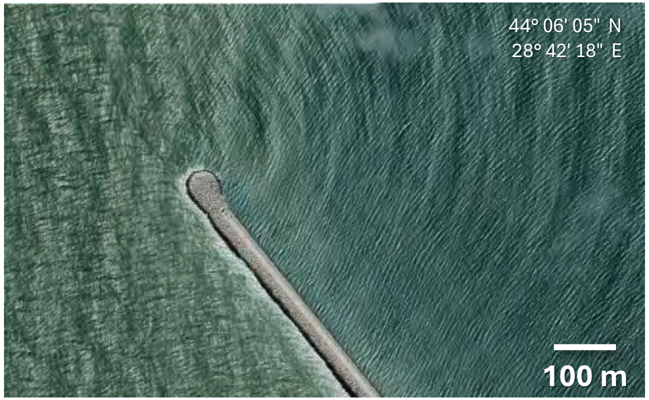
Examples of diffraction in nature

Small opening

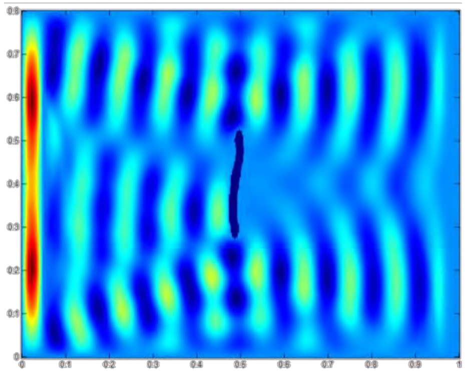


Diffraction at the corner





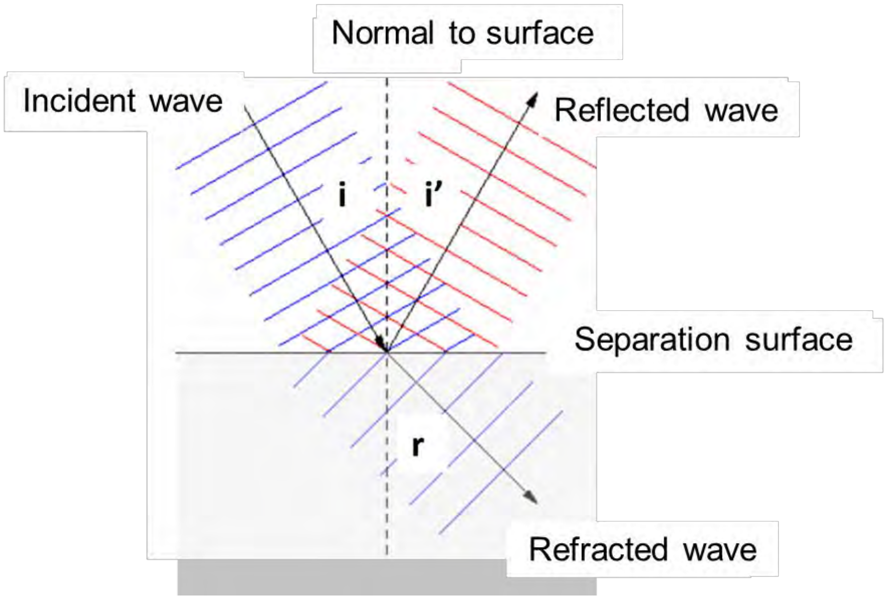
Diffraction around obstacles



F. Logiurato,Teaching waves with Google Earth, Physics Education,47(1), 2012 (https://iopscience.iop.org/article/10.1088/0031-9120/47/1/73)

6. Reflection and refraction of waves

- At the separation surface between two media, an incident wave splits into two waves: the reflected wave and the refracted wave.
- The reflected and refracted (transmitted) waves will have the same frequency as the incident wave, but **the wavelength of the refracted wave is different.**
- The directions in which the two waves propagate are described by the laws of reflection and the laws of refraction, respectively.

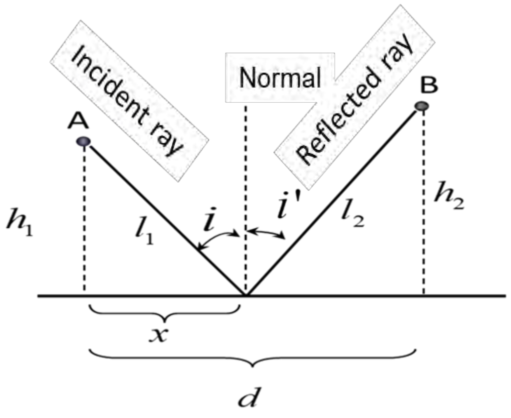


i-angle of incidence;
i'-reflective angle;
r-angle of refraction

Note:
In the following we choose very narrow regions (lines = rays) in the wavefront and refer to them as rays (the case of light) when discussing the laws of reflection and refraction

6.1. The laws of reflection

- There are two laws governing wave reflection:

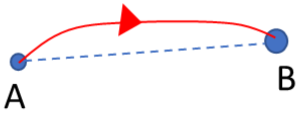


Law I: incident ray, reflected ray and the normal at the point of incidence are in the same plane

Law II: The angle of incidence is equal to the angle of reflection:

$$\boxed{i = i'}$$

- The demonstration of these laws can be made based on the principle of Fermat (1662). In the following we only mathematically demonstrate the second law because the first is obvious.



We calculate the journey time from A to B

$$t = t_1 + t_2$$

$$t_1 = \frac{l_1}{v} = \frac{\sqrt{h_1^2 + x^2}}{v}$$

$$t_2 = \frac{l_2}{v} = \frac{\sqrt{h_2^2 + (d-x)^2}}{v}$$

v – wave's speed;

$$\Rightarrow t = \frac{\sqrt{h_1^2 + x^2}}{v} + \frac{\sqrt{h_2^2 + (d-x)^2}}{v}$$

According to Fermat's P: t=min(t) $\Leftrightarrow \dfrac{dt}{dx} = 0$

$$\Rightarrow \frac{x}{\sqrt{h_1^2 + x^2}} - \frac{d-x}{\sqrt{h_2^2 + (d-x)^2}} = 0$$

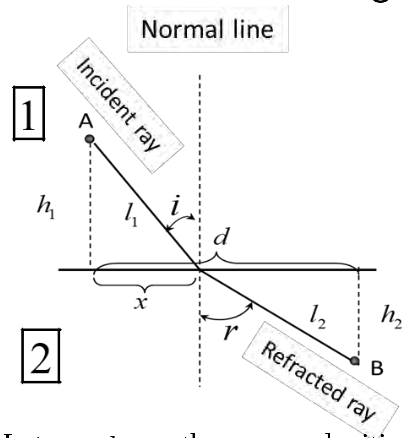
$$\Leftrightarrow \sin i - \sin i' = 0 \Leftrightarrow i = i'$$

Fermat's principle (1662):
A wave will choose the path it travels between two points in such a way as to arrive from A to B in the shortest time.

Note: this is how we choose the route between two cities so that it can be covered in the shortest time (not necessary the shortest distance) that is sometimes on the highway

6.2. Laws of refraction

- There are two laws governing wave refraction:



Law I: incident ray, refracted ray and the normal line at the point of incidence lie in the same plane

Law II: The angle of incidence and the angle of refraction satisfy the relationship:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

- The demonstration of these laws can be made based on the **principle of Fermat** (1662). The proof for the second law:

  We calculate the journey time from A to B:

Let v_1 and v_2 – the wave velocities in medium 1 and 2

$$t = t_1 + t_2$$

$$t_1 = \frac{l_1}{v_1} = \frac{\sqrt{h_1^2 + x^2}}{v_1}$$

$$t_2 = \frac{l_2}{v_2} = \frac{\sqrt{h_2^2 + (d-x)^2}}{v_2}$$

$$\Rightarrow t = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (d-x)^2}}{v_2}$$

Applying Fermat's P: t=min(t)

$$\Leftrightarrow \frac{dt}{dx} = 0$$

$$\Rightarrow \frac{1}{v_1}\frac{x}{\sqrt{h_1^2 + x^2}} - \frac{1}{v_2}\frac{d-x}{\sqrt{h_2^2 + (d-x)^2}} = 0$$

$$\Updownarrow$$

$$\frac{\sin i}{v_1} - \frac{\sin r}{v_2} = 0 \Leftrightarrow \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

The case of light

$$n = \frac{c}{v} \rightarrow n_1 = \frac{c}{v_1} \text{ and } n_2 = \frac{c}{v_2}$$

n_1, n_2 – refractive index of the two media

c - the speed of light in vacuum

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \Leftrightarrow n_1 \sin i = n_2 \sin r$$

Snell's law - The second law of refraction for light and other electromagnetic waves

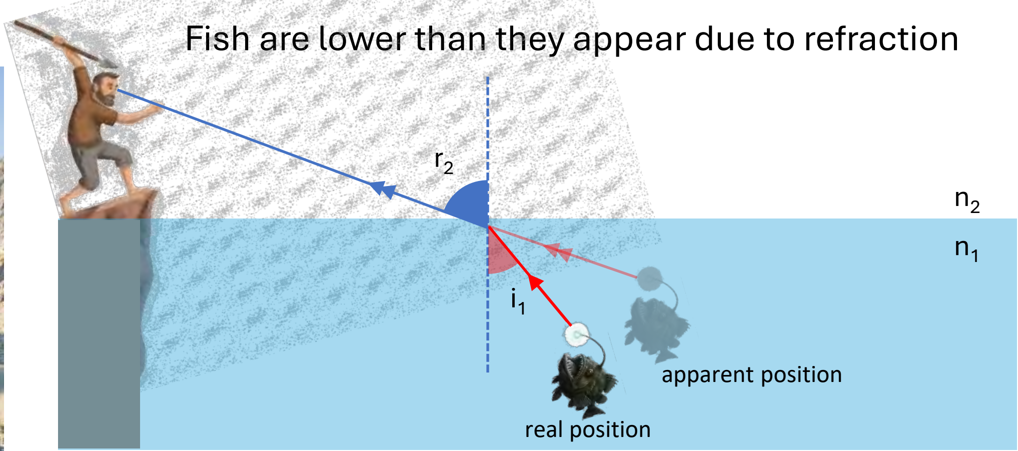
Reflection and refraction in nature

Reflection of mountains in water

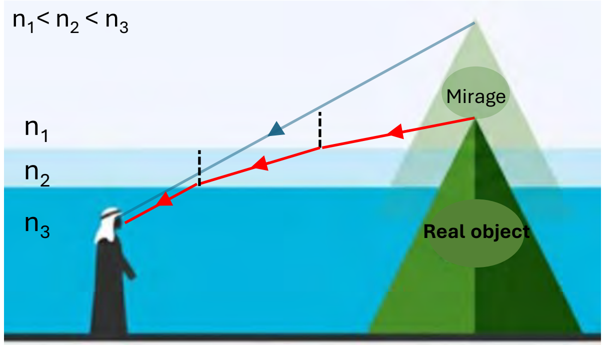
Photo with polarizing filter (reflected light is filtered) | Photo without polarizing filter (reflected light passes)

According to geometric optics, light is reflected from the surface of water. Not only this, but polarization effects will also take place: the reflected light will be fully or partially polarized, having the electric field parallel to the reflecting surface

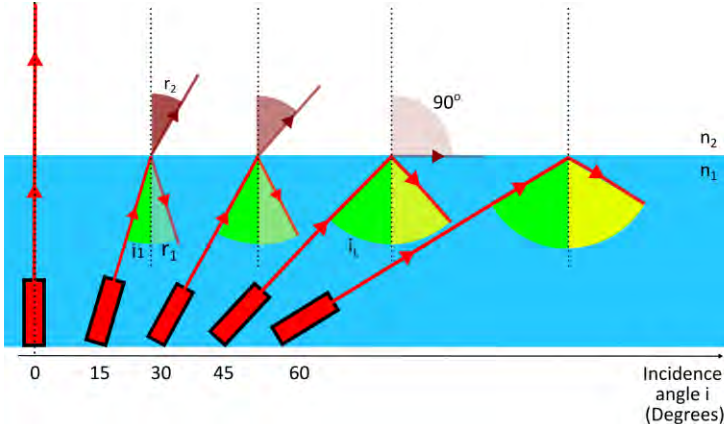
Fish are lower than they appear due to refraction



Fata morgana (mirage)



Total reflection and the optical fiber

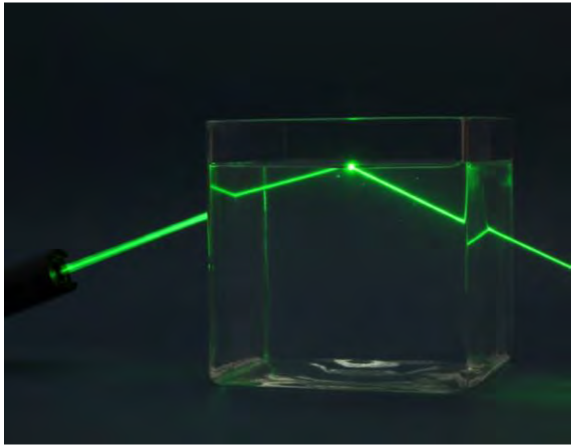


For an angle $i>i_l$ (limit angle) the wave no longer leaves the medium from which it originates, and total reflection occurs. The limit angle can be calculated from the condition r=90°.

$$\frac{\sin i_l}{\sin 90} = \frac{n_2}{n_1} \Leftrightarrow \sin i_l = \frac{n_2}{n_1} \Leftrightarrow \boxed{i_l = \arcsin\left(\frac{n_2}{n_1}\right)}$$

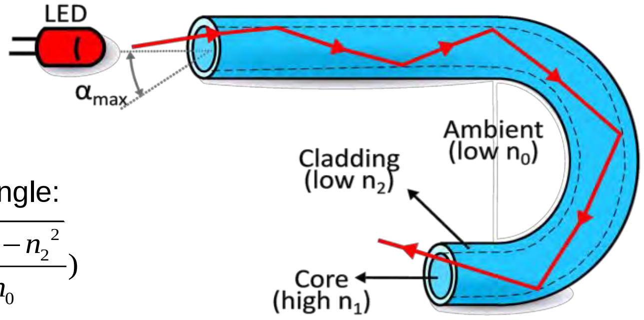
The limit angle above which total reflection occurs

Total internal reflection



The optical fiber

In the case of optical fiber, the angle of incidence is chosen in such a way that it is greater than the boundary angle and multiple total reflections are produced.



Acceptance angle:

$$\alpha_{max} = \arcsin\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$$

Wilson, Hawkes - Optoelectronics. An introduction (Prentice-Hall, 1998)

7. Mechanisms of intensity attenuation

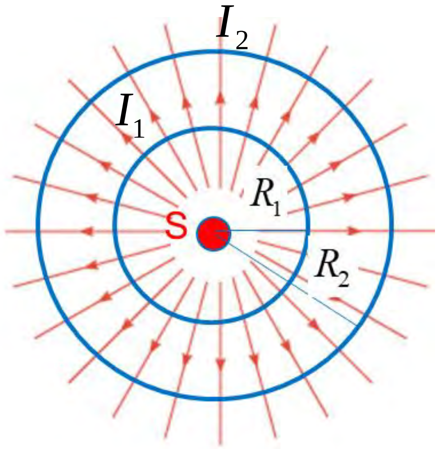
- During their propagation, waves can reduce their intensity.
- There are two mechanisms for attenuating wave intensity that can act independently or simultaneously:

  -geometric attenuation;

  -attenuation by absorption.

7.1. Geometric attenuation of wave intensity

- It occurs due to the scattering of wave energy over larger and larger areas
- To deduce the law of geometric attenuation, we consider a source S emitting waves isotopically, and in addition neglect the absorption of these waves by the medium through which they propagate (see figure)



If we neglect the absorption of waves, it follows that the energy flow through the two spherical surfaces surrounding the source is conserved:

$$\boxed{\Phi_1 = \Phi_2}$$

Since waves fall perpendicular to the two spherical surfaces and the intensity on a given sphere is constant, we have for the two fluxes:

$$\Phi_1 = I_1 S_1 = 4\pi R_1^2 I_1$$

$$\Phi_2 = I_2 S_2 = 4\pi R_2^2 I_2$$

I_1 – intensity at distance R_1

I_2 – intensiy at distance R_2

R_1, R_2 – distances to the wave source

$$4\pi R_1^2 I_1 = 4\pi R_2^2 I_2$$

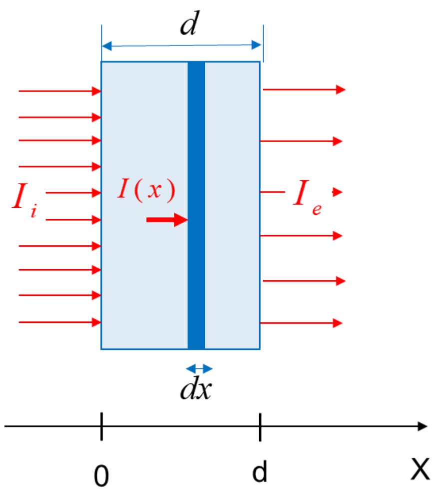
$$\Updownarrow$$

$$\boxed{I_2 = I_1 \left(\frac{R_1}{R_2}\right)^2}$$

The law of geometric attenuation of wave intensity

7.2. Attenuation of wave intensity by absorption

- It occurs due to the conversion of part of the waves' energy into thermal energy (heat)
- To deduce the law of attenuation by absorption we will consider a wall of thickness d and isolate in this wall a slice of infinitesimal thickness dx (see figure)



I_i – intensity at input;

I_e – intensity at exit;

μ – absorbtion coefficient of the wall;

d – wall thickness.

The losses in wave intensity due to absorption through the dx slice are proportional to the thickness dx of the slice, the intensity I(x), at the entrance to the slice and depend on the material from which the slice is constructed.

We can write:

$dI \sim dx$ – intensity reduction proportional with the thicknes of the absorbing layer

$dI \sim I$ – reduction proportional with the intensity

dI – depends on material (via a constant μ)

$$\Rightarrow dI = -\mu I dx \Leftrightarrow \frac{dI}{I} = -\mu dx$$

Integrating the last equality (differential ec.), one obtains:

$$\int_{I_i}^{I_e} \frac{dI}{I} = -\mu \int_0^d dx \Leftrightarrow \ln(I)\Big|_{I_i}^{I_e} = -\mu d$$

$$\Leftrightarrow \ln\left(\frac{I_e}{I_i}\right) = -\mu d \Leftrightarrow \boxed{I_e = I_i e^{-\mu d}}$$

The law of attenuation of wave intensity by absorption

For many materials, the absorption coefficient depends on frequency:

$$\boxed{\mu = \alpha \nu^2}$$

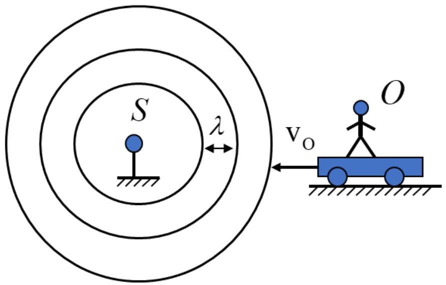
α – constant of material

This means that high-frequency waves are more strongly absorbed than low-frequency ones. This is why ultrasound is absorbed more strongly by materials than sounds

8. The Doppler effect and applications

- *The Doppler effect is the change in the frequency of a wave in an observer relative to that emitted by the source if there is a relative displacement of the source relative to the observer*

Fixed source-mobile observer



Due to the approach of the observer to the source, the period perceived by him (the time between two wave surfaces) will be shorter. This allows the frequency to be calculated at the observer, as it is described below.

$$\left.\begin{aligned} T_O &= \frac{\lambda}{c+\mathrm{v}_O} - \textit{period at observer} \\ \lambda &= cT_S - \textit{wavelength of the source} \end{aligned}\right\} \Rightarrow T_O = \frac{cT_S}{c+\mathrm{v}_O} \Leftrightarrow T_O = T_S \frac{1}{1+\frac{\mathrm{v}_O}{c}} \Leftrightarrow \boxed{\nu_O = \nu_S\left(1 \pm \frac{\mathrm{v}_O}{c}\right)}$$

v_O – *observer velocity*

c – *wave velocity*

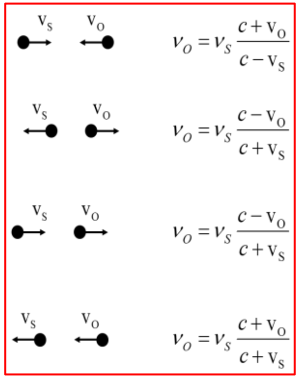
$\nu_S = \frac{1}{T_S}$ – *frequency at source*;

$\nu_O = \frac{1}{T_O}$ – *frequency at observer*;

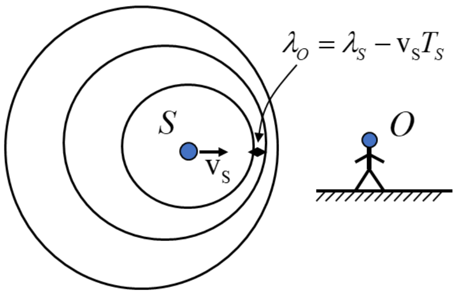
$+$ if the observer approaches

$-$ if the observer moves away

Relative displacements



Mobile source-fixed observer



Due to the approaching of the observer to the source, the wavelength perceived by the observer will be shorter

$$\left.\begin{aligned} \lambda_O &= \lambda_S - \mathrm{v}_S T_S - \textit{wavelenght at observer} \\ \lambda_S &= cT_S - \textit{wavelenght emmited by the source} \\ \lambda_O &= cT_O - \textit{wavelength}\text{ detected by observator} \\ \mathrm{v}_S &- \textit{source}\text{ velocity} \end{aligned}\right\} \Rightarrow cT_O = cT_S - \mathrm{v}_S T_S \Leftrightarrow$$

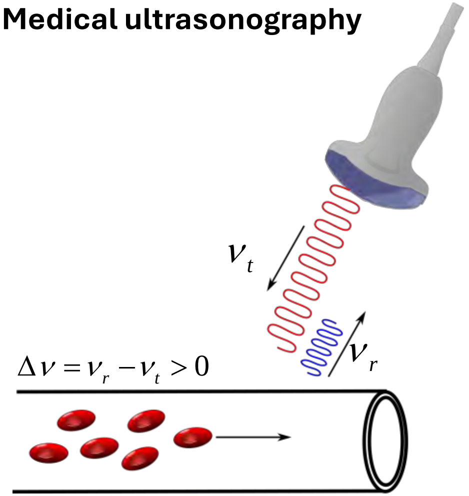
$$\Leftrightarrow T_O = T_S\left(1 - \frac{\mathrm{v}_S}{c}\right) \Leftrightarrow \boxed{\nu_O = \frac{\nu_S}{\left(1 \mp \frac{\mathrm{v}_S}{c}\right)}}$$

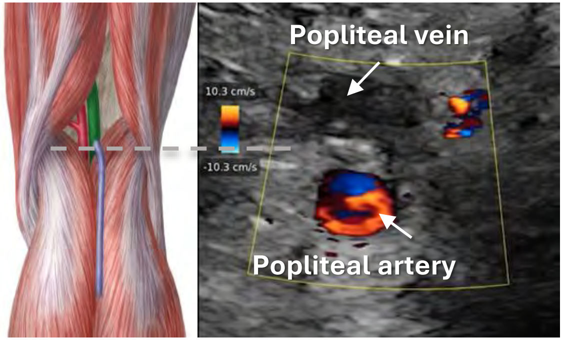
$-$ if the source approaches

$+$ if the source moves away

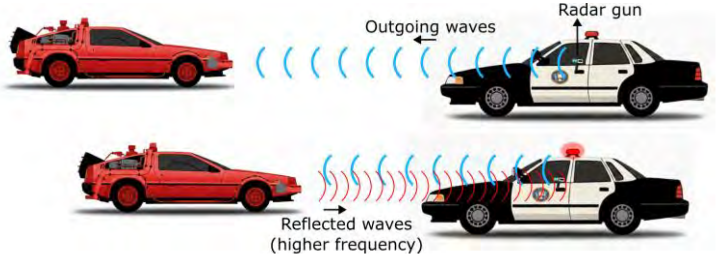
Applications of the Doppler effect

Medical ultrasonography

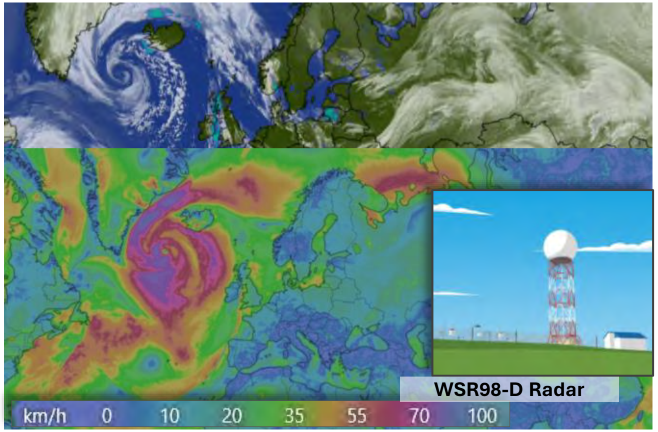




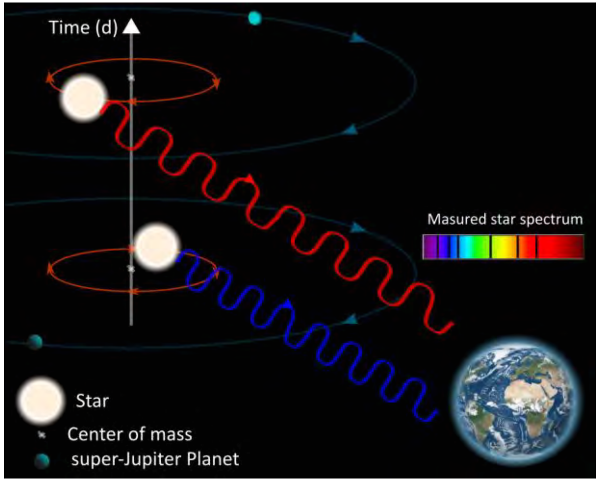
Determining the speed of a car with radars



Determination of cloud velocity



Exo-planet detection (the radial velocity method)



Exo-planet detection using the radial velocity method, based on the Doppler effect. The method enables the measurement of various parameters including orbital velocity, period, excentricity, minimum exo-planet mass etc.

Graphic explanation of the Doppler effect: https://www.youtube.com/watch?v=h4OnBYrbCjY

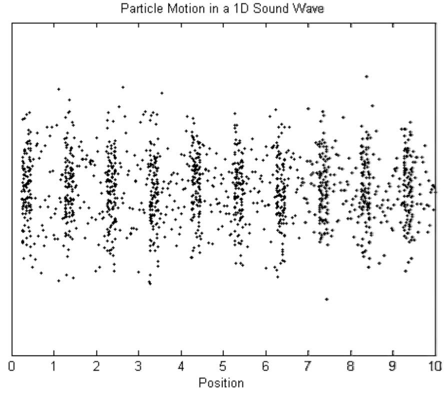
V

Elements of Acoustics and Ultraacoustics

Contents:

1. Sound waves and characteristic physical quantities

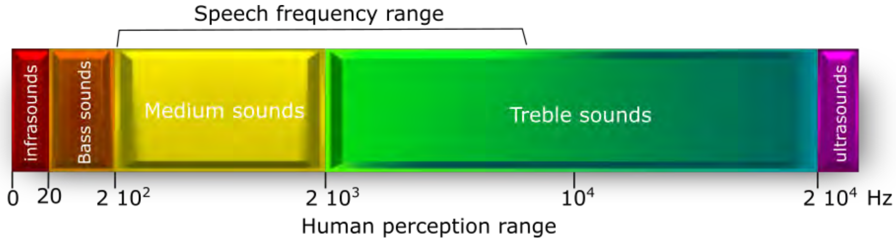
- Sound is a **mechanical wave** that produces a sensation of hearing in the human ear.
- It can be described as an organized propagation of **pressure variations** through a medium (gas, liquid, or solid).



- **General observations:**
  - ✓ Sound cannot propagate through vacuum
  - ✓ Particles oscillate about an equilibrium point;
  - ✓ Mass transport is not the primary mechanism of sound propagation.
  - ✓ Energy is transferred from the sound source

- For an elastic wave to produce hearing sensation it is necessary that it simultaneously meets 3 conditions:
  - ✓ The duration of oscillation must be greater than 0.06 s;
  - ✓ The oscillation frequency should be in the range 20Hz-20kHz
  - ✓ The wave intensity should be in the range of 10^{-12} W/m^2-10^2 W/m^2 (these values may vary from person to

The intensity perceived by the human ear is distributed over 14 orders of magnitude!!!!



The lowest intensity perceived : $I_0 = 10^{-12}$ W/m^2
(the **audibility threshold**)

The highest intensity perceived: $I_p = 10^{2}$ W/m^2
(the **threshold of pain)**

1.1. The acoustic pressure(P_S)

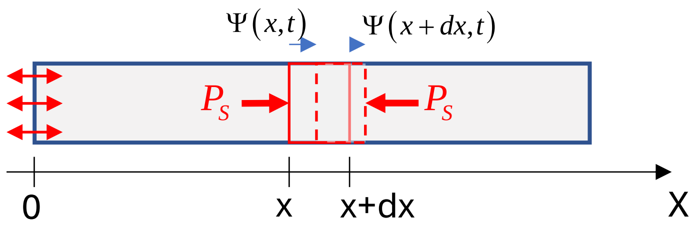
- In the presence of sound, the pressure at a particular location in space varies
- **Sound pressure** (P_S) is defined as the difference between pressure in the presence of a sound wave and in its absence:

$$\boxed{P_S = P - P_0}$$

P – total pressure in the presence of sound

P_0 – total pressure in the absence of sound (ex. atmospheric pressure)

- To deduce the expression of sound pressure we will consider a cylinder (one-dimensional medium) through which a harmonic, elastic wave propagates. Due to the perturbation induced by the wave, different regions in the cylinder undergo successive compressions and decompressions (see figure).



$\Psi(x,t)$ – wave-induced displacement of the x margin

$\Psi(x+dx,t)$ – wave-induced displacement of the $x+dx$ margin

$\Rightarrow d\Psi = \Psi(x+dx,t) - \Psi(x,t)$

Deformation of the selected region in the presence of the wave

$\Psi(x,t) = A\cos(\omega t - kx)$ – the wave function

A - amplitude; ω - pulsation; $k = 2\pi/\lambda$ – wavenumber

- Oscillations produced at one end of the cylinder propagate through it at the speed: $\boxed{c = \sqrt{\dfrac{E}{\rho}}}$

E - Young's modulus

ρ – density of the medium

- Compression and decompression of the selected region occurs due to sound pressure, P_S, which in turn produces an internal force of compression or decompression. If we consider the bar element separately, then, as presented in chapter II, we can apply **Hooke's law** for this element,.

$$\boxed{\frac{F}{S} = E\frac{\Delta l}{l_0}} \text{ – Hooke's law}$$

Analogy

$$\left.\begin{array}{l} \dfrac{F}{S} = P_S; \\ \Delta l \Leftrightarrow d\Psi; \\ l_0 \Leftrightarrow dx \end{array}\right\} \Rightarrow P_S = E\frac{d\Psi}{dx}$$

$$\Psi(x,t) = A\cos(\omega t - kx)$$

$$P_S = kEA\sin(\omega t - kx)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{cT} = \frac{\omega}{c}$$

$$c = \sqrt{\frac{E}{\rho}} \Leftrightarrow E = \rho c^2$$

The acoustic pressure:

$$\boxed{P_S = \rho c \omega A\sin(\omega t - kx)}$$

1.2. The sound intensity and sound flux

- Sound waves are a particular type of elastic wave. Therefore, the intensity and flow of energy carried by them will satisfy the same relationships as those deduced during Chapter IV. Here we simply recall these relationships and definitions with reference to sounds.

- **The Sound intensity (I_s)** represents the energy carried by sound waves per unit area per unit of time:

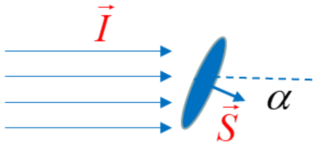
$$I_S = \frac{E_S}{St} \Rightarrow I_S = \rho c \frac{\omega^2 A^2}{2} = Z \frac{\omega^2 A^2}{2}$$

ρ – density of the medium; c-wave velocity; ω-pulsation; A-amplitude of the wave; Z=ρc-acoustic impedance of the medium

- **Sound flux (Φ_S)** represents the energy carried by sound waves through a given surface area S in unit time:

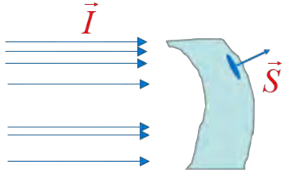
$$\Phi_S = \frac{E_S}{t} \Rightarrow$$

$$\Phi_S = \vec{I}_S \cdot \vec{S} = I_S S \cos\alpha$$



If the intensity is constant on the surface

$$\Phi_S = \iint_S \vec{I}_S \cdot d\vec{s}$$



If the intensity is variable on the surface

1.3. Sound intensity level(N_S)

- As previously discussed, the sound intensity perceived by the human ear is distributed over 14 orders of magnitude, between 10^{-12} W/m² and 10^2 W/m²

  Such a wide distribution of intensity values is difficult to perceive, and therefore it is preferable to characterize the strength of sounds to introduce a new physical quantity, defined logarithmically, called sound level.

- We define the sound intensity level by the relation:

$$N_S = 10\lg\left(\frac{I_S}{I_0}\right) \qquad [N_S]_{SI} = dB = deciBell$$

I_S – intensity of the considered sound;

$I_0 = 10^{-12}$ W/m² – threshold of hearing

The sounds perceived by the human ear will have sound levels ranging from 0 dB to 140 dB

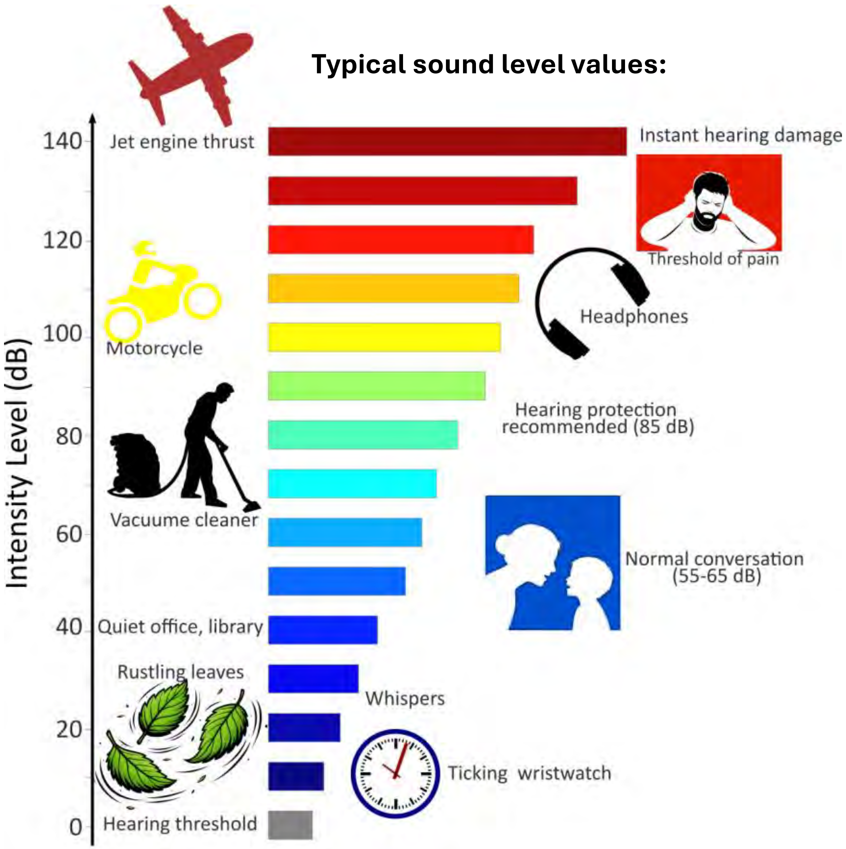
Demonstration:

$$N_{S\min} = 10\lg\frac{10^{-12}}{10^{-12}} = 10\lg 1 = 0\ dB$$

$$N_{S\max} = 10\lg\frac{10^{2}}{10^{-12}} = 10\lg 10^{14} = 140\ dB$$

Sound level meter

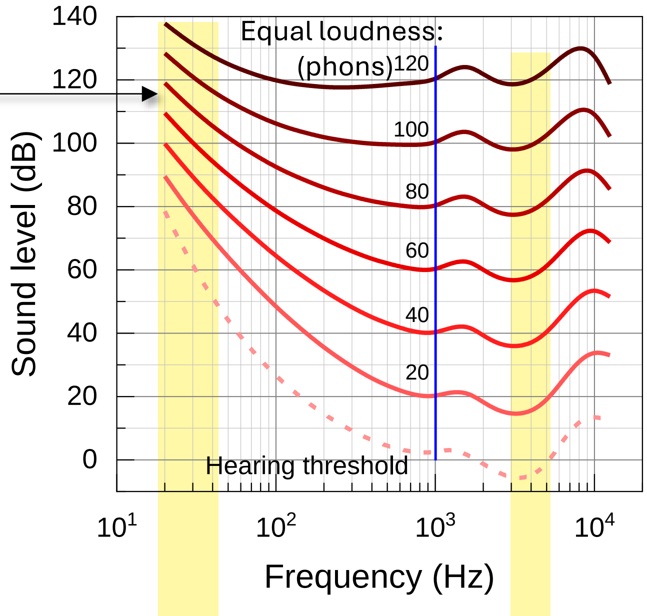
Typical sound level values:



1.4. Sound perception and Loudness level (N_A)

- As seen above, sound level (N_s) is related to the energy carried by sound. However, sounds of different frequencies are perceived differently by the human ear, even though they have the same sound intensity, and therefore the same sound level. Therefore, to better characterize the degree of sound perception, the loudness level is introduced.

Fletcher-Munson type curves (1930) approximating the normal-equal loudness levels according to ISO 226:2003.
A particular person may have deviations from this average.



reduced ear sensitivity to low frequencies

Highest sensitivity is found at 3-4 kHz

- We define the loudness level of a sound by the relationship:

$$N_A = 10\lg\left(\frac{I_A}{I_0}\right) \qquad [N_A]_{SI} = phon$$

$I_0 = 10^{-12}$ W/m^2 – threshold of hearing

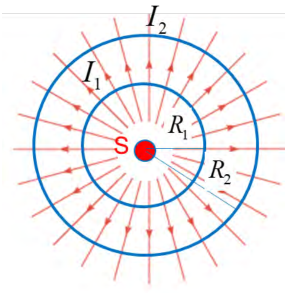
I_A – hearing intensity = the intensity of a sound with the frequency of 1000Hz which would produce the same hearing sensation as the studied sound;

http://hyperphysics.phy-astr.gsu.edu/hbase/Sound/eqloud.html

2. Sound attenuation

- As with all waves, sound waves can reduce their intensity by **geometric attenuation** or via **absorption**. This reduction in intensity has the effect of modifying the sound levels.
- We define the attenuation of the sound level by the relationship: $$A = N_{Sf} - N_{Si}$$ N_{Sf} – final sound level; N_{Si} – initial sound level

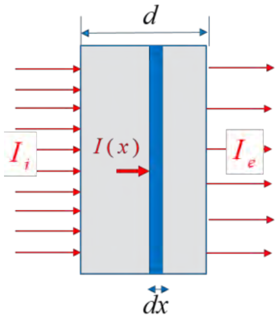
Geometric attenuation



Law of attenuation of intensity with distance from source

$$I_2 = I_1\left(\frac{R_1}{R_2}\right)^2$$

Attenuation by absorption



Law of attenuation of intensity by absorption

$$I_e = I_i e^{-\mu d}$$

μ-absorption coefficient of the material (depends on frequency)

Sound level attenuation by geometric scattering:

$$\left.\begin{array}{l} A = N_{Sf} - N_{Si} = 10\lg\frac{I_2}{I_0} - 10\lg\frac{I_1}{I_0} = 10\lg\frac{I_2}{I_1} \\ \frac{I_2}{I_1} = \left(\frac{R_1}{R_2}\right)^2 \end{array}\right\} \Rightarrow \boxed{A = 10\lg\left(\frac{R_1}{R_2}\right)^2 = 20\lg\frac{R_1}{R_2}} \quad (dB)$$

Attenuation of sound level by absorption:

$$\left.\begin{array}{l} A = N_{Sf} - N_{Si} = 10\lg\frac{I_e}{I_i} \\ \frac{I_e}{I_i} = e^{-\mu d} \end{array}\right\} \Rightarrow \boxed{A = 10\lg e^{-\mu d} = -10\mu d\lg e = -4.3\mu d} \quad (dB)$$

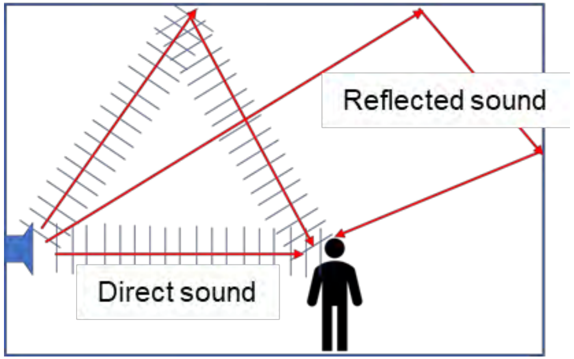
Observation: When a wave passes through a wall, some of the energy will be reflected, thus contributing the reflection to attenuation.

3. Reverberation of sounds

- **Reverberation** is the phenomenon of perceiving a sound in a room even after turning off the sound source. This phenomenon is due to multiple reflections of sound on the walls of the room, which can also exist after cutting off the source.

- Reverberation is characterized by the **reverberation time**, defined as the time for which sound is still heard in the room, after the source is turned off. A more precise definition of reverberation time is that it is the time after which the energy density of room sounds is reduced by 10^6 times or equivalent to 60dB

It can be shown* that in the case of a rectangular enclosure the density of sound energy in the room satisfies the relationship:



Note: if the delay between sounds is long, multiple echoes are perceived (visit Turda Salt Mine for example)

$$w = w_0 e^{-\frac{\alpha c S}{4V} t}$$

w – energy density at an instant t;
w_0 – enegy density at time of turning off the source;
α – absorption coefficient of the walls (0.015 for concrete; 0.8 for felt);
S – surface of the walls; V – volume of the room;
c – speed of sound

Starting from the above relationship, the reverberation time for the rectangular enclosure can be determined:

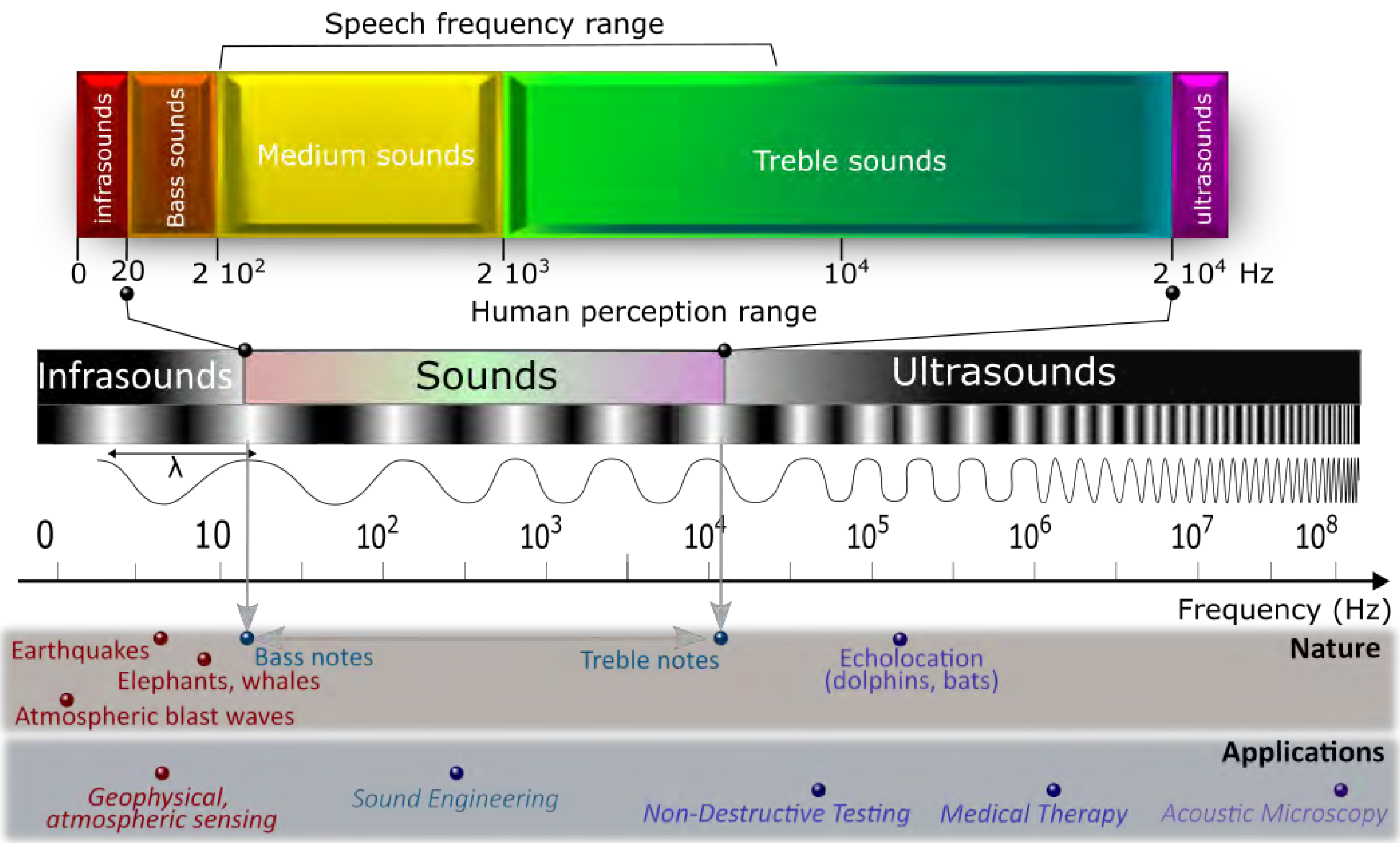
$$w(\tau_R) = \frac{w_0}{10^6} \Leftrightarrow w_0 e^{-\frac{\alpha c S}{4V}\tau_R} = w_0 \cdot 10^{-6} \Leftrightarrow e^{-\frac{\alpha c S}{4V}\tau_R} = 10^{-6} \Leftrightarrow \lg\left(e^{-\frac{\alpha c S}{4V}\tau_R}\right) = \lg\left(10^{-6}\right)$$

$$\Leftrightarrow \frac{\alpha c S}{4V}\tau_R \lg e = 6 \lg 10 \Leftrightarrow \tau_R = \frac{24V}{\alpha c S \lg e} \Leftrightarrow \boxed{\tau_R = 0.16\frac{V}{\alpha S}} \text{ – reverberation time}$$

Note: It is necessary that the reverberation time is generally less than 2.5s.
For a classroom it must be smaller than 1s.
Notre Dame cathedral had a reverberation time of 8.5s

* E. Luca, C. Ciubotaru, Ghe. Zet, A. Păduraru, Fizică generală, EDP București, 1981.

4. The spectrum of acoustic waves and their applications



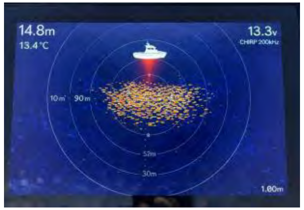
5. Ultrasounds: generation and applications

- Elastic waves with frequency > 20 kHz to 10GHz
- Use: navigation, medicine, biology, chemistry, non-destructive testing (metallurgy, building materials, ...)
- Frequencies used for sound navigation and ranging (SONAR): 1-10 kHz (for long range), 100-500 kHz (for imaging)
- Frequencies used in non-destructive testing of building materials (cement, concrete, wood): 50 kHz-500KHz
- Frequencies used in medicine: 1MHz-3MHz
- Active applications: ultrasounds induce changes to the exposed materials
- Passive applications: ultrasounds are non-invasive

Active applications | **Passive applications**

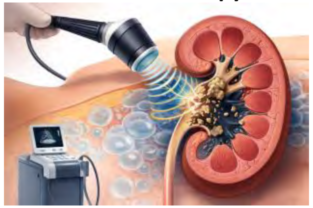
Materials processing

SONAR

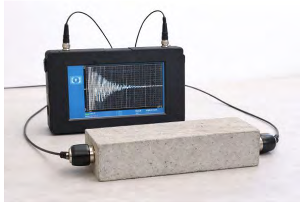


Medical imaging

Medical Therapy



Non-destructive testing



Sensors for robotics

Representative applications of ultrasounds

Cavitation effects

- Consider ***the vapor pressure*** *as the pressure exerted by the vapor when it is found in thermodynamic equilibrium with its coexisting liquid (or solid) phase at a given temperature.*
- The acoustic pressure generated by ultrasounds can cause alternating compression and rarefaction cycles.
- Any existing gas bubbles can grow if the acoustic pressure in the liquid drops bellow the vapor pressure of that liquid.

- ***Cavitation*** *refers to the formation, growth, and collapse of vapor/gas bubbles in a liquid under intense pressure fields. During the collapse of unstable bubbles, the transmitted shockwaves are so intense that the local pressures can reach several gigapascals and temperatures of several thousand Kelvin.*

- The acoustic pressure in the bubble region can be described by:

$$p = p_0 + p_a \sin(\omega t)$$

p =local acoustic pressure

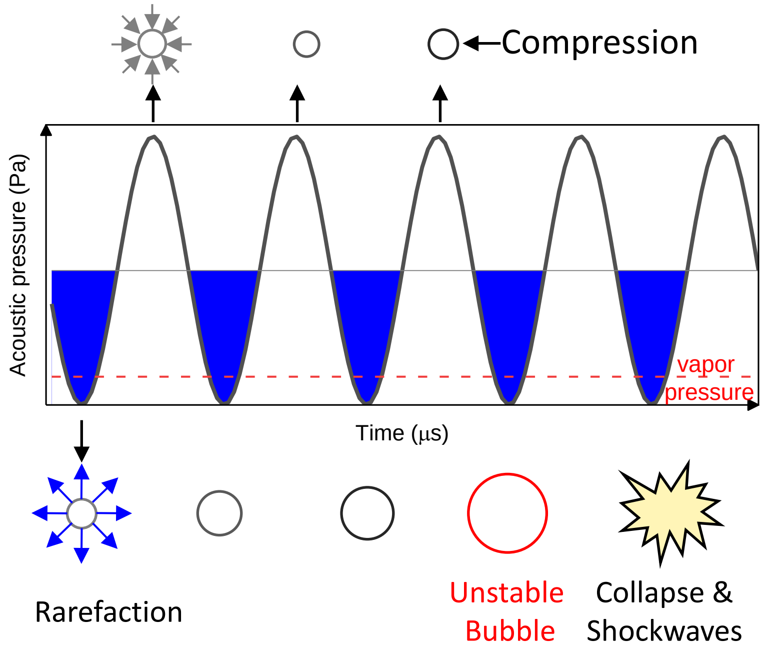
p_0=steady state pressure

p_a=pressure amplitude

ω=angular frequency

Notable applications :

- Ultrasonic cleaning
- Sonochemistry
- Medical lithotripsy
- Emulsification and degassing



Thermal effects

- **Ultrasound absorption** converts acoustic energy into heat.
- Absorption is dominant at higher frequencies, in viscous or highly attenuating or scattering media

$$I = I_0 e^{-\mu x}$$
$$\mu = \alpha \nu^b$$

where μ = absorption coefficient;
α and b depend on the propagation medium

- A monochromatic plane wave can be treated as an effective volumetric heat source Q_{us} [W/m^3]

$$Q_{us} = \mu I$$

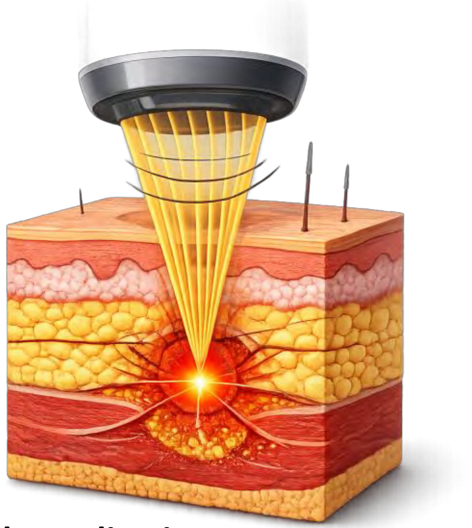
- The temperature rise will depend on intensity, exposure time, and medium properties

Estimations on acoustic properties of various media

Medium	ρ [kg/m^3]	c [m/s]	Z [kg/(m^{-2}s)]	α_0 [dB/(MHzcm)]	b	B/A
Air	1.204	343	10^{-5} 10^6	~2-6	-	~0.7
Water 20 ˚C	1000	1480	1.48 10^6	0.002	2.0	4.96
Blood	1060	1584	1.68 10^6	0.14	1.21	6
Bone	1990	3198	6.36 10^6	3.54	0.9	-
Muscle	1041	1580	1.64 10^6	0.57	1.0	7.43
Fat	928	1430	1.33 10^6	0.6	1.0	10.3

Frequencies selected for medical applications:

- 1 MHz (optimal heating)
- 0.5 MHz (for deep treatments)
- 8 MHZ (shallow treatments)

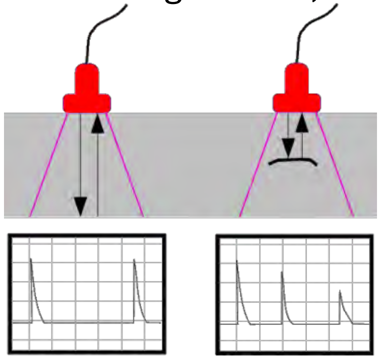


Notable applications:

- High-Intensity Focused Ultrasound (HIFU)
- Physiotherapy and diathermy
- Thermal ablation of tumors
- Materials processing

5.2. Physical principles applied to passive ultrasound applications

- The ultrasound transferred by the transmitter (which can also be a detector) is detected after reflection from inhomogeneities, being subjected to attenuation processes that depend on the investigated medium.



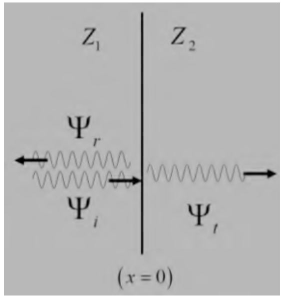
- Due to diffraction phenomena, it is necessary that the wavelength of ultrasound is shorter than the size of the investigated feature (object, tissue, defects, etc.). Otherwise, there is a bypassing of the feature (which generally happens with sounds)

$$\boxed{\lambda << D} - \text{feature dimension}$$

- Due to absorption phenomena, the ultrasound cannot have very short wavelengths (very high frequencies) because it would not penetrate the propagation medium.

Reflective attenuation

The reflected ultrasound reduces its intensity due to partial reflection at the interface

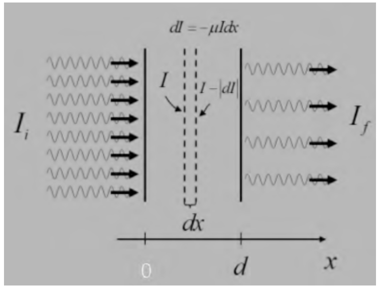


$$\boxed{R = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}} - \text{reflection coefficient}$$

$$Z = \rho c = \text{acoustic impedance}$$

Attenuation by absorption

The transmitted ultrasound reduces its intensity due to absorption.



$$\boxed{I_f = I_i e^{-\mu d} \Leftrightarrow I_f = I_i e^{-\alpha \nu^2 d}}$$

$\mu = \alpha \nu^2$

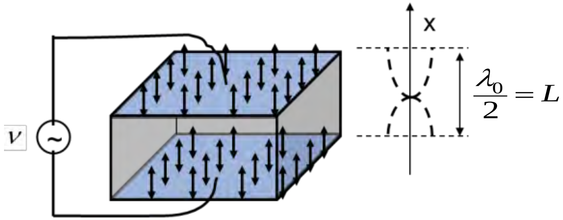
absorption coefficient increases with frequency

5.3. Ultrasound generation

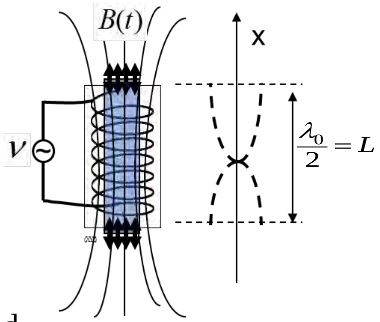
- Ultrasounds can be generated using the piezoelectric effect in certain crystals (i.e. quartz, PZT - lead zirconate titanate, etc) or the magnetostriction effect specific to some magnetic materials

- **Piezoelectric effect** = change in the dimmesion of a crystal (SiO_2) if it is inserted into an electric field.
- For the generation of ultrasound one can silver the sides of a quartz crystal and apply an alternating voltage on them. In this way, oscillations of frequency equal to alternating voltage can occur.
- For the crystal faces to oscillate with maximum amplitude and for the ultrasonic generation process to be more efficient, anti-nodes need to form on the faces, i.e. the crystal thickness should be λ/2. This condition limits the production of very high ultrasound frequencies due to the limited quartz crystal thickness.
- **The magnetostrictive effect** = change in the size of a ferromagnetic material if it is inserted into a magnetic field. The ultrasound generation is similar, only now the magnetic field induces the vibrations

The piezoelectric generator



The magnetostrictive generator



$$\lambda_0 = cT_0 = \frac{c}{\nu_0} \text{ – wavelength of the ultrasound}$$

$$c = \sqrt{\frac{E}{\rho}} \text{ – velocity of the elastic wave (ultrasound)}$$

E –Young modulus
ρ- density of the material
L- thickness of the quartz crystal (bar length**)**

$$\nu_0 = \frac{1}{2L}\sqrt{\frac{E}{\rho}}$$

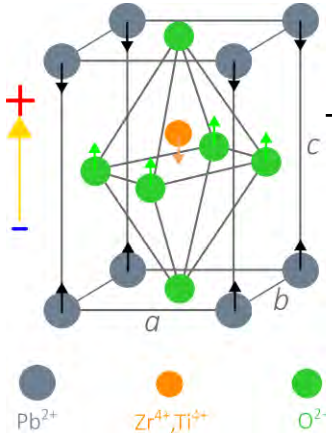
The fundamental frequency generated depends on the thickness of the crystal

Ultrasound transducers based on the piezoelectric effect

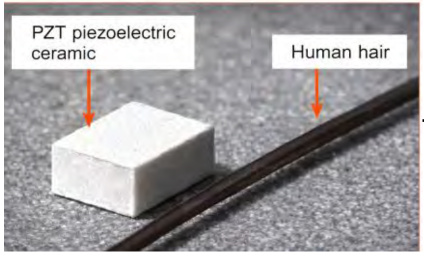
- For a given application, the ultrasound frequency, power and beam geometry are carefully tuned to achieve optimal performance (i.e. resolution, penetration depth, signal to noise ratio, etc.). The contributions of diffraction, reflection, scattering and attenuation must be assessed.
- As shown previously, the main frequency depends mainly on the impedance and thickness of the piezoelectric element.
- The frequency band, the power and beam geometry will be influenced by the probe architecture and electronics.
- Note that ultrasound **transducers** can function both as acoustic emitters and detectors, exploiting the reciprocal nature of the piezoelectric effect.

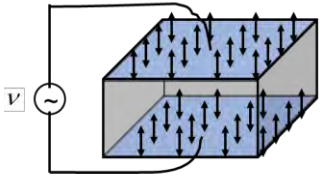
Obstetric sonogram

The piezoelectric effect at atomic scales in a PZT unit cell

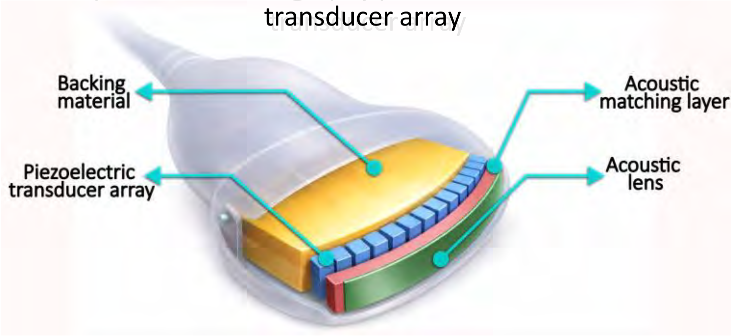


Vibrations in one PZT element





Depiction of a sonography probe based on a convex transducer array



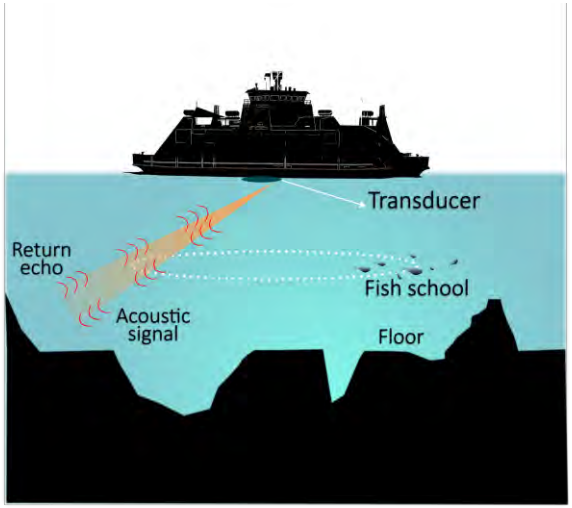
5.4. Passive applications of sound and ultrasounds

Basic Principle of Sound Navigation and Ranging (SONAR)

- In active sonars, a "PING" signal is generated typically by a transducer array at time t=0.
- The signal backscatters from a surface and travels back to the transducers at **Δt**.
- The **time of flight** (Δt), the signal amplitude and phase difference between all transducers is recorded and processed to obtain the position, size and reflective properties of the object.
- The distance r from the target is determined using:

$$r = c\frac{\Delta t}{2}$$

- Passive sonars (hydrophones) are advantageous for stealth conditions.
- In deep sea (~10^3 m), due to pressure and temperature, a natural and horizontal waveguide is formed that makes the signal attenuate by a 1/r law. This is called the Sound Fixing and Ranging (SOFAR) channel.



Principle of SONAR

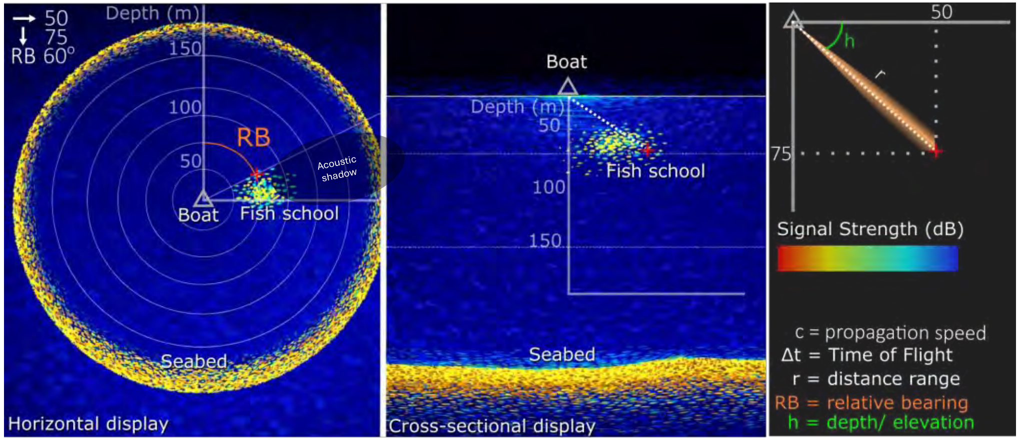
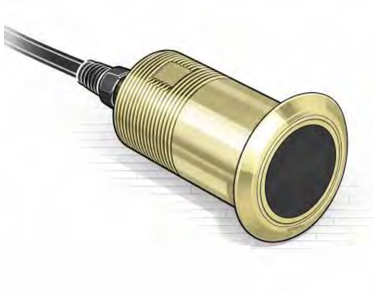
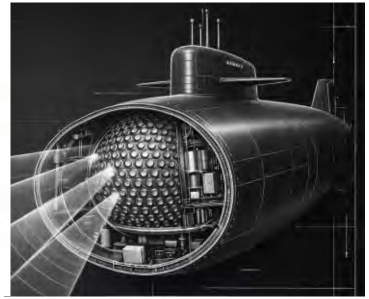


Illustration of sonar displays



Typical transducer for underwater applications



Bow sonar array specialized for military submarines

Non-destructive testing of concrete structures using ultrasounds

- Cement and concrete are heterogeneous building materials with complex microstructures.
- In practice, ultrasound velocity through such media is used as a non-destructive technique for quality assessment of concrete elements. The table indicates the standard classification.
- Small frequencies are used to reduce attenuation (20-150 kHz)

Ultrasound pulse speed

- The transit time needed for an ultrasonic pulse to travel from one probe to another is determined. Measuring the thickness of the investigated sample, one may compute the propagation speed:

$$c = \frac{d}{\Delta t}$$

d = distance traveled

Δt = transit time

- The propagation speed of one-dimensional longitudinal waves is interconnected with the mechanical properties of the material:

$$c = \sqrt{\frac{E}{\rho}}$$

c = wave propagation speed ($\frac{m}{s}$)

E = Young's modulus (Pa)

ρ = bulk volumetric density ($\frac{kg}{m^3}$)

Correlate with stiffness and porosity

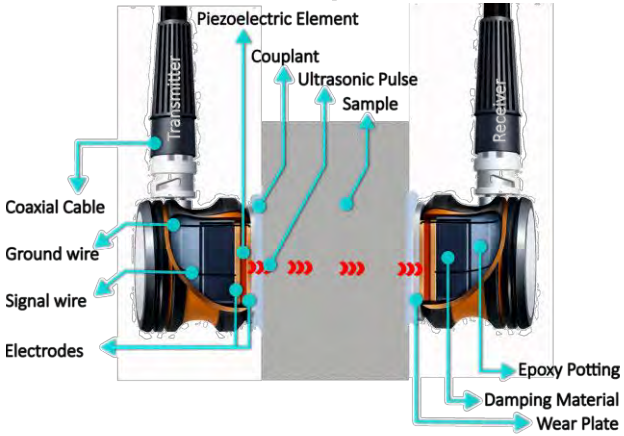
- For bulk, isotropic solids, the local deformation is not purely axial, as the material compresses along the traveling direction and expands laterally. The relation becomes:

$$c = \sqrt{\frac{E}{\rho}\frac{1-\mu}{(1-2\mu)(1+\mu)}}$$

$$\mu = -\frac{Lateral\ \text{Strain}}{Longitudinal\ \text{Strain}} = -\frac{\Delta\Phi}{\Phi_0}\frac{l_0}{\Delta l}$$

(Poisson's ratio)

Ultrasound transducers in transmission set-up



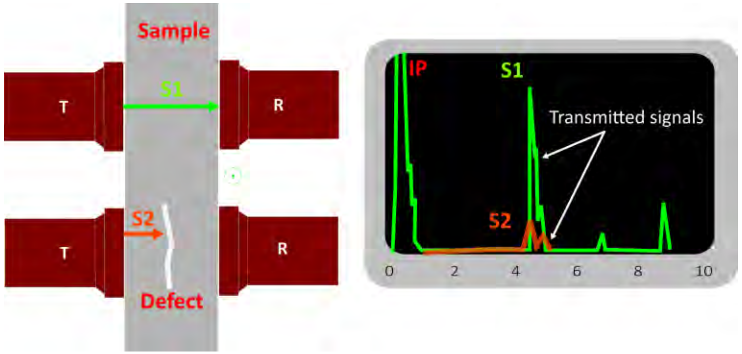
Concrete testing device

Ultrasonic non-destructive testing of steel and other metals

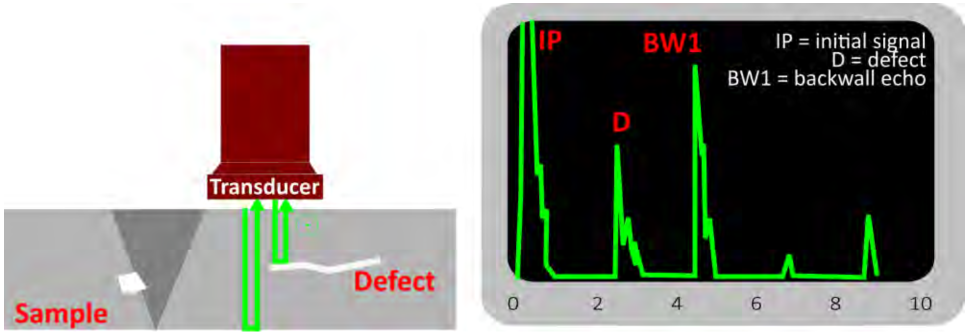
- Metals and alloys are far more homogeneous and less scattering than concrete.
- This allows NDT tesing at higher frequencies (1-10 MHz), hence improved resolution and lower detection limit for defect size.
- The technique enables the non-invasive flaw detection (cracks, inclusions, voids) that are inaccessible to surface sensitive techniques (i.e. liquid penetration)
- The most frequently used operation mode is *pulse-echo* (one probe). The transmission mode, also imaging (*phased array*) are also well represented.

- The operation principles are similar as for the other ultrasound applications
- Both longitudinal and transverse waves are commonly used.
- The measured signal is processed to extract parameters sensitive to various defect features: time of flight (defect depth), amplitude (defect size/reflectivity), signal shape (defect type).

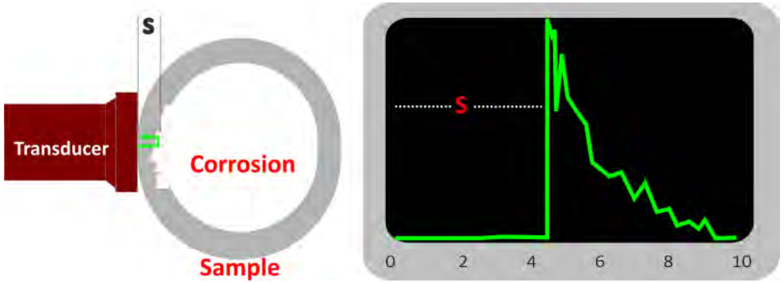
Determination of a flaw in a plate by transmission



Determination of flaws in a plate in pulse-echo mode



Determination of the thickness of a pipe and the corrosion effect



VI

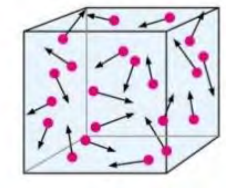
Thermal transport phenomena

Content:

1. Temperature, thermal energy and heat
2. Heat transfer mechanisms. Thermal conduction, convection and radiation

1. Temperature, thermal energy and heat

- ***Temperature (T)*** *is a measure of the thermal agitation energy of atoms and molecules of a physical system (body or gas)*
- If the thermal agitation energy comes only from the translational motion of the molecules of a gas, then between the average kinetic energy of a molecule and the temperature of the gas, there is a relationship:



$$W = m\frac{\overline{v^2}}{2} = \frac{3}{2}k_B T$$

m – mass of a molecule;

$\overline{v^2}$ – mean square velocity of molecules

$k_B = 1.381 \cdot 10^{-23}\, J/K$ – Boltzmann's constant

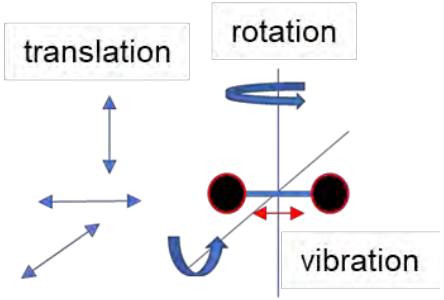
T – absolute temperature (K=Kelvin)

Relation between temperature expressed in Kelvin and temperature expressed in Celsius

$$T(K) = t(°C) + 273.15$$

- If the energy of thermal agitation comes from the **translation, rotation** and **vibration** of molecules of a gas, then between the energy of a molecule and the temperature of the gas there is the relationship:

The case of the bi-atomic molecule



$$W = \frac{t + r + 2v}{2}k_B T$$

t – nr. of degrees of translational freedom;

r – nr. of degrees of rotational freedom;

v – nr. of degrees of vibrational freedom;

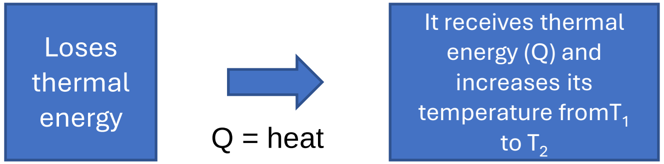
$t = 3$

$r = 2 \quad \Rightarrow \quad W = \frac{7}{2}k_B T$ – energy of a bi-atomic gas molecule (ex. H_2, O_2)

$v = 1$

- In the case of solids, the connection between internal energy and temperature is more complicated, but here we are not interested in energy itself, because we cannot measure it (there is no energy thermometer), but energy transfer, ie heat.

- ***Heat** is a measure of the transfer of thermal energy between two bodies.*



Since heat represents transferred energy, it follows that it is measured in Joule (J)

Other physical quantities that characterize heat received or lost by a body: **heat capacity** (C), **specific heat** (c), and **molar heat** (C_v):

- ***Calorific capacity** (C) is the heat received or lost by a body to change its temperature by 1K*. Therefore, the heat Q received/lost by a body to change its temperature from T_1 to T_2 can be expressed as:

$$\boxed{Q = C\left(T_2 - T_1\right)}$$

C – caloric capacity (J/K);
T_1 – initial temperature (K);
T_2 – final temperature (K)

Note: The calorific capacity of a body depends on its mass and physical characteristics.

- **Specific heat** (c) is the heat received/lost by the unit mass in a body to change its temperature by 1K.

$$\boxed{Q = mc\left(T_2 - T_1\right)}$$

$\boxed{C = mc}$ – relationship between caloric capacity and specific heat
m – mass of the body (kg)
$[c]_{SI} = J/kgK$

- **Molar heat** (C_v) is the heat received/lost by a mole of a material to change its temperature by 1K.

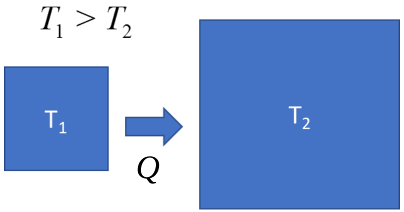
$$\boxed{Q = \nu C_v\left(T_2 - T_1\right)}$$

$\boxed{C = \nu C_v}$ – relationship between caloriccapacity and molar heat
ν – number of mols of substance
$[C_v]_{SI} = J/molK$

2. Heat transfer mechanisms

- The transfer of thermal energy between two bodies can be achieved by three mechanisms:
  1. **Thermal conduction;**
  2. **Thermal convection;**
  3. **Thermal radiation.**

  The three mechanisms can be superposed but can also be analyzed independently

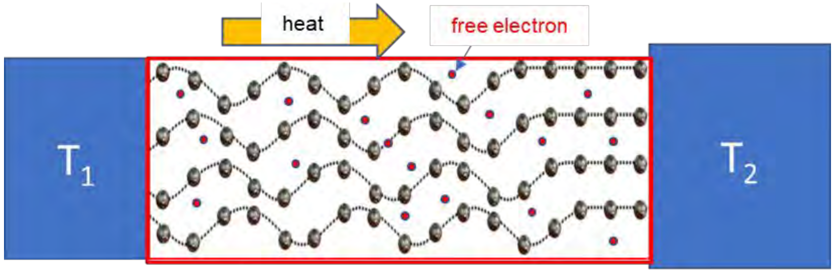


- According to the 2nd principle of thermodynamics, heat always transfers by itself from the body with the higher temperature to the body with the lower temperature

  Note: It is possible to transfer heat from the cold body (thus cooling it further) to the warm body (heating it more), but not by itself, but under the action of an external force (e.g. heat pump)

2.1. Thermal conduction

- ***Thermal conduction*** *is the transfer of energy that occurs by means of atomic collisions, the vibration of the lattice of atoms (phonons) and electronic diffusion, in the case of metals.*
- The phenomenon of thermal conduction is present in all states of aggregation of matter: solids, liquids, gases.



Thermal conductor

Simulation of heat transfer in a metal

https://www.tec-science.com/wp-content/uploads/2020/01/en-thermodynamics-heat-transfer-thermal-conduction-principle-metals.mp4

- ***Thermal current density (j_Q)*** *is defined as the heat energy transported (heat transported) per unit area of the conductor per unit time:*

$$j_Q = \frac{Q}{St}$$

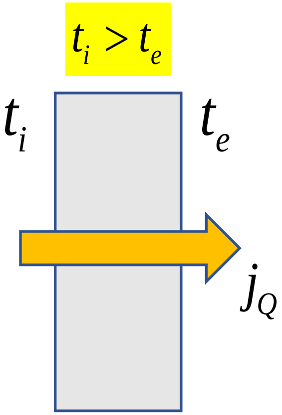
Q-heat (Joule) transported during time t;
S-the cross-surface of the heat-conducting material

⇨ Measuring unit: $\left[j_Q \right]_{SI} = \frac{J}{m^2 s} = \frac{W}{m^2}$

Integral Fourier law

- Fourier's law describes the transport of thermal current (heat) by thermal conduction.
- For the deduction of Fourier's law we consider a wall of thickness d at temperatures t_i and t_e , as in the figure, and make the following experimental observations:

Experimental observations



$j_Q \propto (t_i - t_e)$ – current density proportional with the difference in temperature;

$j_Q \propto \frac{1}{d}$ – current density inversely proportional with the thickness of the wall

j_Q – depends on the material of the wall

Integral Fourier law

$$j_Q = \lambda \frac{t_i - t_e}{d} = \lambda \frac{T_i - T_e}{d}$$

j_Q – thermal current density;

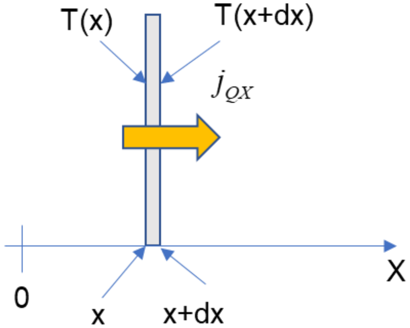
t_i – interior temperature;

t_e – exterior temperature;

d – wall's thickness;

λ – thermal conductivity $[\lambda]_{SI} = W / m \cdot K$

The differential (local) Fourier law



The differential, or local, Fourier law can be obtained from the integral law if we choose the wall as being of very small thickness dx and the temperatures $t_i=t(x)$ and $t_e=t(x+dx)$:

$$j_{QX} = \lambda \frac{t_i - t_e}{d} \Leftrightarrow j_{QX} = \lambda \frac{t(x) - t(x+dx)}{dx}$$

$$j_{QX} = -\lambda \frac{t(x+dx) - t(x)}{dx} = -\lambda \frac{dt}{dx}$$

$$\Leftrightarrow \boxed{j_{QX} = -\lambda \frac{dT}{dx}}$$

Note: Here we took into account that dt=dT because T=t+273.15

The obtained equation describes heat transport only along the direction of the OX, being caused by a temperature difference on the OX axis. If there is a temperature gradient in the material (temperature varies locally in a certain direction) then the above equation can be generalized like this:

$$\boxed{\vec{j}_Q = -\lambda \cdot grad(T) = -\lambda \nabla T}$$

(The Differential (local) Fourier law)

$$where\ grad(T) \equiv \nabla T = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}$$

(the temperature gradient)

Note: The thermal current density vector also *indicates the direction of heat flow*

The thermal conductivity

The thermal conductivity λ depends on the type of the material (state, structure, porosity, etc.). It is higher in metals because of the contribution of free electrons. It is lower in air-filled porous materials.

Material	Thermal conductivity (W/m K)
Copper (pure)	399
Gold (pure)	317
Aluminum (pure)	237
Iron (pure)	80.2
Carbon steel (1 %)	43
Stainless Steel (18/8)	15.1
Glass	0.81
Plastics	0.2 – 0.3
Wood (shredded/cemented)	0.087
Cork	0.039
Water (liquid)	0.6

$\lambda_{aero} \sim 0.012$ W/mK

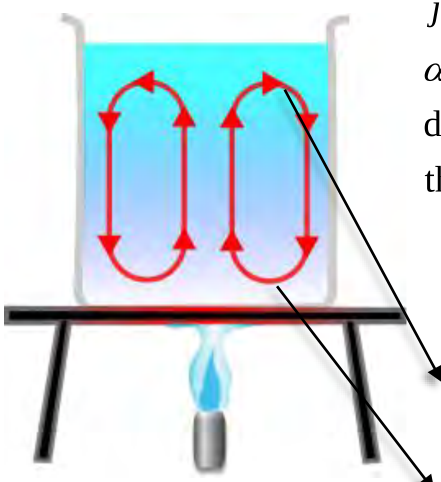
$\lambda_{diamond} \sim 2000$ W/mK

2.2. Thermal convection

- *Thermal convection is the* ***transfer of heat*** *that occurs* ***via a mass transfer***.
- Thermal convection can be free or forced.

Free convection

If the movement of fluids (liquids, gases) that produce heat transfer occurs under the action of gravitational force, due to a difference in density between heated and cold liquid, we speak of **free convection**



$$j_Q = \alpha (T_i - T_e)$$

α – heat transfer coefficient

depending on several factors (i.e. system geometry, the velocity and physical properties of the fluid, etc.)

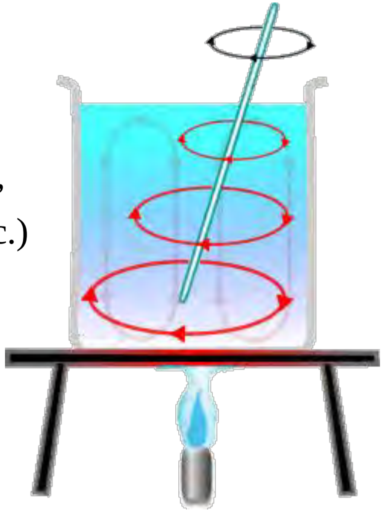
Convective currents

Cold water is more dense
=> Cold layer descends

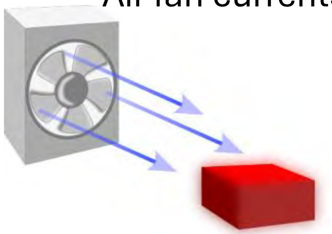
Warm water is more dense
=> Warm layer ascends

Forced convection

If the movement of fluids (liquids, gases) that produce heat transfer occurs under the action of external forces, we speak of **forced convection**



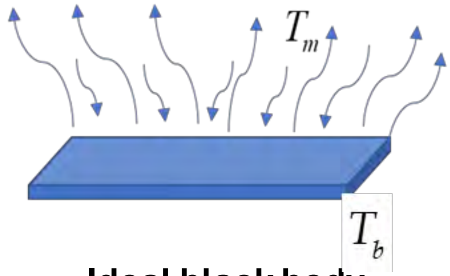
Air fan currents



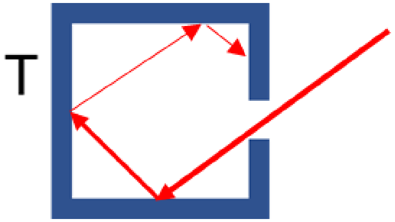
Hot element

2.3. Thermal radiation. The Stefan-Boltzmann Law

Radiation emission-absorption



Ideal black body



T=enclosure temp.=black body temp.

An incident radius on the hole may no longer leave the enclosure due to successive absorptions on its walls. The surface of the hole appears black and is called a black body (because it absorbs all incident radiation)

- All bodies emit heat in the form of thermal radiation and absorb heat from the environment, also in the form of thermal radiation;
- Thermal radiation originates from the vibrations of the atoms and molecules that make up the material from which the surface of that body is formed.
- The emission and absorption of radiation depends on temperature but also on the nature of the surface of the body and is characterized by an emissivity coefficient ε. The best emitter, but also absorbent, is the black body for which ε=1.

The radiative power emitted by the surface S of a body per unit of time is given by the relation:

$$P_{emtted} = \varepsilon \sigma S T_b^4$$

The power absorbed by the surface S per unit time:

$$P_{absorbed} = \varepsilon \sigma S T_m^4$$

The power lost by a body through thermal radiation is given by the difference between the power emitted and the power absorbed:

$$P_{lost} = P_{emitted} - P_{absorbed} = \varepsilon \sigma S \left(T_b^4 - T_m^4\right)$$

ε – surface emissivity;

$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$ – Stefan-Boltzmann constant;

S – surface emitting radiation (m^2);

T_b – temperature of the boby emitting radiation (K);

T_m – temperature of the medium (K)

Emissivity of different surfaces

Material	Emissivity
Polished silver	0.02
Polished copper	0.03
Polished gold	0.03
Aluminum foil	0.07
Wood	0.85
Asphalt pavement	0.9
White paint	0.9
Vegetation	0.94
White paper	0.94
Water	0.95
Black paint	0.98

VII

Fluid transport phenomena in porous media

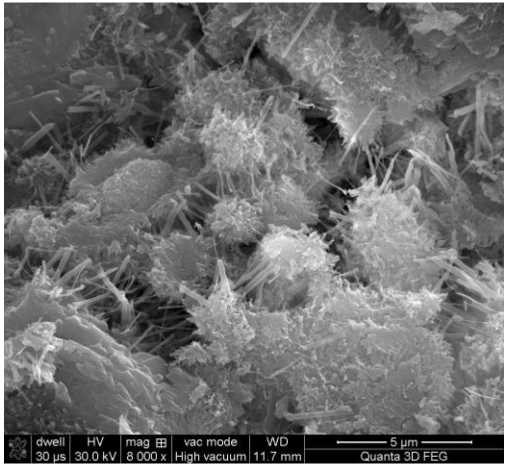
Content:

1. Porous media and characteristic physical quantities
2. Fluid flow as a result of a pressure difference
3. Molecular migration by diffusion;
4. Liquid transport by capillary suction

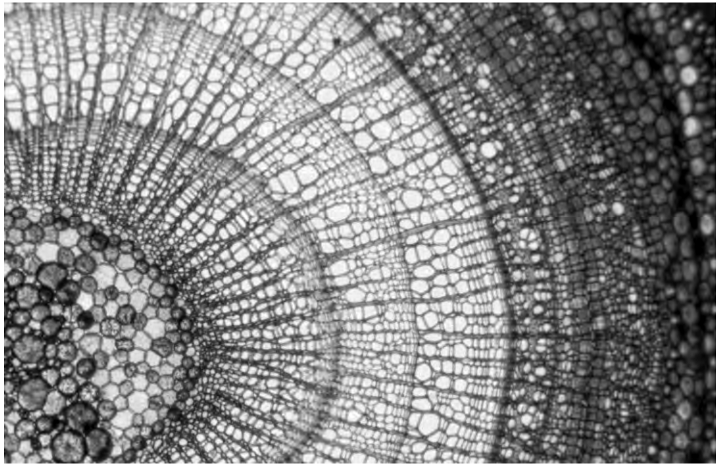
1. Porous media and characteristic physical quantities

- **A porous medium** is a solid structure that has a system of voids (pores) that can be interconnected (open porous media) or isolated (closed pores). Examples of porous media: concrete, BCA, brick, soils, wood, leather, textiles, filters, xerogels, aerogels, etc.

Hydrated cement



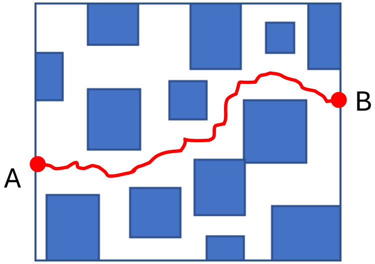
Wood and cellular materials



Aerogels and nanoporous materials

https://upload.wikimedia.org/wikipedia/commons/6/69/Aerogelflower_filtered.jpg

Schematic representation of a 2D porous medium

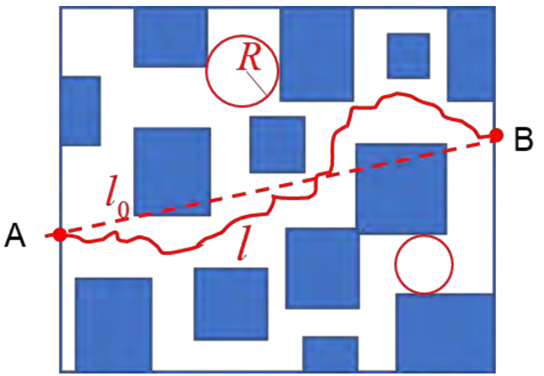


The movement of fluid molecules through a porous medium occurs as through a labyrinth (geometric obstruction) and in addition, for nanometer-sized porous media, it is influenced by the interaction with the surface.

- Porous media are characterized by:

1. **The size (radius) of the pores** = the average radius (R) corresponding to the sphere that can be included in the pores;
2. **Pore volume** = volume of material voids (V_p);
3. **Porosity (Φ)** = the ratio between the pore volume V_p and the volume of the material V:

$$\Phi = \frac{V_P}{V}$$



4. **Specific surface area** = the surface area corresponding to the unit mass of the material

$$s_m = \frac{S_P}{m}$$

S_p – total pore surface;
m – mass of the material; $[s_m]_{SI} = m^2 / kg$

5. **Tortuosity** = the ratio of the average particle path length through the material (l) and the length of the direct path (l_0)

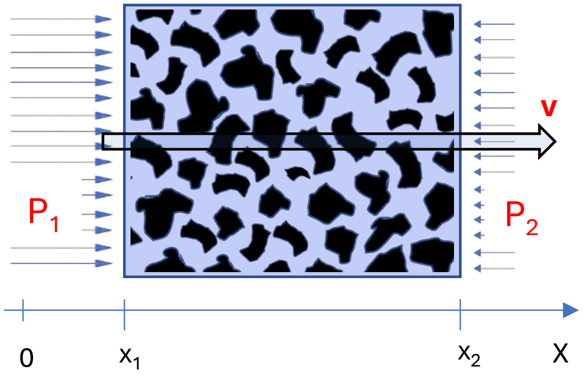
$$\tau = \frac{l}{l_0}$$

l – average length of the path between two points A and B hhrough the porous system;
l_0 – length of the direct path;

Note: All these parameters determine the molecular transport and the flow of fluids through the porous medium.

2. Fluid flow as a result of a pressure difference

- If there is a pressure difference, ΔP, between the surfaces of a porous medium, saturated with a fluid (liquid or gas), then this will determine an average flow velocity of the fluid that satisfies **Darcy's law**:



$$\mathrm{v} = -k\frac{1}{\eta}\frac{\Delta P}{\Delta x}$$

v – mean flow velocity of the fluid (m/s);

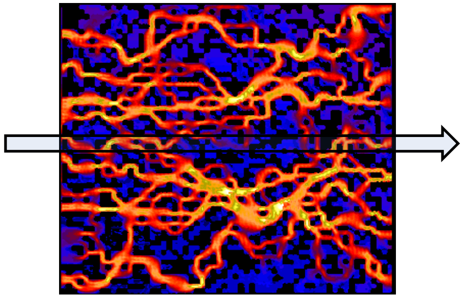
$\Delta P = P_2 - P_1$ -pressure difference (N/m^2);

$\Delta x = x_2 - x_1$ -thickness of the porous medium (m);

η – viscosity coefficient of the fluid (N·s/m^2);

k – permeability of the porous medium (m^2)

Velocity Coding MRI Image



Prof. R. Kimmich, Uni-Ulm, Germania

Observations:

- In the above relation **the viscosity coefficient (η)** *of the fluid is a characteristic quantity that depends on the friction between the molecules*

$$\eta_{water} = 10^{-3}\,\mathrm{Ns/m^2};$$

$$\eta_{oil}(5W40) = 287.23\cdot10^{-3}\,\mathrm{Ns/m^2} \text{ la } 20°\mathrm{C}$$

- The viscosity coefficient decreases with increasing liquid temperature.

- The **permeability** k corresponding to the pore medium depends on porosity, pore size and tortuosity. It is considered that, in general, permeability scales with the square of the average pore dimension d:

$$k = Cd^2$$

The measuring unit for permeability is: $[k]_{SI} = m^2$

Alternatively, it is expressed in Darcy: $1Darcy \equiv 1D = 10^{-12}m^2$

3. Molecular migration by diffusion

- ***Diffusion*** = *the random movement of atoms or molecules of a fluid (gas, liquid) as a result of their thermal agitation energy.*

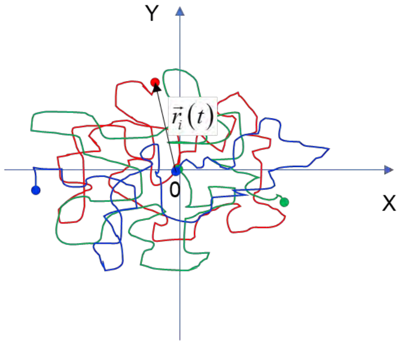
Due to diffusion, the atom moves from its initial position, and the displacement follows a random trajectory. If we consider the environment through which the particles move as isotropic (the same properties in all directions) and unrestricted, then the average deviation of molecules from their initial position at a given point in time is:

$$\langle \vec{r}(t) \rangle = 0$$

On the other hand, the mean square displacement of the molecules, after a certain diffusion time t satisfies the law:

$$\boxed{\langle \vec{r}^{\,2}(t) \rangle = 6Dt}$$ -Einstein's law

Displacement of 3 molecules from their initial position



Here we have: $\langle \vec{r}^{\,2}(t) \rangle = \frac{1}{N}\sum_{i=1}^{N} \vec{r}_i^{\,2}(t)$ – mean squared displacement relative to the initial position;

$\vec{r}_i$ – position of the atom i at an instant t;

D – diffusion coefficient (m^2/s). $\rightarrow D_{H_2O} = 2.3 \cdot 10^{-9}\ m^2/s$

Observations:

- The **diffusion coefficient (D)** plays an essential role in describing the random movement of molecules as a result of their thermal agitation energy. D depends on the nature of the medium through which the diffusing molecules move and on the geometric characteristics of these molecules (size and shape)
- In the case of pure liquids, the diffusion coefficient satisfies the relationship:

$$\boxed{D = \frac{kT}{f}}$$

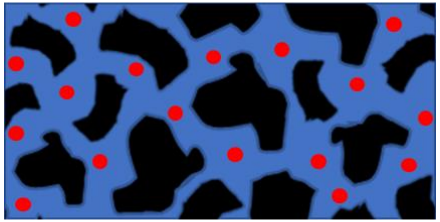
k – Boltzmann's constant; T – temperature (K);

$f = 6\pi\eta R$ – friction factor in the case of spherical molecules of radius R;

η – viscosity coefficient of the liquid in which the molecule displaces

Self-diffusion in porous media

- If a liquid uniformly fills a porous medium and a 2nd liquid (molecules drawn in red) is dissolved in this liquid, then the molecules of the 2nd liquid will not be characterized by a net transport of the substance but will execute random movements characterized by an average quadratic deviation depending on the porous medium, the geometric characteristics of the molecules that diffuse and the liquid (blue) medium in which they diffuse.
- The self-diffusion coefficient of the molecules of the 2nd liquid (red) through the porous medium depends on the characteristics of the porous medium as follows:



$$D = D_0 \Phi^m$$ -Archie's Law

D – diffusion coefficient inside the porous medium;

D_0 – diffusion coefficient in the bulk liquid;

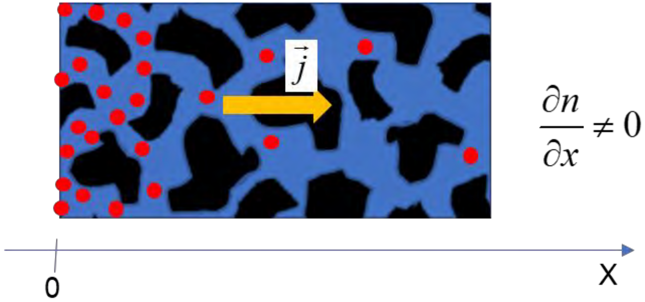
$\Phi = \frac{V_P}{V}$ – porosity of the material;

m – empiric coefficient (experimentaly determined).

Obs.: measurement of the diffusion coefficient D (e.g. by NMR techniques) allows the extraction of information about the porous medium to be studied

Diffusion induced by a concentration gradient

- If the molecules of a liquid (drawn in red) are concentrated more in a certain region of the porous medium, then they will leave that region, producing a molecular current density described by Fick's laws:



$$\frac{\partial n}{\partial x} \neq 0$$

$$\vec{j} = -D\nabla n(\vec{r},t)$$ – low I of Fick

$$\frac{\partial n(\vec{r},t)}{\partial t} = D\nabla^2 n(\vec{r},t)$$ – low II of Fick

$\vec{j}$ – molecular current density

(nr. of molecules through unit area in unit time)

D – diffusion coefficient inside porous medium;

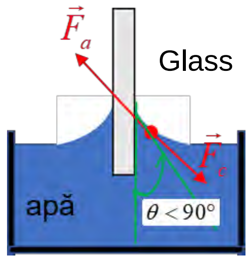
n – molecule concentration in a certain region

Obs.: These equations describe the ingress of pollutants through a porous medium soaked with a liquid or even the ingress of that liquid.

4. Liquid transport by capillary suction

It is experimentally observed that some liquids adhere to the surface of solids (wet the surface) and others do not adhere (do not wet). The explanation of this phenomenon lies in the balance of forces in the system and in the difference between the **adhesion** and **cohesion forces**, as illustrated in the figure below.

The liquid wets the surface:



$$F_a > F_c \Leftrightarrow \theta < 90°$$

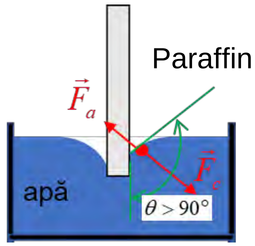
F_a – adhesion force from the solid;

F_c – cohesion force from other water molecules;

θ – contact angle

The liquid wets the surface if the adhesion forces are greater than the cohesion forces and the contact angle is less than 90°

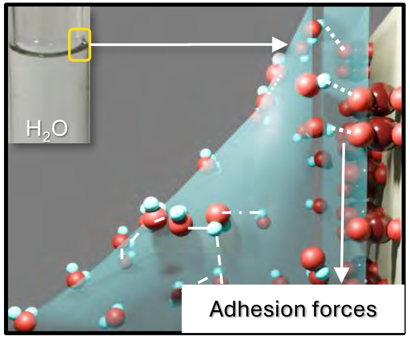
The liquid does not wet the surface:

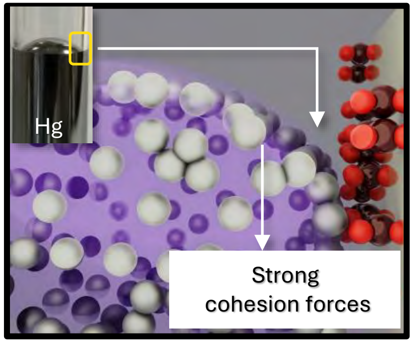


$$F_a < F_c \Leftrightarrow \theta > 90°$$

The liquid does not wet the surface if the adhesion forces are less than the cohesion forces and the contact angle is greater than 90°

Adhesion and cohesion in tubes

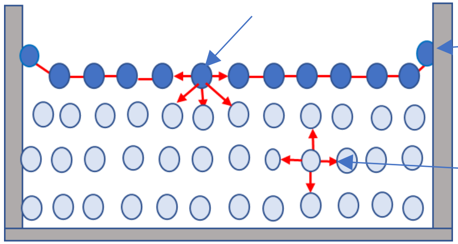




4.1 Surface tension and Laplace's law

- Cohesion forces (dipole or hydrogen bonds) act between the liquid molecules, the effect of which is to create a surface layer that acts as a **membrane** that compresses the liquid (surface tension is exerted). That is why liquids, unlike gases, are already compressed and changing their volume is possible (additional compression) only under the action of extreme pressures.

A net downward cohesive force acts on the molecules on the surface, compressing the liquid like a membrane



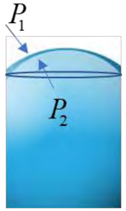
Both adhesion and cohesion forces act

Cohesion forces that cancel each other out

If we put a paper clip on that imaginary membrane, it will float. There are also insects that can walk on water (e.g. the water spider), moving like a membrane.

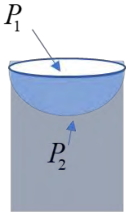
- Due to the curvature of the liquid surface, the membrane formed by the surface layer **produces an increase or decrease in the pressure** of the liquid depending on how it is oriented. The modification of the pressure under the membrane satisfies **Laplace's law**:

The pressure increases under the membrane



$P_2 = P_1 + \Delta P$

The pressure drops under the membrane



$P_2 = P_1 - \Delta P$

$$\Delta P = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$\Delta P = P_2 - P_1$ -pressure difference induced by the surface layer (membrane);

R_1, R_2 – curvarure radii defining the surface;

σ – surface tension coefficient of the liquid;

i.e. $\sigma_{H_2O} = 72.7 \cdot 10^{-3}$ N/m; $\sigma_{Hg} = 472 \cdot 10^{-3}$ N/m;

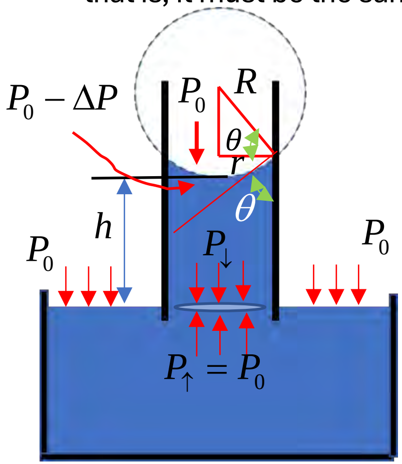
If the membrane is spherical: $R_1 = R_2 = R$

$$\Delta P = \frac{2\sigma}{R}$$

Laplace's law governs the transport of fluids through porous media based on the phenomenon of capillary suction

4.2. Ascent of fluids in capillary tubes. Jurin's Law

- Starting from Laplace's law, which describes the changes in pressure introduced by the curved surface of a liquid, we can calculate the **capillary ascension** h (or depression if the liquid does not wet the walls of the tube) of the liquid through the tube.
- To calculate capillary ascension we observe that the pressure at the base of the tube exerted on an imaginary surface must be equal, that is, it must be the same from bottom to top as from top to bottom. It is written mathematically as follows:



R – meniscus radius (m);

r – tube radius (m);

h – capillary ascension (m);

P_0 – athmospheric pressure (N/m^2);

$\rho g h$ – hydrostatic pressure (N/m^2); (exerted by the liquid column)

θ-contact angle (rad);

(at very thin tubes θ=0)

$$\left.\begin{array}{l} \boxed{P_\uparrow = P_\downarrow} \\ P_\uparrow = P_0 \\ P_\downarrow = P_0 - \Delta P + \rho g h \end{array}\right\} \Rightarrow P_0 = P_0 - \Delta P + \rho g h \Leftrightarrow \Delta P = \rho g h \Leftrightarrow h = \frac{\Delta P}{\rho g}$$

According to Laplace's law, the change in pressure induced by the concavity of the spherical surface of radius R is:

$$\Delta P = \frac{2\sigma}{R} = \frac{2\sigma}{r}\cos\theta$$

We consider the relationship between the radius of the meniscus, R, the radius of the capillary tube, r, and the contact angle θ:

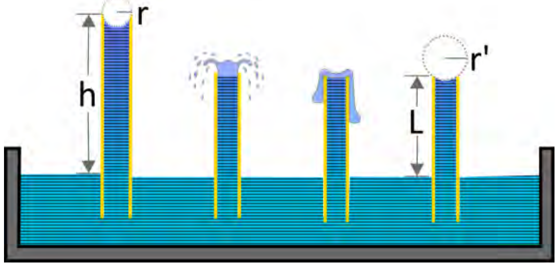
$$R = \frac{r}{\cos\theta}$$

Jurin's Law

$$\boxed{h = \frac{2\sigma}{\rho r g}\cos\theta}$$

Obs. : Jurin's law describes the height to which the liquid rises through the capillary tube and is obtained because of the compensation of the gravitational force by the forces of surface tension. It is observed that h depends on the contact angle. If it is higher than 90 degrees, we are talking about capillary depression. In the case of very thin tubes θ=0.

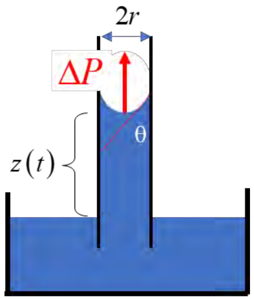
Question:
What happens to the fluid if the capillary is shorter than h?



4.3 Velocity of displacement by capillary suction. The case of porous materials

- The movement of the liquid through the capillary tube as a result of capillary suction does not occur instantaneously but over time, being characterized by a certain speed of advancement.
- If we consider the pressure difference ΔP, described by Laplace's law, as the driving force that pulls the liquid into the tube, we can calculate its **average velocity** by applying Darcy's law and then calculate the **penetration depth** (distance traveled) over time.

Capillary tube



In the case of the capillary tube, Darcy's law is written:

$$\mathrm{v} = k\frac{1}{\eta}\frac{\Delta P}{\Delta x} - \text{Darcy's law}$$

$$\Delta P = \frac{2\sigma}{R} = \frac{2\sigma}{r}\cos\theta - \text{Laplace's law}$$

$$\Delta x = z(t) - \text{height of the liquid inside the tube}$$

$$\Rightarrow \boxed{\mathrm{v} = \frac{1}{z}\frac{k}{\eta}\frac{2\sigma}{r}\cos\theta}$$

Liquid velocity through the capillary tube becomes smaller and smaller as the fluid is absorbed into the tube (z increases)

$$k = C\,r^2 - \text{relationship permeability pore radius}$$

$$C = 1/8 - \text{for a cpillary tube}$$

If we take into account the definition of speed, in the last equation we have:

$$\frac{dz}{dt} = \frac{1}{z}\frac{2Cr\sigma}{\eta}\cos\theta \Leftrightarrow zdz = \left(\frac{k}{\eta}\frac{2\sigma}{r}\cos\theta\right)dt \Leftrightarrow \int_0^{z_\tau} zdz = \left(\frac{k}{\eta}\frac{2\sigma}{r}\cos\theta\right)\int_0^{\tau} dt \Leftrightarrow$$

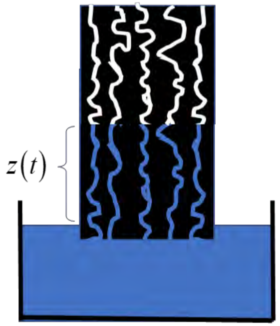
$$\frac{z_\tau^2}{2} = \left(\frac{k}{\eta}\frac{2\sigma}{r}\cos\theta\right)\tau \Leftrightarrow z_\tau = 2\sqrt{\frac{k\sigma\cos\theta}{\eta r}\tau} \Leftrightarrow z(t) = 2\sqrt{\frac{k\sigma\cos\theta}{\eta r}t} \Leftrightarrow z(t) = 2\sqrt{\frac{C\sigma r\cos\theta}{\eta}t} \Leftrightarrow \boxed{z(t) = S\sqrt{t}}$$

Penetration depth as a function of time

$$S = 2\sqrt{\frac{C\sigma r\cos\theta}{\eta}}$$

- suction constant - can be calculated exactly in the case of a capillary (as seen above)

Porous material



In the case of a **porous material**, the **penetration depth** depends on time via the same relationship, $z = S\sqrt{t}$, only in this case the suction constant S must be determined experimentally.

Part II

I
Elements of electrostatics

Content:
1. Introduction
2. Coulomb's Law
3. The electric field intensity vector
4. The electric potential
5. Gauss's Law
6. Applications of Gauss’s law

1. Introduction

- **Electrostatics** is that part of physics that studies electric fields produced by charges at rest**;**
- **Electric charge** is an intrinsic property of elementary particles. These particles modify the space around them, producing **electric fields**;
    - The measuring unit of charge is Coulomb (C)
    - In SI we have: $1C = 1A \cdot 1s$
- There are only two elementary particles that are charged with electric charge: the **proton** (p) and the **electron** (e);

  The electric charge of the **proton** is **positive** and equal to: $1e^{+} = 1.6 \cdot 10^{-19} C$

  The electric charge of the **electron** is negative and equal to: $1e^{-} = -1.6 \cdot 10^{-19} C$

  Although the mass of the proton is much greater than that of the electron ($m_p = 1836 m_e$), the charge is identical in modulus; ($m_p = 1.67 \cdot 10^{-27}$Kg; $m_e = 9.1 \cdot 10^{-31}$Kg;)
- All bodies will have multiple charges of these elementary charges. We can say that **electric charge is quantized**

$$\boxed{Q = ne}$$

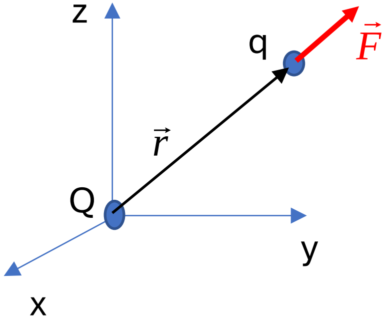
$n -$ no. of excess protons, or electrons

Observations:

1. The total charge in the Universe is 0, that is, there is as much positive charge as there is a negative charge;
2. The total charge of a closed system shall be conserved similarly as the total energy of the system shall be conserved;
3. There are other charged particles (positrons, muons, quarks) but these do not manifest themselves in ordinary energy physics, of interest in engineering;
4. Charging with electric charge of a body is done by removing electrons or bringing electrons onto it;
5. Bringing or removing electric charges can be done by physical contact between two differently charged bodies, by friction of the bodies, or by radiation (including visible light)

2. Coulomb’s law

- ***The force*** *of interaction between two point charges Q and q at a distance r from each other is given by Coulomb's law (1780, French physicist Charles Augustin de Coulomb):*



$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r}$$

$\vec{F}$ – force of Q acting on q

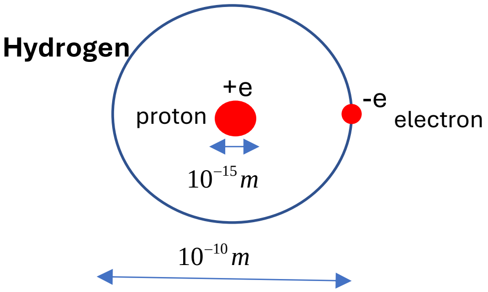
r-distance between charges;

$\hat{r}$- unit vector of the pozition vector $\vec{r}$ pointing from Q to q;

$\varepsilon_0 = 8.85\cdot10^{-12} C^2/Nm$ – electric permitivity of vacuum

$$\frac{1}{4\pi\varepsilon_0} \cong 9\cdot10^9 \frac{Nm^2}{C^2}$$ – approximation used in applications

The electric force is much stronger than the gravitational force:



Observations:

- If the charges are of the same sign (++or +-) the force will be repulsive;
- If the charges are of different signs (- + or + -) the force will be of attraction;
- According to the principle of **action and reaction**, an equal force in the opposite sense will also act on the charge Q:

$$\left.\begin{array}{l} F_{el} \simeq 8.2\cdot10^{-8} N \\ F_{gr} \simeq 3.6\cdot10^{-47} N \end{array}\right\} \Rightarrow \frac{F_{el}}{F_{gr}} \simeq 2.2\cdot10^{39}$$

⇨ Gravitational forces can be neglected in the study of the atom

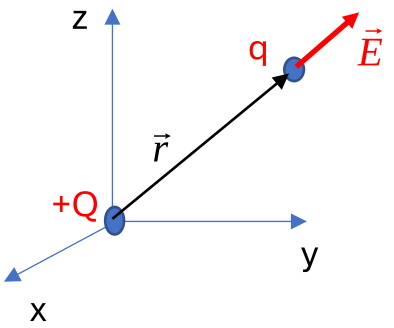
3. The electric field intensity vector ($\vec{E}$)

- The **electric field** is a **modified state of space** that manifests itself by **exerting a force on electric charges**.
- It **can be produced by electric charges** or **variable magnetic fields** (we will see in the following chapters);
- The electric field is characterized by the vectorial physical quantity called **electric intensity ($\vec{E}$)**

Definition: ***The electric field intensity ($\vec{E}$)*** *is the force acting on the unit, positive charge, placed in the field (Electric force on a +1C of charge)*

- The electric force acting on the charge q placed in the electric field is: $\boxed{\vec{F}_e = q\vec{E}} \Rightarrow [E]_{SI} = \frac{N}{C} = \frac{V}{m}$

3.1. The intensity of the electric field produced by a point charge Q



The force exerted by charge Q on charge q is given by Coulomb's law:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r}$$

The field intensity E is the force exerted on the unit charge:

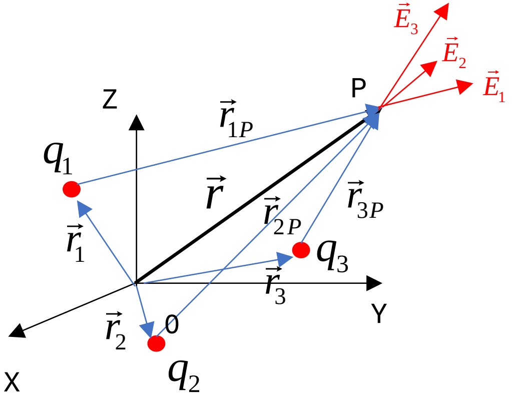
$$\vec{E} = \frac{\vec{F}}{q}$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}}$$

Electric field intensity

3.2. The intensity of the electric field produced by a system of point charges

In the case of a system of point charges, the **total intensity** produced by all charges at a point P shall be calculated as the vector sum of the intensities produced by each load in the system:



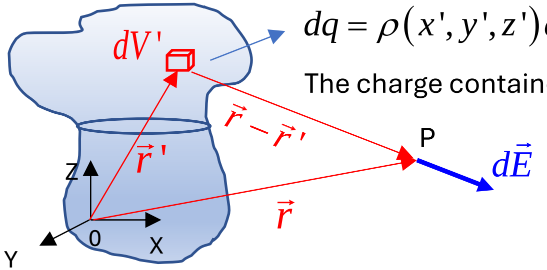
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}^2}\hat{r}_{3P} + ...$$

q_1, q_2, q_3 -charges in the system;

$\vec{r}_{1P} = \vec{r} - \vec{r}_1;\ \ \vec{r}_{2P} = \vec{r} - \vec{r}_2;\ \ \vec{r}_{3P} = \vec{r} - \vec{r}_3$ -vectors connecting the charges with P

3.3. The intensity of the electric field produced by a charge distribution

If the electric charge is continuously distributed in space, then the electric field it produces at the P-point can be calculated as a vector sum of the electric fields produced by the volume elements into which the given distribution can be decomposed.



$dq = \rho(x', y', z')dV'$

The charge contained inside the volume element

$\rho(x', y', z')$ – charge density in the position (x', y', z') =

= charge of the unit volume in the position (x', y', z') (C/m^3)

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(\vec{r} - \vec{r}\,')^2}\vec{r} - \vec{r}\,'$$

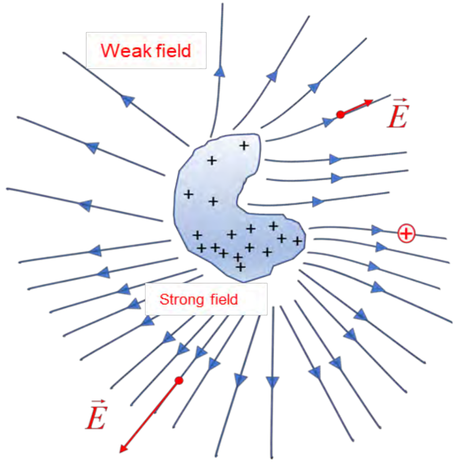
- the elementary electric field produced by the volume element dV'

3.4. Representation of the electric field by field lines

The electric field produced by a system of charges or by a single charge may be represented by **field lines** if the following rules are considered:

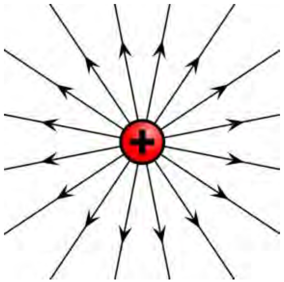
1. The *density* of the field lines *is proportional to the strength of the field* (magnitude/magnitude of vector E);
2. The *orientation and direction* of a field line indicate *the path along which a positive charge placed in the field would travel*;
3. The electric field intensity vector ***E*** *is always tangent to field lines*.

Positive charge distribution

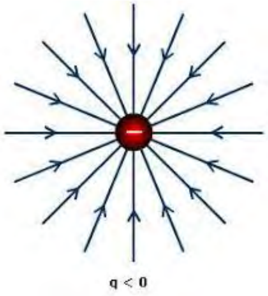


The arrow indicates the direction of movement of a positive charge

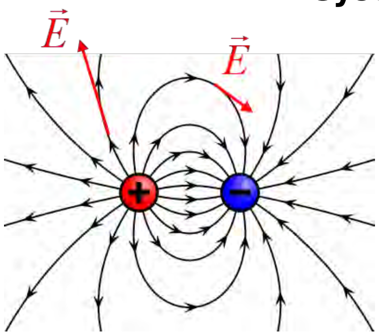
Positive charge

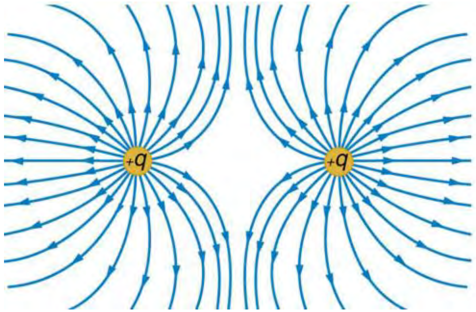


Negative charge



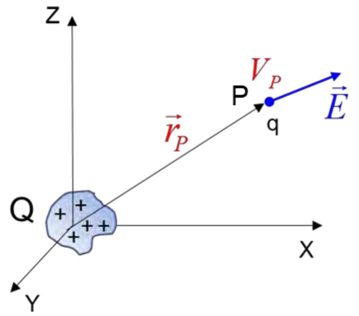
System of two charges





4. The electric potential (V)

- We define the electric potential (V) produced by charge Q at point P as the work done by the electric field to move the unit charge of from point P to infinity



$$\boxed{V_p = \frac{L_{P\infty}}{q}} = \frac{\int_P^\infty \vec{F}\cdot d\vec{r}}{q} = \frac{\int_P^\infty q\vec{E}\cdot d\vec{r}}{q} = \int_P^\infty \vec{E}\cdot d\vec{r} \Leftrightarrow \boxed{V_p = \int_P^\infty \vec{E}\cdot d\vec{r}}$$

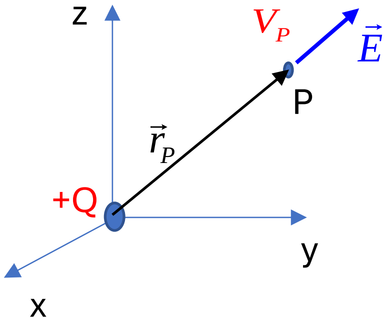
The potential of the electric field at a certain point in space

Here we considered that: $\vec{F} = q\vec{E} - electric\ force$

The electric field potential is a **scalar** quantity and is measured in Volt (V)

4.1. The potential of a point charge

- The potential produced by a point charge Q at a point P can be calculated from the relation of definition, in which we take into account the field strength produced by the point charge:



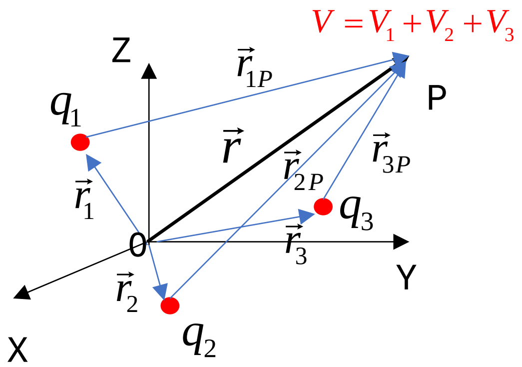
$$\left.\begin{array}{l} V_p = \int_P^\infty \vec{E}\cdot d\vec{r} \\ \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \end{array}\right\} \Rightarrow V_p = \frac{Q}{4\pi\varepsilon_0}\int_{r_P}^\infty \frac{1}{r^2}dr \Leftrightarrow V_p = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_P} \Leftrightarrow \boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

The electric potential produced by the point charge Q at distance r from it.

Here we took into account that: $\hat{r}\cdot d\vec{r} = dr$

4.2. The potential of the electric field produced by a system of point charges

In the case of a point charge system, the total potential produced by charges at a point P is calculated as the algebraic sum of the potentials produced by each charge in the system:



$$V = V_1 + V_2 + V_2 + \ldots = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}} + \ldots$$

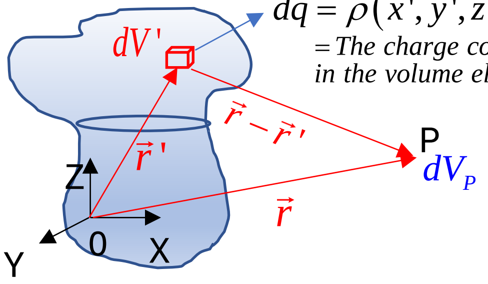
q_1, q_2, q_3 - charges of the system;

$r_{1P} = |\vec{r} - \vec{r}_1|; \;\; r_{2P} = |\vec{r} - \vec{r}_2|; \;\; r_{3P} = |\vec{r} - \vec{r}_3|$

(the magnitude of the vectors connecting the charges q_i with point P)

4.3. The electric field potential produced by a charge distribution

If electric charge is continuously distributed in space, then the electric field potential it produces at point P can be calculated as an algebraic sum of the electric fields produced by volume elements into which the given distribution can be decomposed.



$dq = \rho(x', y', z')dV'$

= *The charge contained in the volume element dV'*

$\rho(x', y', z')$ – charge density in the position (x', y', z') =

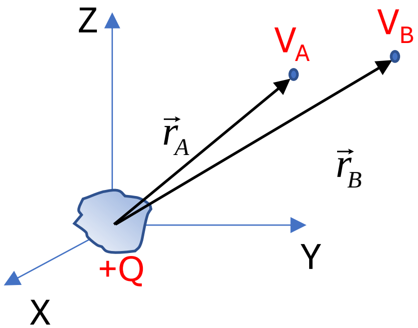
= charge of the unit volume in the position (x', y', z')(C/m^3)

$$dV_P = \frac{1}{4\pi\varepsilon_0}\frac{dq}{|\vec{r} - \vec{r}\,'|} = \frac{1}{4\pi\varepsilon_0}\frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}dx'dy'dz'$$

(the elementary electric potential produced by the volume element dV')

4.4. Potential difference. Relation between intensity (E) and potential (V)

Consider two points in space A and B and aim to calculate the potential difference between the two points.



$$\Delta V = V_B - V_A = \int_{r_B}^{\infty} E\cdot dr - \int_{r_A}^{\infty} E\cdot dr = -\int_{\infty}^{r_B} E\cdot dr - \int_{r_A}^{\infty} E\cdot dr = -\int_{r_A}^{r_B} E\cdot dr \Leftrightarrow \boxed{\Delta V = -\int_{r_A}^{r_B} E\cdot dr}$$

The potential difference between two points A and B

If the two points A and B are very close then:

$$\Delta V \rightarrow dV = -\vec{E}\cdot d\vec{r}$$

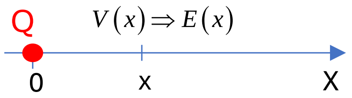
$$\Rightarrow \boxed{\vec{E} = -\frac{dV}{d\vec{r}} = -grad\left(V\right) = -\nabla V}$$

The relation between the electric field intensity and the electric potential

Note: the relationship between electric field intensity **E** and the electric potential V allows to obtain the intensity (vector quantity) as the spatial derivative of the potential (scalar magnitude)

Example:

The calculation of the electric field strength produced by a point charge Q at position x

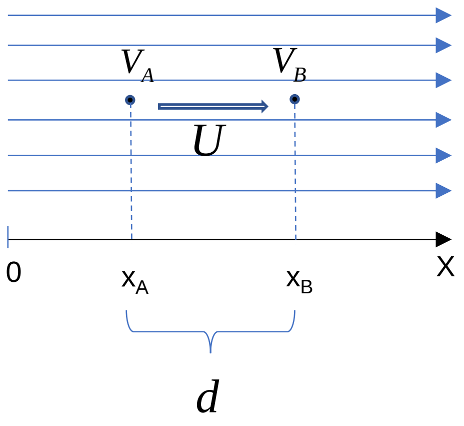


$$\left.\begin{array}{l} V(x) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x} \\ E(x) = -\dfrac{\partial V(x)}{\partial x} \end{array}\right\} \Rightarrow E(x) = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$$

4.5. The potential difference in the case of a constant electric field. Electric tension (Voltage)

- The **potential difference** between two points in space in a constant electric field, as shown in the figure, is:

$\vec{E} = cnst.$



$$\Delta V = V_B - V_A = -\int_{x_A}^{x_B} \vec{E}\cdot d\vec{r} = -\int_{x_A}^{x_B} E dx = -E\left(x_B - x_A\right) = -Ed$$

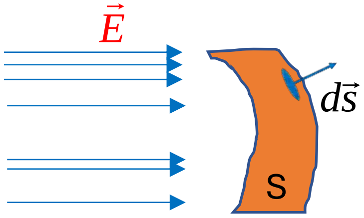
Since the difference is negative, it follows that we have a potential drop from point A to point B. We denote that potential drop with U and call it electric tension (or voltage)

$$U = -\Delta V = Ed \quad \Leftrightarrow \quad \boxed{U = Ed} \quad \Leftrightarrow \quad \boxed{E = \frac{U}{d}}$$

The connection between electric tension (U) and electric field strength (E) in the case of a constant electric field

4.6. The flux of the electric field through a certain surface

- The flux of the electric field through a given surface is defined by the integral of the inner product **E** · d**s** on that surface:

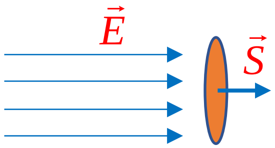


$$\Phi = \iint_S \vec{E} \cdot d\vec{s} \qquad [\Phi]_{SI} = \frac{V}{m} m^2 = Vm$$

S- The total surface (orange region)
$d\vec{s}$-surface element, oriented

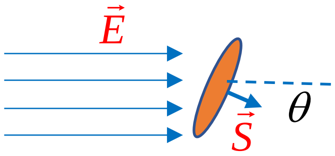
Observations:

- If the electric field intensity vector is constant on the surface S and additionally perpendicular to it, then the electric field flux is calculated as follows:



$$\Phi = ES$$

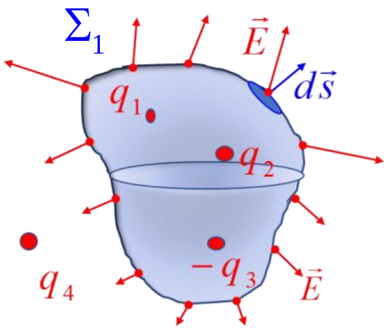
- If the electric field strength vector is constant on the surface S but oriented at an angle θ to it, then the electric field flux is calculated as follows:



$$\Phi = \vec{E} \cdot \vec{S} = ES\cos\theta$$

5. Gauss's law

- Gauss's law allows the calculation of the strength of the electric field E produced by symmetric configurations of electric charge for which a Gaussian surface **Σ** can be chosen so that the flux **Φ** of the electric field **E** throulg this area to be easily calculated;
- Gauss's law states that the flux of electric field E through a closed surface Σ_1 (also called Gaussian) depends only on the electric charge contained inside that surface, and satisfies the relationship:



$$\Phi_\Sigma = \frac{Q_{\text{int}}}{\varepsilon_0}$$

-the mathematical formulation of Gauss's law

Q_{int} – the total charge inside surface Σ

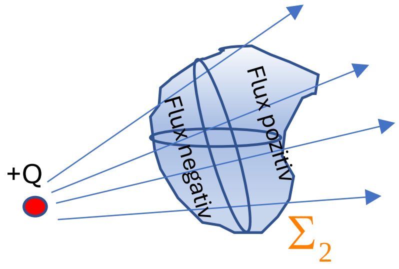
$$\Phi_\Sigma = \oiint_{\Sigma_1} \vec{E} \cdot d\vec{s}$$

-the electric field flux through the enclosed surface Σ

-represents an infinite sum of elementary fluxes

$$d\Phi = \vec{E} \cdot d\vec{s}$$

The charge outside the surface Σ_2 does not produce a flux through this surface

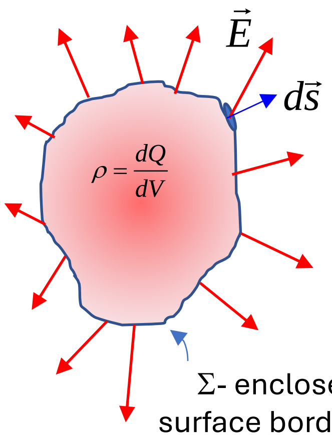


Observations:

- The total charge inside Σ_1 in the case represented in the figure: $Q_{int}=q_1+q_2-q_3$;
- One can observe that q_4, which is located outside Σ_1 does not contribute to the electric field flux
- The law described by the above formula will be applied in the following for some practical applications

The integral and differential form of Gauss's law

Gauss's law can easily be put into integral form if one expresses the charge inside the closed surface as a function of the charge density in volume V bounded by the surface.



Σ- enclosed surface bordering volume V

$$\Phi_\Sigma = \frac{Q_{int}}{\varepsilon_0} - \text{Gauss law}$$

$$\Phi_\Sigma = \oiint_\Sigma \vec{E}\cdot d\vec{s} - \text{flux through } \Sigma$$

$$Q_{int} = \iiint_V \rho dV - \text{charge inside V}$$

The integral form of Gauss's law

$$\oiint_\Sigma \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\iiint_V \rho dV$$

$$\rho = \frac{dQ}{dV} - \text{charge density} \qquad [\rho]_{SI} = \text{C/m}^3$$

If in the integral form of Gauss's law one takes into account the Gauss-Green-Ostrogradsky formula:

$$\oiint_\Sigma \vec{E}\cdot d\vec{s} = \iiint_V \left(\nabla\cdot\vec{E}\right) dV$$

which converts an integral on the enclosed surface Σ into an integral on volume V bounded by that surface, yields:

$$\iiint_V \left(\nabla\cdot\vec{E}\right) dV = \frac{1}{\varepsilon_0}\iiint_V \rho dV$$

The differential form of Gauss's law

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\text{where } \nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

(the divergence of the $\vec{E}$ vector)

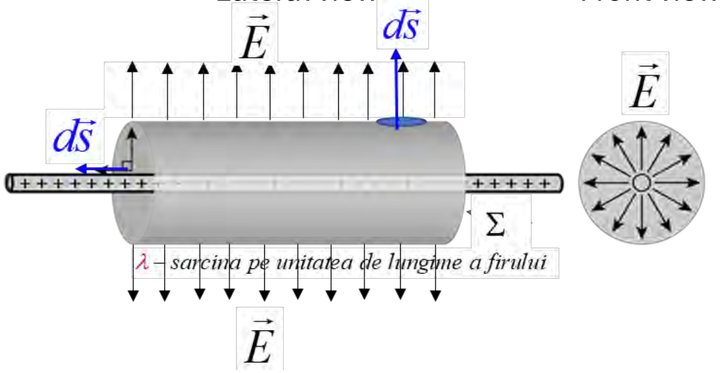
The differential form of Gauss's law shows that electric field lines diverge

6. Applications of Gauss's law

6.1 The intensity of the electric field produced by an infinitely long wire

- Consider an infinitely long wire charged with the linear density of electric charge λ. The unit of λ is C/m.
- We ask ourselves: what is the electric field strength at distance r from the wire?
- To answer this question, we draw a cylindrical Gaussian surface wrapping the wire and observe that the electric field strength on this surface is constant and perpendicular to the surface Σ

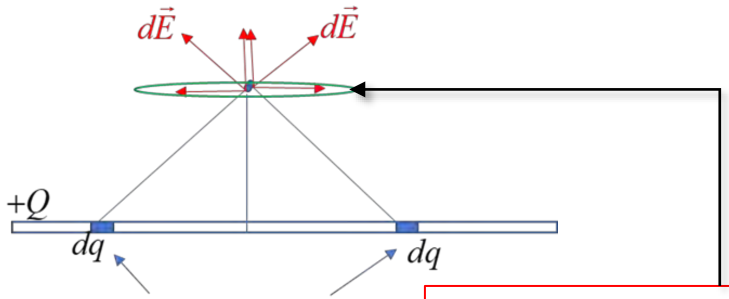
Lateral view Front view



Observations:

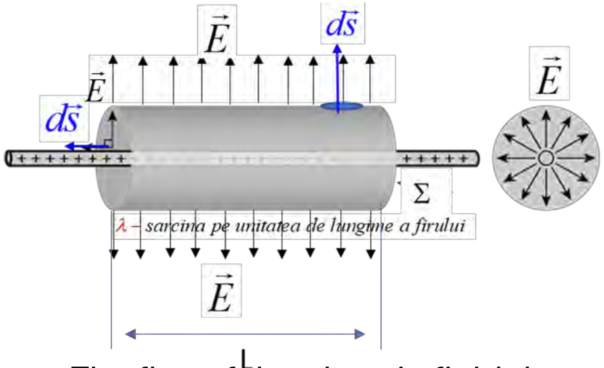
- The electric field lines around the wire are radially oriented and perpendicular to the wire.
- This happens *only* for an infinite wire because the components of vector E parallel to the wire cancel each other out

Demonstration that the electric field vector is perpendicular to the wire



Symmetrically chosen wire elements

These components of the electric field produced by two symmetrical elements in the wire cancel each other out. It follows that only the component perpendicular to the wire survives, which gives an electric field perpendicular to the wire.



- To calculate the strength of the electric field, we apply Gauss's law, in which we consider the following observations:

$$\Phi_\Sigma = \frac{Q_{int}}{\varepsilon_0}$$

The Mathematical formulation of Gauss's law

- The flux of the electric field through the enclosed surface Σ is calculated as the sum of the flux through the lateral surface and the flux through the two bases:

$$\Phi_\Sigma = \iint_\Sigma E\cdot ds = \iint_{\Sigma lateral} E\cdot ds + \oiint_{\Sigma base1} \vec{E}\cdot d\vec{s} + \oiint_{\Sigma base2} \vec{E}\cdot d\vec{s} = \oiint_{\Sigma lateral} E\cdot ds = \oiint_{\Sigma lateral} Eds = E\iint_{\Sigma lateral} ds = E2\pi rL$$

(they are zero because $\vec{E}\perp d\vec{s}$) (here we used $\vec{E}||d\vec{s}$ si E=cnst. on the lateral surface)

- The charge contained inside the Gaussian surface is that contained in the length L of the wire. It can be expressed as it follows:

$$Q_{int} = \lambda L$$

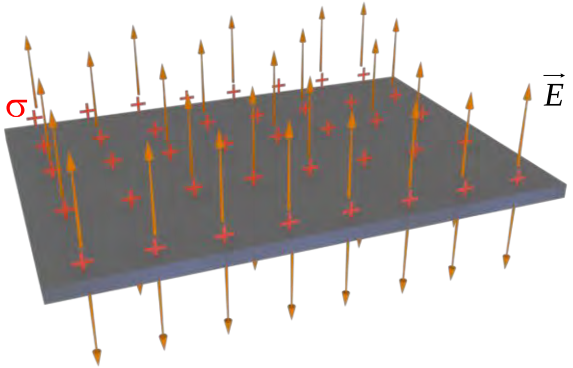
Substituting Φ_Σ and Q_{int} into the mathematical formula of Gauss's law, we get:

$$E2\pi rL = \frac{\lambda L}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The electric field intensity at distance r from an infinitely long and straight wire

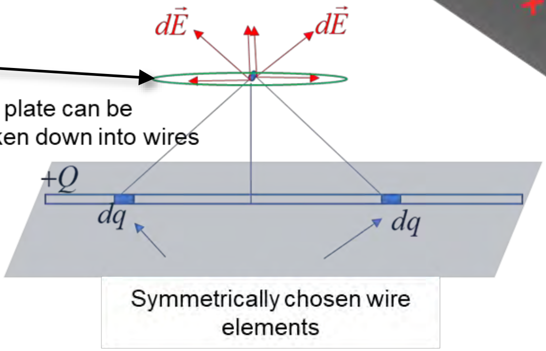
6.2.The intensity of the electric field produced by an infinite plate

- Consider an infinite plate charged with the surface density of electric charge σ. The unit of measurement for σ is C/m^2.
- We ask ourselves: what is the electric field strength at distance h from the plate?
- To answer this question, we draw a Gaussian surface Σ, cylindrical, which cuts the plate and observe that the electric field strength on the two bases is constant and perpendicular to the surface of the bases



Demonstration that the electric field vector is perpendicular to the plate

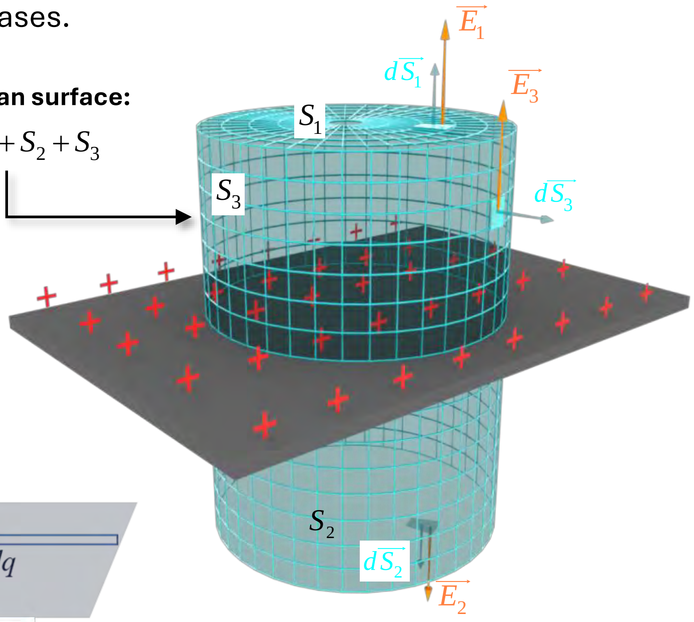
These components of the electric field produced by two symmetrical elements in the wire cancel each other out. It follows that only the component perpendicular to the wire survives, which gives an electric field perpendicular to the wire. Since the plate can be broken down as a sum of wires, it follows that the electric field ($\vec{E}$) produced by the plate is perpendicular to it



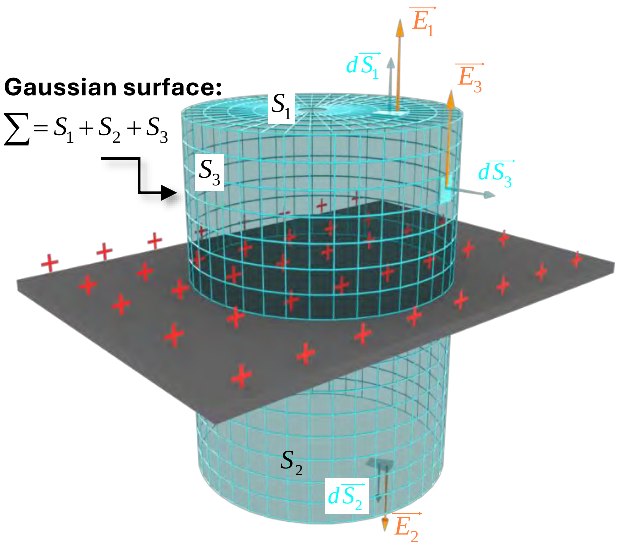
- We ask ourselves: what is the electric field strength at distance h from the plate?
- To answer this question, we draw a Gaussian cylindrical surface Σ, which cuts the plate and observe that the electric field strength on the two bases is constant and perpendicular to the surface of the bases.

Gaussian surface:

$$\sum = S_1 + S_2 + S_3$$



- To calculate the intensity of the electric field, we apply Gauss's law, in which we take into account the observations below.



$$\Phi_\Sigma = \frac{Q_{int}}{\varepsilon_0}$$

The Mathematical formulation of Gauss's law

- The flux of the electric field through the enclosed surface Σ is calculated as the sum of the flux through the lateral surface and the flux through the two bases:

$$\Phi_\Sigma = \iint_\Sigma E \cdot ds = \iint_{\Sigma lateral} E \cdot ds + \iint_{\Sigma base1} E \cdot ds_1 + \iint_{\Sigma base2} E \cdot ds_2 = 2 \oiint_{\Sigma base1} E \cdot ds_1 = 2E \oiint_{\Sigma base1} ds_1 = 2ES_1$$

(this integral is zero because $\vec{E} \perp d\vec{s}_3 \Leftrightarrow$ electic field lines do not cross this surface)

(the flux through these two basis is equal)

(here $\vec{E} \| d\vec{s}_1$ si E=cnst. on basis surface)

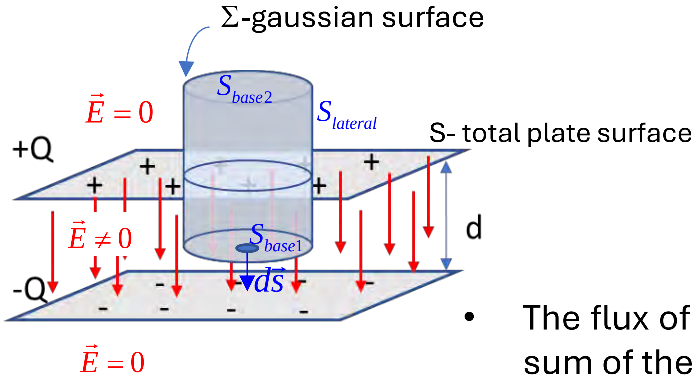
- The charge inside the Gaussian surface (cylinder) is that contained in the cut section of the cylinder from the plane. It can be expressed as follows: $Q_{int} = \sigma S_1$

Substituting Φ_Σ and Q_{int}, obtained above, in the mathematical expression of Gauss's law, one obtains:

$$E = \frac{\sigma}{2\varepsilon_0}$$

-the intensity of the electric field produced by the plate;
-it is observed that the field strength does not depend on the distance from the plate

6.3. The intensity of the electric field between the plates of a plane capacitor. Electric capacity



- For calculating the intensity of the electric field, we apply Gauss's law, in which we take into account the following observations:

$$\Phi_\Sigma = \frac{Q_{int}}{\varepsilon_0}$$

- the mathematical formulation of Gauss's law

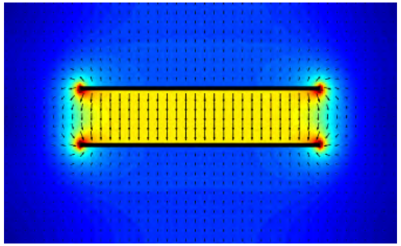
- The flux of the electric field through the enclosed surface Σ is calculated as the sum of the flux through the lateral surface and the flux through the two bases:

$$\Phi_\Sigma = \iint_\Sigma E\cdot ds = \oiint_{Slateral} \vec{E}\cdot d\vec{s} + \iint_{Sbase1} E\cdot d\vec{s} + \oiint_{Sbase2} \vec{E}\cdot d\vec{s}_2 = \iint_{Sbase1} E\cdot ds == E\oiint_{Sbase1} ds = ES_{base1} = ES_{base}$$

(the integral is zero because $\vec{E}||Slateral \Leftrightarrow$ electric field lines do not cross this surface)

(zero because E=0 in exterior)

($\vec{E}||d\vec{s}_1$ and E=cnst. on the surface of base1)



Simulation of intensity inside and outside the capacitor. Edge effects are observed. If the capacitor is very large (S>>d), edge effects are neglected and Gauss's law can be applied to field calculation. Also, outside the capacitor can be considered E=0

http://www.drjamesnagel.com/EM_Beauty.htm

- The charge inside the Gaussian surface (cylinder) is that contained in the cut section of the cylinder from the plane. It can be expressed as follows:

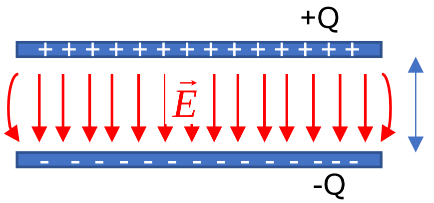
$$Q_{int} = \sigma S_{base}$$

Substituting Φ_Σ si Q_{int} , obtained above, in the mathematical formulation of Gauss's law, one obtains:

$$E = \frac{\sigma}{\varepsilon_0}$$

- the intensity of the electric field produced between the capacitor plates if edge effects are neglected;

The capacitance of the plane capacitor



Q = the charge on one plate

S = the total surface of one plate;

d = the distance between plates;

U = electrical voltage between plates;

$\varepsilon_0 = 8.854 \times 10^{-12}$ [F/m]

(the electrical permittivity of vacuum)

- ***Electric capacitance*** *(C) is a physical quantity expressing the ability of a body to store electric charge.*
- ***In the case of an insulated conductor,*** *the electrical capacitance is defined by the ratio of the electric charge Q of the conductor to its potential V: C = Q / V*
- ***In the case of a plane capacitor,*** *the capacitance is defined by the ratio of the capacitor's electric charge Q to the potential difference U between the plates:*

$$C = \frac{Q}{U}$$

The measuring unit for the electrical capacitance in SI is Farad: **1F=1C/1V**

Note*: most capacitors used in practice have capacities ranging from pico-Farad to hundreds of micro-Farad. However, in recent years, supercapacitors have also been developed with capacities in the order of thousands of Farads that can replace batteries at some point.*

- Starting from the definition relation for plane capacitor capacitance and using the relationships below (previously deduced), we can calculate the capacitance of a plane capacitor as follows:

$C = \frac{Q}{U}$ - capacitance;

$U = Ed$ - the voltage between the plates;

$E = \frac{\sigma}{\varepsilon_0}$ -electric field intensity between plates;

$\sigma = \frac{Q}{S}$ -surface charge density;

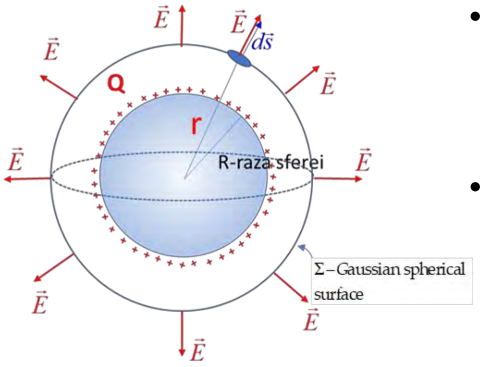
$$\Rightarrow C = \frac{Q}{U} = \frac{Q}{Ed} = \frac{Q}{\frac{\sigma}{\varepsilon_0} d} = \frac{Q}{\frac{Q}{\varepsilon_0 S} d} = \varepsilon_0 \frac{S}{d} \Leftrightarrow \boxed{C = \varepsilon_0 \frac{S}{d}}$$

The capacitance of a plane capacitor having vacuum (or air) between plates

Note: *if a dielectric with the electric permittivity ε is found between the plates, then the capacitance is given by the relation C= εS/d*

6.4. Conductive sphere (metallic)

- In the case of a conductive sphere, charge Q is evenly distributed over the surface of the sphere because the charges repel each other, thus reaching the state of minimum energy;
- We want to calculate the strength of the electric field at distance r from the center of the sphere;



Note: The intensity is perpendicular to Σ and has the same value at each point on the surface

- For calculating the strength of the electric field, we apply Gauss's law, considering:

$$\boxed{\Phi_\Sigma = \frac{Q_{int}}{\varepsilon_0}}$$ *- The mathematical formulation of Gauss's law*

- The electric field flux through the enclosed surface is:

$$\Phi_\Sigma = \oiint_\Sigma \vec{E} \cdot d\vec{s} = \quad E\oiint_\Sigma ds = E4\pi r^2$$

(here $\vec{E} \| d\vec{s}_1$ and E=cnst. on surface Σ)

$$\oiint_\Sigma ds = 4\pi r^2 \text{ represents the area of } \Sigma$$

- The entire charge is found inside the Gaussian surface Σ: $\quad Q_{int} = Q$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

Intensity at distance r from the center of the sphere

Observations:

- The intensity produced by an electric charge Q evenly distributed over the surface of the sphere is the same as the intensity that the same charge would produce if concentrated in the center of the sphere;
- The **intensity of the electric field inside the sphere is zero**. This can be demonstrated by choosing the surface Σ inside the sphere and observing the lack of electric charge inside it.

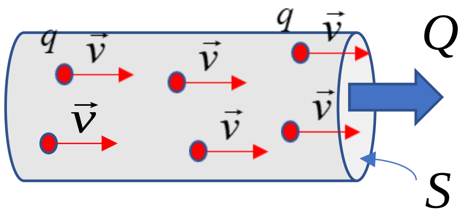
II
Electric current. Ohm's Law

Content:

1. Electric current intensity
2. Microscopic description of intensity. Local and integral Ohm's law
3. The continuity equation

1. Electric current intensity

- Electric current is the **coherent movement** of charge carriers (electrons, protons, ions)
- ***The intensity*** *of electric current represents the electric charge carried by current through the surface S in unit time:*



$$I = \frac{Q}{t}$$

Q - total charge transported through surface S;
t -the time interval during which this charge is transported;
S – the cross-section of the cylinder

The SI unit of intensity is the Ampere: $1A = \frac{1C}{1s}$

Note: The above definition of electric current intensity applies only to stationary currents (does not vary over time). It is an average current over the time interval t.

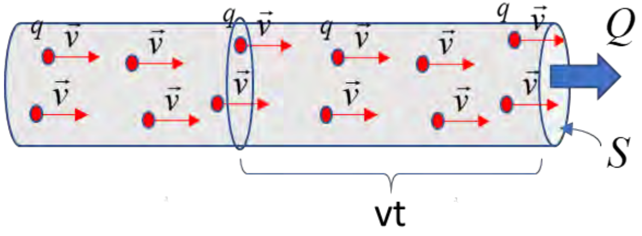
If the current varies over time, we define the **instantaneous current** intensity by the relation:

$$i = \frac{dQ}{dt}$$

dQ - the total elementary charge carried through the surface S in the elementary time interval dt

2. Microscopic description of electric current intensity

- We consider a cylinder (it can be a wire) through which electric charge is transported;
- The charge carriers in this case have each charge q
- We ask ourselves: how can we express the intensity of electric current in terms of charge q, the average speed of the carriers v and the number of carriers in the unit volume n (these are all microscopic characteristics)?



q- charge of a carrier (q=-e in the case of electrons);
n- number of carriers per unit of volume;
S- section of the cylinder (wire);
Q- charge transported during t through section S
v-average (drift) speed of a carrier.

$$I = \frac{Q}{t}$$ - we start from the definition of intensity

the charge Q crossing the section S during a time t is contained in the volume V of a cylinder having the base area S and the height h=vt (same reasoning as in the calculation of wave intensity):

$$Q = qnV = qnSvt$$

$$nV = \text{ nr. of carriers inside V}$$

$$\Rightarrow \boxed{I = qnvS}$$

(The microscopic expression of electrical intensity)

Note:
- The law is written in a vectorial manner to describe local charge transport;
- In the case of electrons q=-e and therefore the current density is oriented in the opposite direction to the speed of movement of electrons. It follows that electrons move through conductors (wires) in the opposite direction of flow of current.

- ***The density of the electric current*** *represents the electric charge carried by current per unit area in unit time:* $\boxed{j = \frac{Q}{St}}$ -the SI unit of current density is **A/m²**

If we use definition of intensity in the expression of current density, we obtain:

$$\left.\begin{aligned} j &= \frac{Q}{St} = \frac{I}{S} \\ I &= qnvS \end{aligned}\right\} \Rightarrow \boxed{\vec{j} = qn\vec{v}}$$

(The microscopic expression for the current density)

2.1. Local Ohm's law

- Starting from the expression of current density: $\vec{j} = qn\vec{v}$

- The average transport speed v of the charge carriers (also called the drift speed) depends on the strength E of the applied electric field (electric force, F=qE).
- It is found that average speed is related to the intensity by the relation:

$$\vec{v} = \mu\vec{E}$$

$$\Rightarrow \vec{j} = qn\mu\vec{E}$$

We denote: $\sigma = nq\mu$

The local form of Ohm's describes the local current density produced by a macroscopic electric field:

$$\Rightarrow \vec{j} = \sigma\vec{E}$$

The mobility of charge carriers (μ)

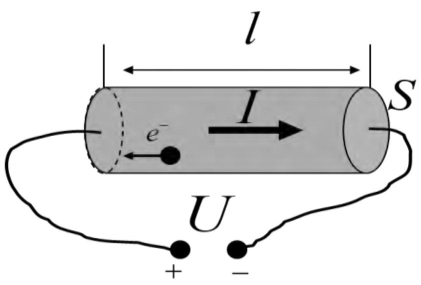
- *It* depends on the characteristics of the material through which the charge carriers (e.g. electrons) are transported, but also on temperature.
- The measuring unit for mobility is m^2/Vs

The electrical conductivity of the material (σ)

- It depends on the characteristics of the material through which the charge carriers (e.g. electrons) are transported, but also on the temperature.
- The unit of measurement for conductivity is $\mathbf{\Omega^{-1}m^{-1}}$

2.2. Integral form of Ohm's law

- Starting from the local Ohm's law and applying it to a conductor of section S and length l between the ends of which to apply a potential difference U (electric voltage), one can obtain the integral Ohm's law.



$I = jS$ -relation between intensity and current density ($j = I / S$)

$j = \sigma E$ -Ohm's local law;

$E = \dfrac{U}{l}$ -intensity of the electric field is constant;

$$\Rightarrow I = \sigma S \frac{U}{l} = \frac{S}{\rho l} U = \frac{U}{\rho \frac{l}{S}} = \frac{U}{R} \Leftrightarrow \boxed{I = \frac{U}{R}}$$

The integral form of Ohm's law —one of the most important laws in electronics

- In obtaining the integral form of Ohm’s law, we introduced two new physical quantities: the **electrical resistivity** and the electrical **resistance**:

$\boxed{\rho = \dfrac{1}{\sigma}}$ -electrical resisitivity

$\boxed{R = \rho \dfrac{l}{S}}$ -electrical resistance

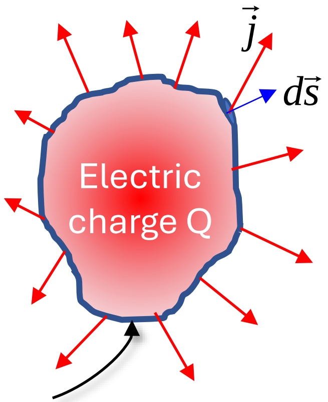
Measuring units:

$[\rho]_{SI} = \Omega \cdot m$

$[R]_{SI} = \Omega = Ohm$

3. The continuity equation

- It describes the transport of electric charge to a specific region of space.
- It is based on the law of conservation of electric charge, one of the fundamental laws of physics



Σ- the closed surface bordering the volume V

The loss of electric charge through the surface Σ in the unit of time is given by relation:

$$\frac{dQ}{dt} = -\oint\!\!\oint_{\Sigma} \vec{j}\cdot d\vec{s}$$ *-conservation of electric charge*

$\vec{j}\cdot d\vec{s}$ - the electrical charge loss through $d\vec{s}$

The charge inside volume V is expressed as a function of the charge density ρ

$$Q = \iiint_V \rho(x,y,z)\,dV$$

$\rho(x,y,z)$ – electric charge density depends on location inside volume V

$$\frac{d}{dt}\iiint_V \rho\,dV = -\oint\!\!\oint_{\Sigma} \vec{j}\cdot d\vec{s}$$

(the integral form of charge conservation law)

We consider the Gauss-Green-Ostrogradsky theorem

$$\oint\!\!\oint_{\Sigma} \vec{j}\cdot d\vec{s} = \iiint_V (\nabla\cdot\vec{j})\,dV \text{ ,where } \nabla\cdot\vec{j} = \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z}$$

$$\frac{d}{dt}\iiint_V \rho\,dV = \iiint_V (\nabla\cdot\vec{j})\,dV$$

$$\nabla\cdot\vec{j} + \frac{d\rho}{dt} = 0$$

- *The continuity equation for the electric charge is contained in Maxwell's set of equations that form the theoretical foundations of electromagnetism*

III
Elements of Magnetostatics

Content:

1. Introduction
2. Characterization of the magnetic field by field lines
3. The Lorentz Force
4. Ampere’s Force
5. The Biot-Savart Law
6. Ampere’s Law
7. Applications of Ampere’s law

1. Introduction

- Magnetostatics studies magnetic fields that do not vary in time (they are static);
- Magnetic field is a modified state of space manifested by the action of a magnetic force on a moving electric charge;
- The magnetic field can be produced by moving electric charges (electric currents) and by magnetic particles (electron, proton). Variable magnetic fields can also be produced by variable electric fields (as presented in next chapter)
- The physical quantity describing the magnetic field is called **magnetic flux density** (B) (or **magnetic induction vector**).
- The measuring unit (SI) for magnetic flux density (or magnetic induction) is Tesla (T). An alternative unit is Gauss ($1T=10^4G$).

Source	Approximate Magnetic Field Strength
Human brain	1-10 pT
Residential electric distribution lines	100 – 500 nT
Microwave oven (30 cm away)	6 μT
Earth’s magnetic field at the surface	25 μT – 65 μT
Refrigerator magnet	50 G (5 mT)
Junkyard electromagnet	1 T
Clinical MRI scanners	0.5 – 3.0 T (typical)
Research MRI scanners (human)	7.0 T – 11.7 T
Laboratory NMR spectrometers	6 – 23 T
Largest pulsed field created in lab	97 T (nondestructively)
Largest pulsed field created in lab	730 T (destructively)

http://mriquestions.com/how-strong-is-30t.html

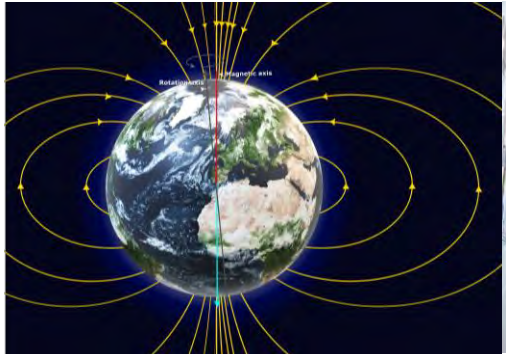


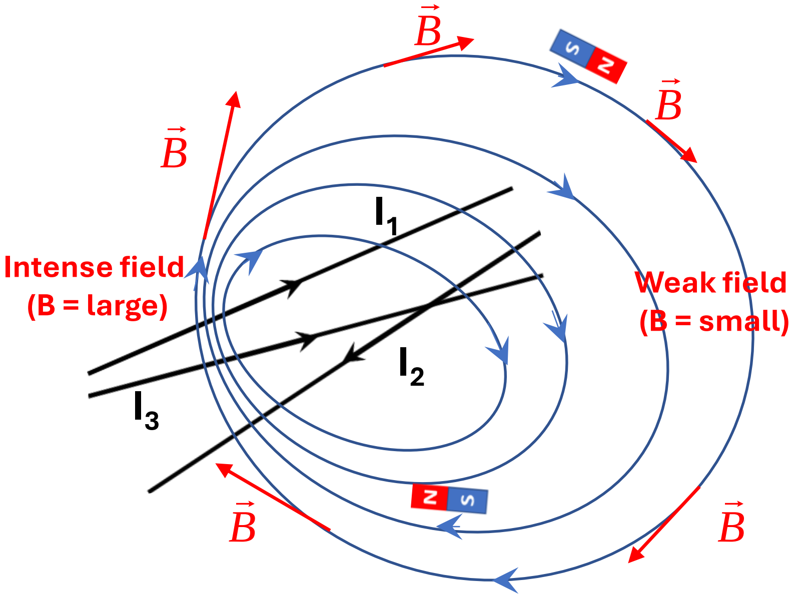
Illustration of Earth's magnetic field (between 25μT at the equator and 62μT at the north pole)

The most powerful electromagnet (45 T))
https://nationalmaglab.org

2. Characterization of the magnetic field by field lines

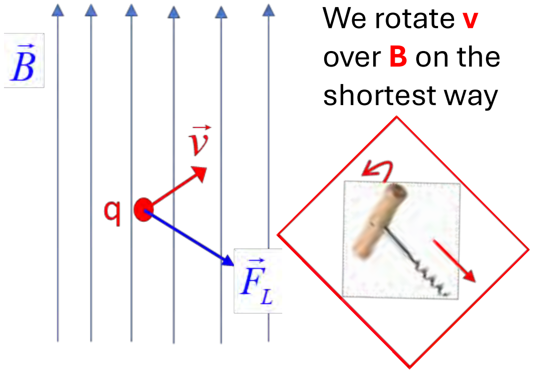
- As with the electric field, we can characterize the strength of the magnetic field as well as the orientation of the magnetic induction vector B with the help of field lines.

- **Rules related to field lines:**
    1. The magnetic field lines are *closed loops* surrounding the source of the magnetic field. Mathematically, this means that: $\nabla \cdot \vec{B} = 0$
    2. The density of the field lines *is proportional to the strength* of the field (modulus of the magnetic induction vector **B**);
    3. The local orientation of the field lines *is parallel to the north pole* of a small magnet placed in the field;
    4. Magnetic *induction vector* $\vec{B}$ is *tangent* to the field line.



3. The Lorentz force

- On a point charge q moving at velocity v through a magnetic field of induction **B** acts a magnetic force (also called the Lorentz force):



$$\vec{F}_L = q\left(\vec{v} \times \vec{B}\right)$$

$\vec{F}_L$ - Lorentz force (N);
q - charge (C);
$\vec{v}$ -velocity of moving charge (m/s);
$\vec{B}$ - magnetic flux density (T)

Observations:

- The Lorentz force is perpendicular to the plane formed by the vectors $\vec{v}$ and $\vec{B}$;
- The sense of the Lorentz force is given by the direction in which the drill (the screw) advances if we rotate $\vec{v}$ towards $\vec{B}$ along the shortest path (the cork screw rule);

- Analyzing the expression of the Lorentz force yields:

$$\vec{F}_L = \begin{cases} 0 & \text{-if } \vec{v} \parallel B \Leftrightarrow \text{the magnetic field does not act on charges moving parallel with B} \\ qvB & \text{-if } \vec{v} \perp B \end{cases}$$

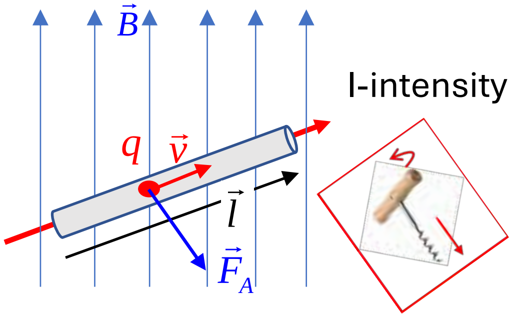
Note: *The Lorentz force is the most important force in magnetism that allows deducing many laws and explaining many phenomena*

- The elementary work performed by the Lorentz force is zero, i.e. **the magnetic force does not change the energy of charged particles** moving in the magnetic field:

$$dL_{F_L} = \vec{F}_L \cdot d\vec{r} = q\left(\vec{v} \times \vec{B}\right) \cdot \frac{d\vec{r}}{dt} dt = q\left(\vec{v} \times \vec{B}\right) \cdot \vec{v} dt = 0$$

4. Ampere's force

- On a conductor of length l placed in magnetic field **B** and crossed by a current of intensity I acts a magnetic force, of the form:



$$\boxed{\vec{F}_A = I\left(\vec{l}\times\vec{B}\right)}$$ - Ampere force

$\vec{F}_A$ - Ampere force (N);

I - current intensity (A);

$\vec{l}$ - the wire length, orriented parallel with I;

$\vec{B}$ - magnetic flux density (T)

Obs.: Dealing with a vector product, the sense of the Ampere force is also obtained with the **corkscrew** rule.

- **Demonstration:**

The total magnetic force (Ampere) on the wire is the sum of the Lorentz forces exerted on the charges that form the current flowing through the wire. So, we can deduce the Ampere force as a summation of Lorentz forces, as seen below.

On a charge q in the wire acts the Lorentz force:

$$\vec{F}_L = q\left(\vec{v}\times\vec{B}\right)$$

On all charges acts the Ampere force:

$$\vec{F}_A = N\vec{F}_L$$

$N = nV = nSl$ -nr. of charges inside the wire;

$n -$ nr. of charges inside unit volume;

$V -$ volume of the wire; $S -$ section of the wire

$$\Rightarrow \vec{F}_A = nqSl\left(\vec{v}\times\vec{B}\right)$$

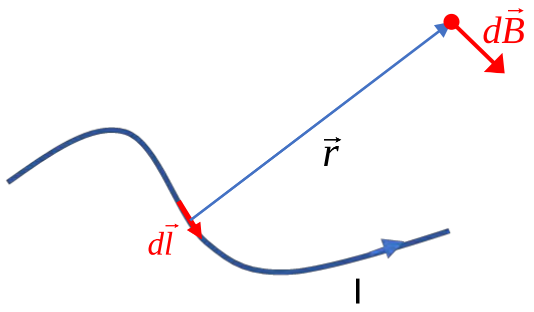
$\vec{I} = qn\vec{v}S$ - the microscopic expresion of the intensity

$$\Rightarrow \vec{F}_A = l\left(\vec{I}\times\vec{B}\right) \Leftrightarrow \vec{F}_A = I\left(\vec{l}\times\vec{B}\right)$$

Here the vector was transferred to the length of the wire

5. The Biot-Savart law

- **Magnetic induction** created by a wire element $d\vec{l}$ carrying the electric current of intensity I is given by **Biot-Savart's law:**



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$$

- The differential form of the Biot-Savart law

$d\vec{B}$ -magnetic flux density generated by the element $d\vec{l}$ (T)

$\vec{r}$ - the position vector where the magnetic field is calculated;

$d\vec{l}$ - the wire element (m);

I - the current intensity (A);

$\mu_0 = 4\pi \cdot 10^{-7}$ - the magnetic permeability of vacuum (H/m);

$$1H/1m = 1Henry/1m = 1N/1A^2$$

- Orientation of the elementary magnetic field $d\vec{B}$ it is given by the cork-skrew rule, as follows from the defining relation
- The total magnetic field $\vec{B}$, produced by the whole wire, can be calculated as an infinite (integral) sum of the contributions of elements into which the wire can be decomposed:

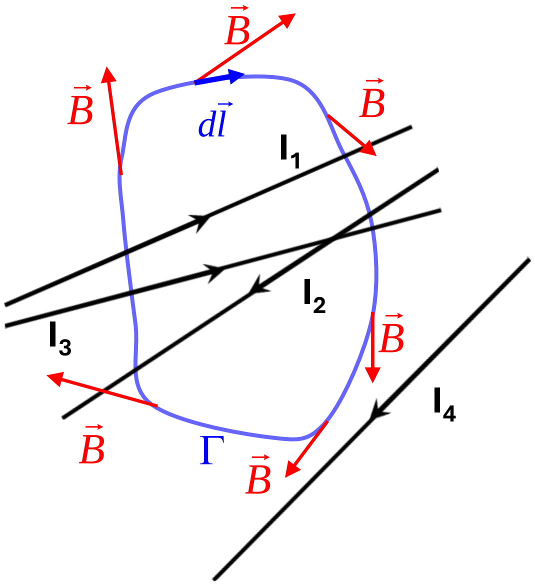
$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi} \int_{\substack{wire\\length}} \frac{d\vec{l} \times \vec{r}}{r^3}$$

- integral form for the Biot-Savart law

- The Biot-Savart law is used to calculate magnetic fields produced by wires or various coils.

6. Ampere's law

- It is a law that allows simple calculation of the magnetic field **produced by symmetric current** configurations. It is a law similar to Gauss's for the electric field.
- Ampere's law can be deduced from Biot-Savart's law, which has a more general usage.
- ***Ampere's law*** *states that the* ***circulation of the magnetic induction vector on a closed loop Γ depends only on the intensity of current passing through the closed loop****:*



$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{int}}$$ (Ampere's law)

$\oint_\Gamma \vec{B}\cdot d\vec{l} =$ the circulation of vector $\vec{B}$ on the closed loop Γ

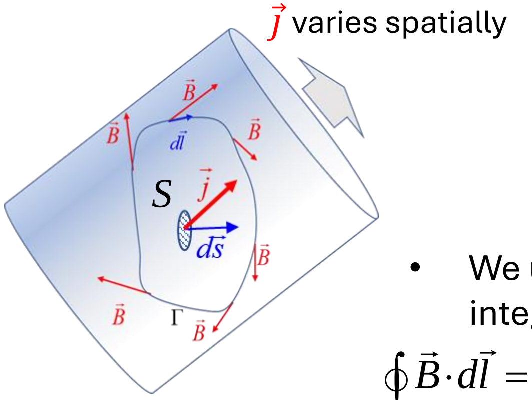
In the case of the current configuration in the figure, it is observed that the current I_4 **does not contribute**

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \mu_0\left(I_1 - I_2 + I_3\right)$$

Note: *Ampere's law can be written in integral and differential form, as we will see on the following pages.*

6.1. The Integral form and the differential form of Ampere's law

- Consider a region of space traversed by a current whose current density varies spatially.



- Ampere's law for that region reads:

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{int}}$$

where

$$I_{\text{int}} = \iint_S \vec{j}\cdot d\vec{s}$$

$$\Rightarrow \boxed{\oint_\Gamma \vec{B}\cdot d\vec{l} = \mu_0 \iint_S \vec{j}\cdot d\vec{s}} \text{ – integral form of Ampere's law}$$

- We use **Stokes' theorem** of transforming an integral on a closed curve Γ into an integral on the surface S bounded by the curve:

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \iint_S \left(\nabla\times\vec{B}\right)\cdot d\vec{s} \quad \textit{where} \quad \nabla\times B = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)\times\left(B_x\vec{i} + B_y\vec{j} + B_z\vec{k}\right)$$

the curl (*rotor*) of vector B

$$\Rightarrow \iint_S \left(\nabla\times\vec{B}\right)\cdot d\vec{s} = \mu_0 \iint_S \vec{j}\cdot d\vec{s} \Leftrightarrow \boxed{\nabla\times\vec{B} = \mu_0\vec{j}}$$

- the differential form of Ampere's law

The presence of a current produces a rotational magnetic field around that current

Observation: The differential form of Ampere's law *is contained in* <u>*Maxwell's set of equations*</u>. However, this form is incomplete. It is valid only for stationary currents (constant over time). If we are dealing with variations of the electric field in the same region where we also have electric currents, then the local form will be described by the equation:

$$\boxed{\nabla\times\vec{B} = \mu_0\vec{j} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}}$$

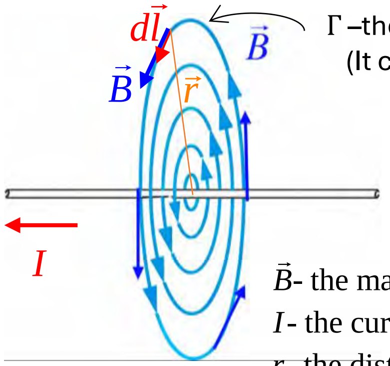
- Ampere-Farraday law

The presence of a current or variation in the electric field produces a rotational magnetic field

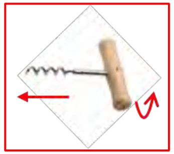
7. Applications of Ampere's law

- Ampere's law can be applied to calculating the **magnetic field produced by symmetric current configurations**. Here we will choose some more important applications. Other applications will be shown at the seminar.

7.1. The magnetic field produced by a straight, infinite wire



Γ –the loop around the wire.
(It can match a field line)



$\vec{B}$- the magnetic flux density;
I- the current intensity in the wire;
r- the distance from the wire to the point of interest;

We apply Ampere's law $\oint_\Gamma \vec{B}\cdot d\vec{l} = \mu_0 I_{int}$

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \underset{(\vec{B}\,\|\,d\vec{l}\,)}{\oint_\Gamma B\,dl} = \underset{(B\text{ is constant on }\Gamma)}{B\oint_\Gamma dl} = \underset{(\text{the integral over the loop is the circle length})}{B2\pi r}$$

The electric current intensity inside loop Γ:

$$I_{int} = I$$

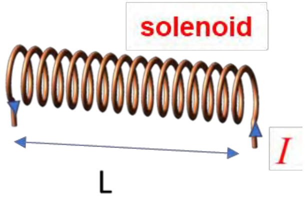
$$\Rightarrow B2\pi r = \mu_0 I \Leftrightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

The magnetic field at distance r from an infinite wire crossed by current I

Observations: Infinite wire ⇔ neglect the edge effects

- The magnetic field lines around an infinite wire are concentric circles. This can be easily demonstrated using the Biot-Savart law;
- $\vec{B}$ vector is always **tangent** to the field lines ($\vec{B} \parallel d\vec{l}$);
- The meaning of field lines is given by the direction in which a drill (corkscrew) should be rotated so that it moves forward just like the current

7.2. The magnetic field produced by a solenoid



- Ampere's law can also be applied for calculating the magnetic field produced by a **solenoid**. The calculation is based on the fact that one can neglect the magnetic field outside the solenoid if it is long enough. Also, the magnetic field inside the solenoid is constant.

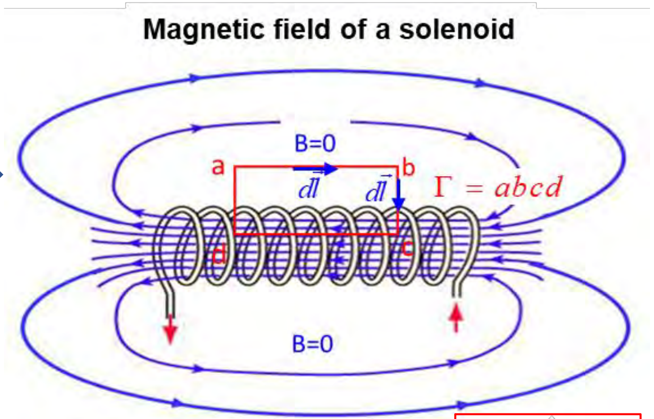
L - the length of the solenoid;
N – the nr. of solenoid turns;
N/L-nr. of turns per unit length;

- We apply Ampere's law on rectangle Γ : $\oint_{\Gamma} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{int}}$

Since $\Gamma = abcd \implies$

$$\oint_{\Gamma} \vec{B}\cdot dl = \underbrace{\int_a^b \vec{B}\cdot d\vec{l}}_{=0} + \underbrace{\int_b^c \vec{B}\cdot d\vec{l}}_{=0} + \int_c^d \vec{B}\cdot dl + \underbrace{\int_d^a \vec{B}\cdot d\vec{l}}_{=0} = \int_c^d \vec{B}\cdot dl = \int_c^d Bdl = Bl$$

because B=0 (a→b); because B=0 or $\vec{B}\perp d\vec{l}$ (b→c); because B=0 or $\vec{B}\perp d\vec{l}$ (d→a); $\vec{B} \parallel d\vec{l}$ and B=cnst. on the cd side



- The current intensity inside the Γ loop is obtained by multiplying the number of turns inside the loop by the intensity I carried through each turn: $I_{\text{int}} = \frac{N}{L} l \cdot I$

l - the length of sides ab and cd;

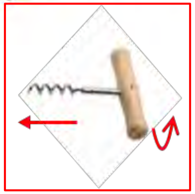
$\frac{N}{L} l$ - the number of loops per length l;

$d\vec{l}$ – the elementary lenght of the loop.

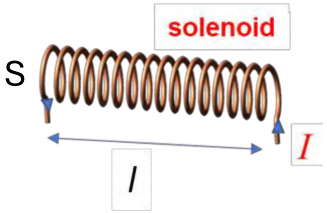
$$Bl = \mu_0 \frac{NI}{L} l \iff \boxed{B = \mu_0 \frac{NI}{L}}$$

The magnetic field inside the solenoid

We rotate the corkscrew in the direction of current. The forward direction gives the sense of field lines.



7.3. The inductance of a solenoid



The inductance *of a solenoid (or coil) is defined as the ratio of the flow of the magnetic field through that solenoid (or coil) to the strength of the current passing through it:*

$$L = \frac{\Phi}{I}$$ - The solenoid inductance;

$[L]_{SI} = Henry\ (H)$

$\Phi_1 = BS$ – flux through a single loop if B=cnst. and $\perp$ on loop;

$\Phi = N\Phi_1 = BSN$ – flux through all loops;

$B = \mu_0 \frac{NI}{l}$ – magnetic field induction inside solenoid;

$$\Rightarrow L = \mu_0 \frac{N^2 S}{l}$$

l- the length of the solenoid;
N- the nr. of turns of the solenoid;
S- the section of the solenoid;
I- the current intensity through the solenoid.

Note:

- The inductance of the solenoid depends only on its geometric characteristics and the number of turns;
- The inductance formula was deduced for the case when inside the solenoid we have vacuum or air. If a material with magnetic properties is found inside the solenoid, then the formula becomes:

$$L = \mu_0 \mu_r \frac{N^2 S}{l} = \mu \frac{N^2 S}{l}$$

$\mu = \mu_0 \mu_r$ – permeability of the medium filling the core;
μ_r – relative permeability of the medium.

IV

Elements of electrodynamics

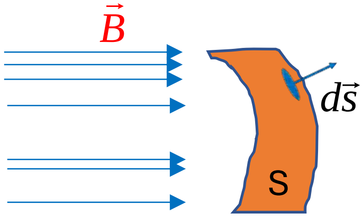
Content:

1. Magnetic field flux
2. Law of electromagnetic induction
3. Law of magneto-electric induction
4. Maxwell's equations

1. The flux of the magnetic field through a certain surface

- **The flux of the magnetic induction** vector through a given surface is defined by the integral of the scalar product $\overrightarrow{B} \cdot d\vec{s}$ on that surface:

$$\Phi_B = \iint_S \vec{B} \cdot d\vec{s}$$



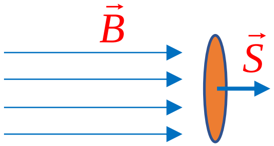
S- the surface (in orange)
$d\vec{s}$-oriented surface element

$$[\Phi_B]_{SI} = Tm^2 = Wb = Weber$$

- **Observations:**

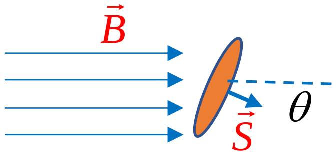
1. If the magnetic induction vector is constant on the surface S and additionally perpendicular to it, then the magnetic field flux is calculated as follows:

$$\Phi_B = BS$$



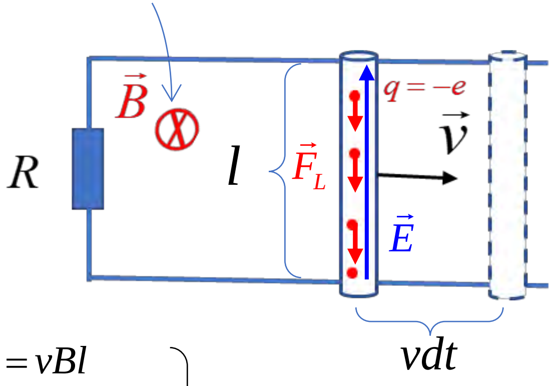
2. If the magnetic flux flux vector is constant on the surface S but oriented at an angle θ relative to this, then the flux of the magnetic field is calculated as follows:

$$\Phi_B = \vec{B} \cdot \vec{S} = BS\cos\theta$$



2. Law of electromagnetic induction

- **Electrodynamics** studies electric and magnetic fields that vary over time and generate each other
- **Electromagnetic induction** consists of the generation of a rotational electric field by a variable magnetic field. To understand this, we will consider the experiment in the figure below, in which a conducting wire can move over a conducting frame. The frame is placed perpendicular to the lines of a magnetic field.



- ❑ On the charges q=-e (electrons) moving in the magnetic field will act the Lorentz force: $\vec{F}_L = q\left(\vec{v}\times\vec{B}\right)$
- ❑ This force can be associated with an electric force having the same effect: $\vec{F}_e = q\vec{E}$
- ❑ From the equality of the two forces, it follows that the electric field generated inside the conductor moving at velocity v is expressed as: $E = vB$
- ❑ If the electric field is uniform, we can write: $E = \dfrac{u}{l}$

  where u = the electrical voltage between conductor ends (along E)

$\Rightarrow u = vBl$

- ❑ The variation in magnetic field flux produced by conductor displacement during elementary time dt is: $d\Phi = Blvdt \Rightarrow Blv = \dfrac{d\Phi}{dt}$

$\Rightarrow u = \dfrac{d\Phi}{dt} \Rightarrow$

$$\Rightarrow \boxed{u = -\frac{d\Phi}{dt}}$$

Faraday's law of electromagnetic induction (1831)

Note: *The "– " sign takes into account Lentz's rule*

Statement of the law of electromagnetic induction:

The electromotive voltage u induced in a circuit is equal to the variation of magnetic field flux through that circuit in unit time:

$$u = -\frac{d\Phi_B}{dt}$$

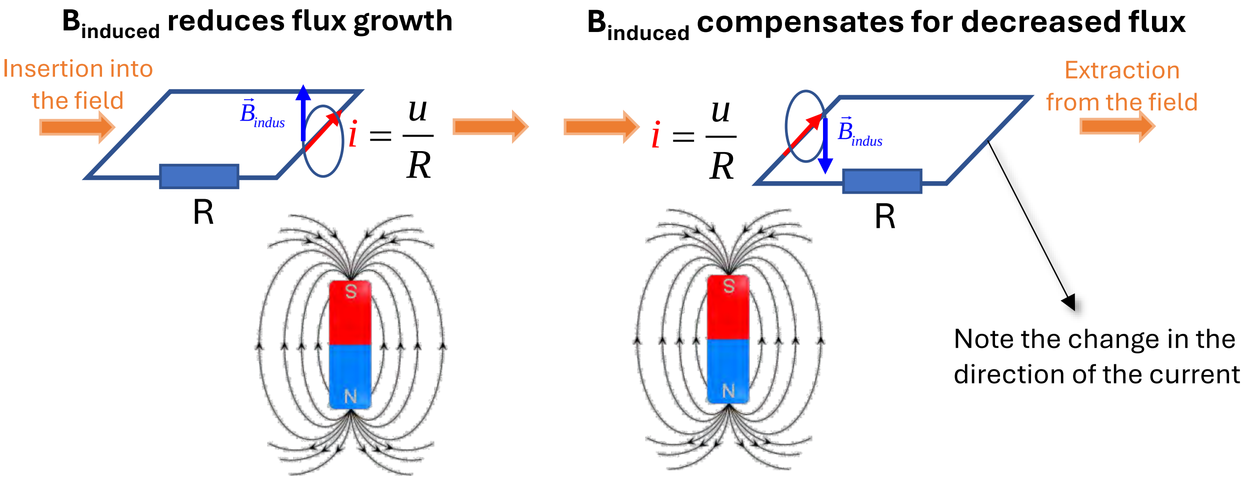
u – electromotive voltage (V);

$d\Phi_B$ = magnetic flux variation $(\mathrm{Weber=Wb=T \cdot m^2})$;

dt – elementary time (s)

The "–" sign takes into account Lentz's rule

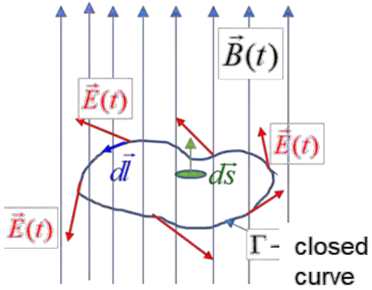
Lentz's rule: *The electric current i induced by the electromotive voltage u has such a sense that the magnetic field produced by it opposes the variation in flux that created it (this is a kind of action-reaction principle for electromagnetism).* This is illustrated by inserting and extracting a circuit from a magnetic field and measuring obtained the electric current, as shown below:



Applications:
- AC power generation;
- operation of transformers;
- explains eddy currents;
- magnetic braking.

2.1. Integral and differential form of the electromagnetic induction law

- The law of electromagnetic induction can be written in another form if we consider the existence of the electric field to be unconditioned by the presence of a conductive material.
- So, consider a region of space where we have a variable magnetic field, and apply the law of electromagnetic induction to that region



Law of electromagnetic induction: $$u = -\frac{d\Phi}{dt}$$

Electromotive voltage *u* is the circulation of the vector E on Γ curve: $$u = \oint_{\Gamma} \vec{E} \cdot d\vec{l}$$

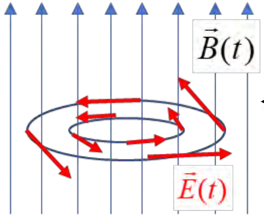
The flux *Φ* of the magnetic induction B through S bounded by the curve Γ: $$\Phi = \iint_{S} \vec{B} \cdot d\vec{s}$$

Note: If B is uniformly distributed over the surface S, and forms an angle with it, then: $$\Phi = BS\cos\theta$$

$$\oint_{\Gamma} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_{S} \vec{B} \cdot d\vec{s}$$

(the integral form of Faraday's law)

We further use **Stokes**' theorem for transforming a closed curve integral over contour Γ into a surface integral over the bounded surface S: $$\oint_{\Gamma} \vec{B} \cdot d\vec{l} = \iint_{S} \left(\nabla \times \vec{B}\right) \cdot d\vec{s}$$



According to Faraday's law, any variable magnetic field, B(t), produces a rotational electric field, E(t) (even in vacuum).

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

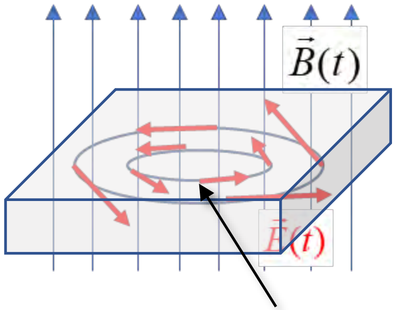
(Faraday's Law in differential) form

2.2. Eddy currents

- Eddy currents (also called Foucault currents) occur in conductors as a **direct consequence of the law of electromagnetic induction**:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- As seen above, a variable magnetic field produces a rotational electric field in a particular region of space. This electric field generates an **orderly movement of electrons** (current) inside the conductors because it acts with an electric force: F=qE



The E field generates eddy currents also called Foucault currents

Effects of eddy currents:

- Eddy currents may be used in train braking systems or electromagnetic induction furnaces;
- They reduce the efficiency of transformers because they are associated with energy losses. To reduce eddy currents, transformers are made of thin parts (sheets)

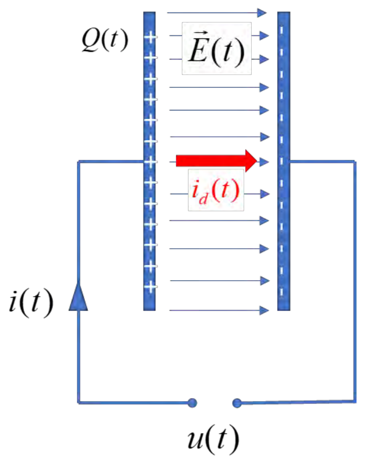
Links where eddy current based braking mechanism is illustrated:

https://www.youtube.com/watch?v=H31K9qcmeMU

https://www.youtube.com/watch?v=Yu1uRvErM80

3. Law of magneto-electric induction

- **Magneto-electric induction** consists of generating a rotational magnetic field in a region of space where there is a variable electric field
- To derive the law of magneto-electric induction consider a plane capacitor connected to a time-varying voltage u(t) (alternating voltage). An i(t) current arises in the circuit, which formally passes through the capacitor plates, although there is no electric charge there.



According to the definition of intensity: $i(t)=\dfrac{dQ}{dt}$

This current passes virtually (due to the change in polarity of the plates: +-, -+,+-,...) and among the plates: $i_d(t)=i(t)$ – the displacement current

$\Rightarrow \; i_d(t)=\dfrac{dQ}{dt}$

In the case of the plane capacitor, we have :

$$\left.\begin{aligned} Q(t)&=Cu(t)\\ C&=\varepsilon_0\frac{S}{d}\\ u(t)&=E(t)d\end{aligned}\right\}\Rightarrow Q(t)=\varepsilon_0 E(t)S$$

$\Rightarrow$

$$\left.\begin{aligned} i_d(t)&=\varepsilon_0 S\frac{dE}{dt}\text{ – intensity of the displacement current;}\\ j_d&=\frac{i_d}{S}\text{ – current density}\end{aligned}\right\}\Rightarrow \boxed{\vec{j}_d(t)=\varepsilon_0\frac{d\vec{E}}{dt}}$$

S-surface of plates;
d-distance between plates;
C=$\varepsilon\frac{S}{d}$-electric capacitance of a plane capacitor

The displacement current density produced by a variation in electric field strength.
This **current density** is a virtual one, it is not associated with an electric charge transport

The Ampere-Maxwell law

- If in a certain region of space we have, in addition to a load current density $\vec{j}$ a displacement current density $\vec{j}_d$, induced by an electric field variation, we can rewrite **Ampere's law** as:

$$\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \vec{j}_d \right)$$

$\vec{j}$ – charge current density;

$$\vec{j}_d = \varepsilon_0 \frac{d\vec{E}}{dt} \text{ – displacement current density;}$$

$$\Rightarrow \quad \boxed{\nabla \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{d\vec{E}}{dt}}$$

Ampere-Maxwell law

4. Maxwell's equations

- The set of equations below was found by several physicists (Gauss, Faraday, Ampere, Maxwell), but was synthesized by Maxwell, and is therefore called **Maxwell's equations**. In previous courses we have shown how these equations were obtained. The equations below also describe the classical electromagnetism.

Maxwell's equations (1862)

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ – Gauss law for electric field intensity;

$$\nabla \cdot \vec{B} = 0$$ – Gauss law for magnetic induction;

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ – Faraday's law;

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$ – Maxwell-Ampere law.

To these equations one can add

$$\vec{F} = q\vec{E} + q\left(\vec{v} \times \vec{B}\right)$$ – the electro-magnetic force on a point charge q;

$$\vec{j} = \sigma \vec{E}$$ – the local form of Ohm's law;

$$\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0$$ – the continuity equation

The above equations were written for vacuum. In the case of dielectric or magnetic materials, the following shall be replaced:

$$\varepsilon_0 \to \varepsilon = \varepsilon_0 \varepsilon_r$$
$$\mu_0 \to \mu = \mu_0 \mu_r$$

V

Electromagnetic waves in vacuum

Content:
1. What are electromagnetic waves and how are they generated?
2. The equation of electromagnetic waves
3. The energy transported by e.m.w
4. Polarization of light

1. What are electromagnetic waves and how are they generated?

- **Electromagnetic waves** can be generated by **accelerating electric charges** (antennas, oscillating dipoles) or **by atoms** (visible light, X-rays or gamma rays)

- **Electromagnetic waves spectrum** contains wavelengths between 10^{-5}nm and 10^{3}m and frequencies from 10^{2}Hz up to 10^{24}Hz
- **Visible spectrum** contains wavelengths between 400 nm and 700 nm, i.e. **only a tiny fraction** of the entire electromagnetic wave spectrum;

Obs: In the interpretation of quantum physics, light and other radiation are made up of photons, and the energy of one photon is given by the relation:

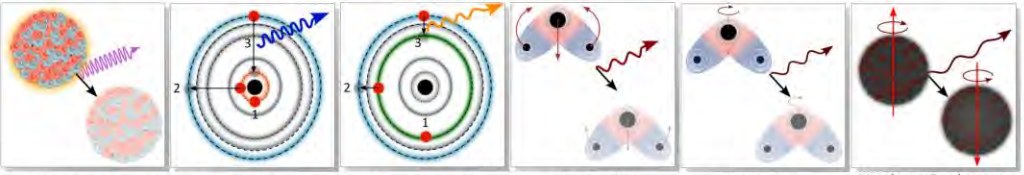
$$E_f = h\nu$$

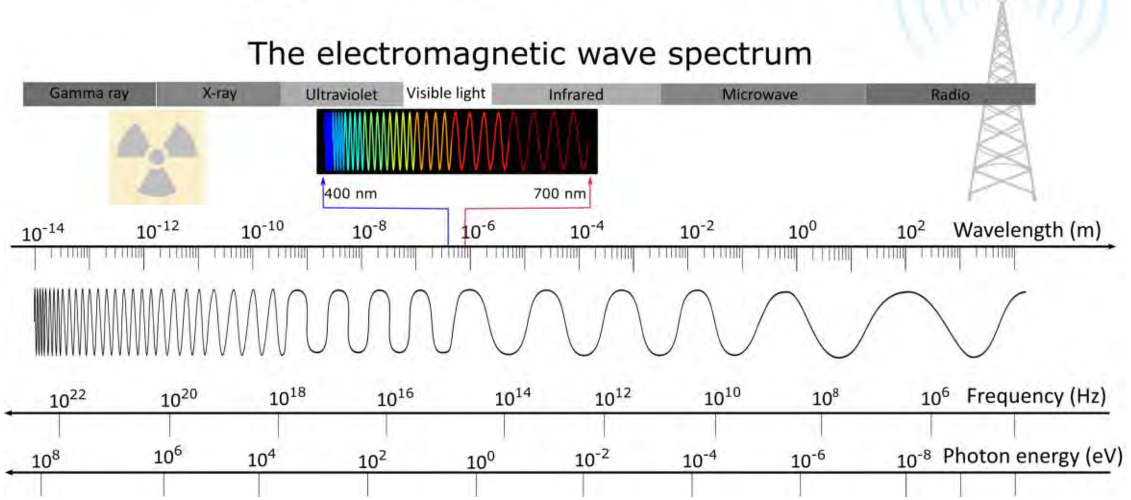
$h = 6.626 \cdot 10^{-34}\, Js$ - Planck's constant

$\nu -$ radiation frequency (Hz)

The energy of radiation increases with frequency. For example, X-rays and gamma rays are so energetic, they can ionize atoms and may induce harmful effects.

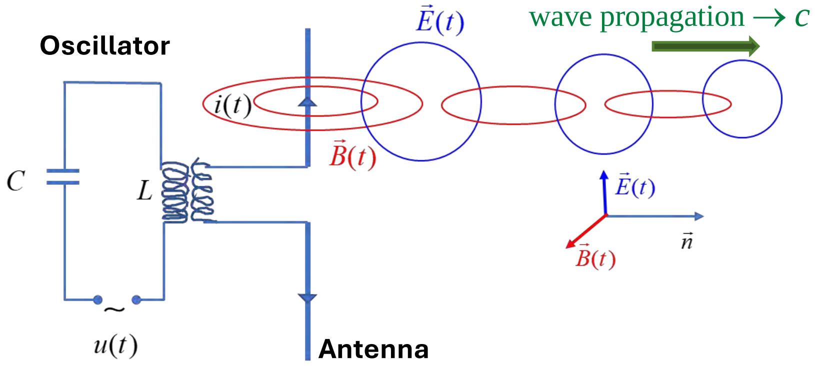
Fundamental processes in matter that emit/absorb electromagnetic waves



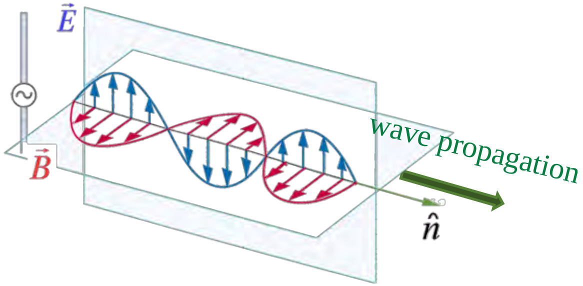


2. The equation of electromagnetic waves in vacuum

- If electromagnetic waves (e.m.w.) are produced by accelerating electric charges (variable currents, electric dipoles) then their generation and propagation is described using Maxwell's equations. The principle of generation of e.m.w. by an antenna and their propagation is schematically described in the figure below.



Electric field vector **E** and the magnetic field vector **B** which make up the electromagnetic wave are ⊥ on $\hat{n}$ - unit vector of the propagation direction



The equations describing the reciprocal generation of a rotational electric field by a variable magnetic field and a rotational magnetic field by a variable electric field are:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{ – Faraday's law;}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \text{ – Maxwell-Ampere law}$$

$$\underset{because}{\Rightarrow} \quad \nabla \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

j=0 in vacuum

In the following, these equations will be used to derive the wave equation for the electrical component and the magnetic component of the electromagnetic field

2.1. The wave equation for the electrical component in e.mw.

- We start from Maxwell's equations: $\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t};$

$$\nabla\times\vec{B}=\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}.$$

- Applying to the first equation the vector product with the ∇ operator, we get:

$$\left.\begin{aligned}&\nabla\times\left(\nabla\times\vec{E}\right)=-\frac{\partial\left(\nabla\times\vec{B}\right)}{\partial t}\\ &\text{-but}\quad \nabla\times\vec{B}=\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\end{aligned}\right\}\Rightarrow\nabla\times\left(\nabla\times\vec{E}\right)=-\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

-here we use: $\nabla\times\left(\nabla\times\vec{E}\right)=\nabla\left(\nabla\cdot\vec{E}\right)-\nabla^2\vec{E}=-\nabla^2\vec{E}$

$\nabla\cdot\vec{E}=0$ -because in vacuum we do not have electric charge (ρ=0)

$$\Rightarrow\ \boxed{\nabla^2\vec{E}-\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=0}$$

The wave equation for the electrical component

- We can further compare the obtained equation with the plane harmonic wave equation (the course on elastic waves Sem. I):

$$\nabla^2\Psi\left(\vec{r},t\right)-\frac{1}{c^2}\frac{\partial^2\Psi\left(\vec{r},t\right)}{\partial t^2}=0$$

$$\Rightarrow\ \boxed{c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}}\cong 3\cdot10^8 m/s$$

The propagation speed of e.m.w. (light) in vacuum

2.2. Wave equation for the magnetic component in e.m.w.

- We start from Maxwell's equations: $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t};$

$$\nabla\times\vec{B}=\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}.$$

- Applying the vector product to the left to the 2nd equation with the ∇ operator, we get:

$$\left.\begin{aligned}&\nabla\times\left(\nabla\times\vec{B}\right)=\varepsilon_0\mu_0\frac{\partial\left(\nabla\times\vec{E}\right)}{\partial t}\\&\text{-but}\quad\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\end{aligned}\right\}\Rightarrow\nabla\times\left(\nabla\times\vec{B}\right)=-\varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}$$

-here we use: $\nabla\times\left(\nabla\times\vec{B}\right)=\nabla\left(\nabla\cdot\vec{B}\right)-\nabla^2\vec{B}=-\nabla^2\vec{B}$

$\nabla\cdot\vec{B}=0$ -allways true for the magnetic field

$$\Rightarrow\quad\boxed{\nabla^2\vec{B}-\varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}=0}$$

The wave equation for the magnetic component

- When comparing the obtained equation with the plane harmonic wave equation, we reach the same result:

$$\boxed{c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}}\cong3\cdot10^8 m/s$$

The propagation speed of e.m.w. (light) in vacuum

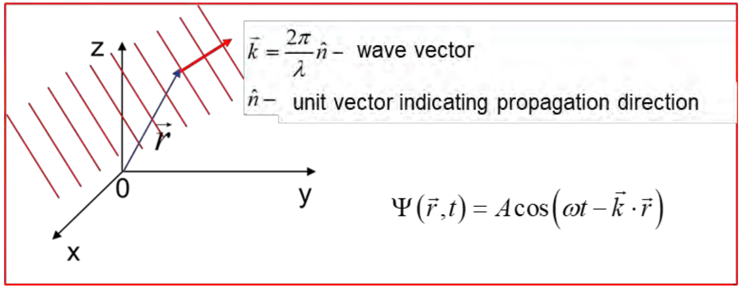
2.3. Solutions of differential equations

- In the mechanics lectures (1st semester) it has been proven that the solution of the differential equation describing the propagation of the plane harmonic wave is: $\nabla^2\Psi(\vec{r},t)-\frac{1}{c^2}\frac{\partial^2\Psi(\vec{r},t)}{\partial t^2}=0 \longrightarrow \Psi(\vec{r},t)=A\cos(\omega t-\vec{k}\cdot\vec{r})$
- We will have the same type of solution for electromagnetic waves:

Equation	Solution	The solution in complex form
$\nabla^2\vec{E}-\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=0$ →	$\vec{E}=\vec{E}_0\cos(\omega t-\vec{k}\cdot\vec{r})$ →	$\vec{E}=\vec{E}_0 e^{i(\omega t-\vec{k}\cdot\vec{r})}$
$\nabla^2\vec{B}-\varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}=0$ →	$\vec{B}=\vec{B}_0\cos(\omega t-\vec{k}\cdot\vec{r})$ →	$\vec{B}=\vec{B}_0 e^{i(\omega t-\vec{k}\cdot\vec{r})}$

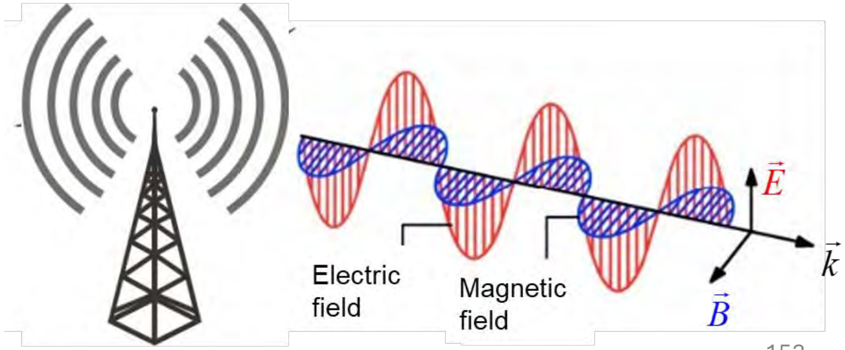
$\vec{E}_0,\vec{B}_0$ – amplitudes of the electric and magnetic field components;
$\omega=2\pi\nu$ – angular frequency of the wave;
$\vec{k}=\frac{2\pi}{\lambda}\hat{n}$ – wave vector;
$\lambda=cT$ – wave length;
$\hat{n}$ – unit vector indicating the propagation direction of the wave.

***Note**: The complex solution is used for ease of calculations. Only the real part is kept in the final result*



Course summary of the elastic wave equation (Sem. I)

Graphical representation of solutions



2.4. The nabla operator (∇) for plane, harmonic, e.m.w.

- When working with e.m.w. the operator ∇ often appears applied to the vectors $\vec{E}$ and $\vec{B}$, as for instance:

$$\left.\begin{array}{l}\nabla\cdot\vec{E}=\nabla\cdot\vec{E}_0 e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\\ but\\ \nabla=\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\end{array}\right\}\Rightarrow\nabla\cdot\vec{E}=E_{0x}\frac{\partial}{\partial x}\left[e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\right]\hat{x}+E_{0y}\frac{\partial}{\partial y}\left[e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\right]\hat{y}+E_{0z}\frac{\partial}{\partial z}\left[e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\right]\hat{z}$$

$$=-ik_xE_{0x}e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\hat{x}-ik_yE_{0y}e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\hat{y}-ik_zE_{0z}e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\hat{z}$$

To avoid any confusion between the complex number i and the vector $\vec{\imath}$. To avoid further confusion, here we have noted the unit vectors of the 3 directions as $\hat{x},\hat{y},\hat{z}$

we have: $\frac{\partial}{\partial x}\left[e^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\right]=\frac{\partial}{\partial x}e^{i\left(\omega t-k_xx-k_yy-k_zz\right)}=-ik_xe^{i\left(\omega t-\vec{k}\cdot\vec{r}\right)}$

-similar for other components

$$\left.\begin{array}{l}\nabla\cdot\vec{E}=-i\vec{k}\cdot\vec{E}\\ but\\ \vec{k}=k\vec{n}\end{array}\right\}\Rightarrow\nabla\cdot\vec{E}=-ik\vec{n}\cdot\vec{E}$$

$k=\frac{2\pi}{\lambda}$ – wave number;

$\vec{n}$ – unit vector indicating the propagation direction

This property will be used to demonstrate the transverse nature of e.m.w. and the relation between E and B in an electromagnetic wave.

$$\nabla=-ik\vec{n}$$ - definition valid only for harmonic plane waves !!!

2.5. Transversality of e.m.w.

- The electromagnetic waves are transverse waves: $\vec{E} \perp \vec{n}$ si $\vec{B} \perp \vec{n}$ ($\vec{n}$ – unit vector indicationg the propagation direction)
- To prove the transversality of e.m.w. we start from the Gauss equations for the electric and magnetic field **in vacuum** and consider the property of the ∇ operator, as previously demonstrated:

$$\left.\begin{array}{l}\nabla \cdot \vec{E} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla = -ik\vec{n}\end{array}\right\} \Rightarrow \begin{array}{l}-ik\vec{n} \cdot \vec{E} = 0 \Leftrightarrow \vec{E} \perp \vec{n} \\ -ik\vec{n} \cdot \vec{B} = 0 \Leftrightarrow \vec{B} \perp \vec{n}\end{array} \Rightarrow$$

The two vectors are perpendicular to the direction of propagation (transverse wave).

2.6. The connection between E and B in e.m.w.

- We start from the Faraday equation for the electric field and take into account the previously demonstrated ∇ operator property:

$$\left.\begin{array}{l}\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ B = B_0 e^{i\left(\omega t - \vec{k}\cdot\vec{r}\right)} \Rightarrow \dfrac{\partial \vec{B}}{\partial t} = i\omega B_0 e^{i\left(\omega t - \vec{k}\cdot\vec{r}\right)} = i\omega B \\ \nabla = -ik\vec{n}\end{array}\right\} \Rightarrow k\left(\vec{n} \times E\right) = \omega B \Leftrightarrow \vec{n} \times E = \frac{\omega}{k} B \Leftrightarrow \boxed{\vec{n} \times \vec{E} = c\vec{B}} \Rightarrow \boxed{E = cB}$$

but $\dfrac{\omega}{k} = \dfrac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \dfrac{\frac{2\pi}{T}}{\frac{2\pi}{cT}} = c$ – speed of light in vacuum

The connection between the electrical and magnetic component forming the electromagnetic wave; It is noted that $\vec{E} \perp \vec{B}$

3. The energy carried by electromagnetic waves

- Electromagnetic waves carry energy and this energy is stored in the electrical component E of the field and in the magnetic component B (or H)
- It can be shown that the energy densities of the electric and magnetic field satisfy the formula:

$$w_e = \varepsilon_0 \frac{E^2}{2}$$ – energy density of the electric field (J/m³);

$$w_m = \mu_0 \frac{H^2}{2}$$ – energy density of the magnetic field;

$\vec{E}$-intensity of the electric field (V/m);

$\vec{H} = \frac{\vec{B}}{\mu_0}$ – intensity of the magnetic field (A/m);

$$\Rightarrow w = w_e + w_m = \varepsilon_0 \frac{E^2}{2} + \mu_0 \frac{H^2}{2} \Leftrightarrow w = \varepsilon_0 E^2 = \mu_0 H^2$$

the energy density of e.m. waves

$$\left.\begin{aligned} B &= \frac{E}{c} \\ H &= \frac{B}{\mu_0} \end{aligned}\right\} \Rightarrow H = \frac{1}{\mu_0}\frac{E}{c}$$

$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ – velocity of e.m. waves

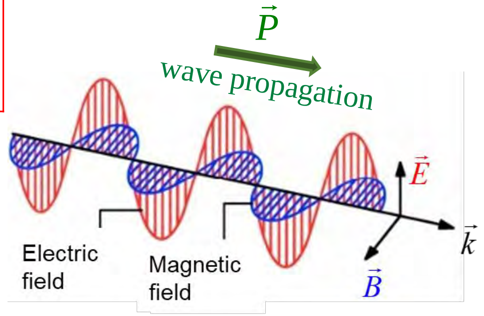
- Knowing the energy density allows the calculation of **wave intensity** (as for the sound waves):
- *The wave intensity is the energy carried by the wave through the unit area in the unit time (W/m²)* →

$$I = cw = c\varepsilon_0 E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}} E^2$$

- The intensity of the wave can also be written vectorially, thus indicating the direction of energy propagation. In this case, ***I*** it is replaced by **Poynting vector**:

$$\vec{P} = \vec{E} \times \vec{H} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Note: the magnitude of the Poynting vector is the intensity I



4. Polarization of light

- Light is an electromagnetic wave that can be perceived by the human eye, just as sound is an elastic wave that can be perceived by the human ear. From the electromagnetic wave, **only the electric field E creates visual sensation**, which interacts with the electrical charges in the retina.

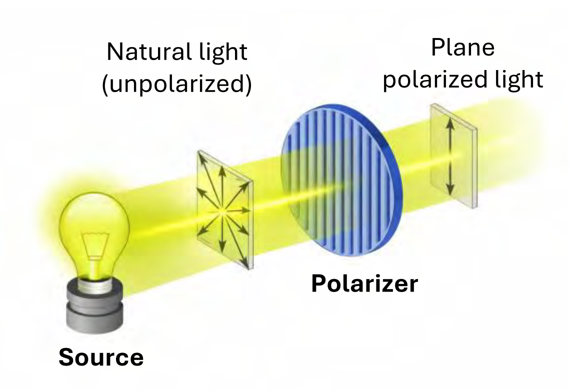
- Because light is produced by atoms that have a random orientation in the substance→ **the electric field E in the light beam is isotopically oriented**. It is said that in this case we are dealing with **natural light**.

- **Polarized light** has the electric field vector **E** **oriented only in a certain direction**, called **the polarization direction**

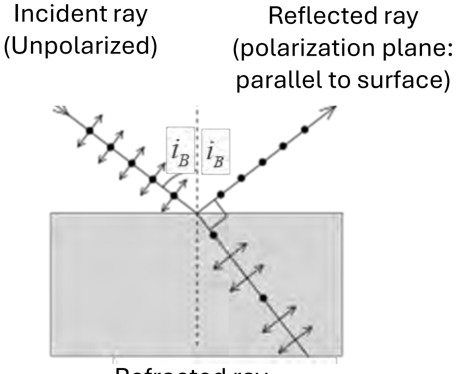
- The polarization of light can be achieved in several ways using: **(1) reflection, (2) double refraction** (bi-refringence), **(3) selective absorption** (dichroism)

- **Birefringence** is the property of certain materials that separates surface incident waves into ordinary waves (O) and extraordinary waves (E) that have perpendicular electric field vectors and travel at different speeds.

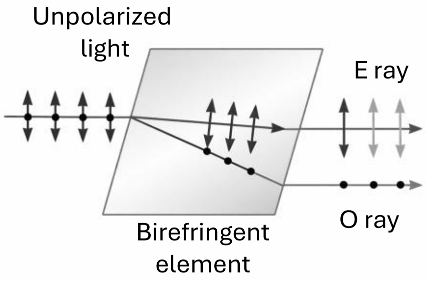
Polarization by selective absorption



Polarization by reflection



Birefringence



Brewster's angle: The beam reflected at Brewster angle is totally polarized

$$i_B = \arctan\left(\frac{n_2}{n_1}\right)$$

VI

Materials in electric field

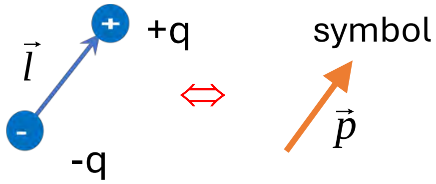
Content:

1. Electric dipole and field interaction
2. Dielectrics in electric field

1. The electric dipole

- By introducing an insulating material (dielectric) into the electric field, the charge distribution within its atoms or molecules is altered, resulting in a modification of the electric field inside the material. However, this redistribution of charge preserves the overall electrical neutrality of the material ($Q_{total}=0$), causing only a displacement of the center of positive charge relative to the center of negative charge. The material is then said to become electrically polarized.
- To understand the polarization of the dielectric it is first necessary to define the **electric dipole** and quantify its interaction with an external electric field.

- ***Definition:*** *An electric dipole consists of two equal and opposite charges,* ***+q*** *and* ***–q*** *separated by a distance* ***l****.*



The dipole moment for a two-charge system is defined by the relation:

$$\boxed{\vec{p} = q\vec{l}}$$

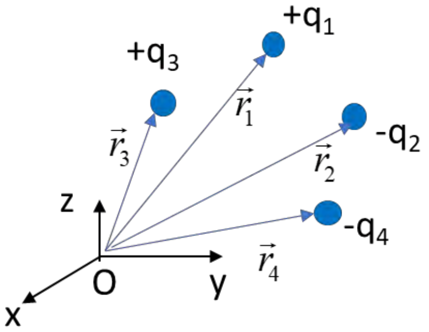
$\vec{p}$ – dipol moment (Cm);

q – electric charge (C);

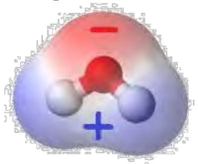
$\vec{l}$ – displacement vector (m) – oriented from – to +

The dipole moment for a system of charges is defined by the relation:

$$\boxed{\vec{p} = \sum_{i=1}^{N} q_i \vec{r}_i}, \text{ where } \sum_{i=1}^{N} q_i = 0 \text{ - the system is electrically neutral}$$



Example: The water molecule



⇔ $\vec{p}_{H_2O}$

$p_{H_2O} = 1.84D$

$1D = Debye = 3.34 \cdot 10^{-30} Cm$

1.1. The electric field created by the dipole

Although electrically neutral, an electric dipole will create an electric field around it given by the formula:

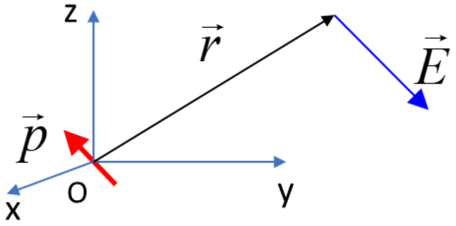
$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5}-\frac{\vec{p}}{r^3}\right]$$

$\vec{E}$ – intensity of the electric field

$\vec{p}$ – dipole moment

$\vec{r}$ – position vector

ε_0 – electric permitivity of vacuum



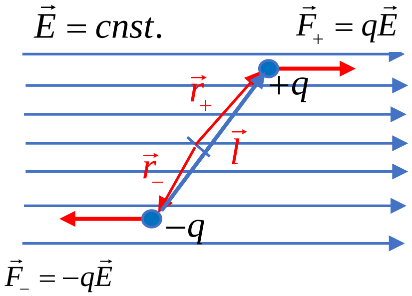
***Note:** this formula can be deduced by adding the electric fields produced by the two-point charges that form the dipole*

1.2. The torque of the electric field force on a dipole

We consider a dipole placed in a constant electric field. The torque of force (see the Part. I) what acts on the dipole is calculated as the sum of the torques acting on the two charges:

$$\left.\begin{array}{l}\vec{M}=\vec{M}_+ +\vec{M}_- =\vec{r}_+\times\vec{F}_+ +\vec{r}_-\times\vec{F}_-;\\ F_+=qE;\\ \vec{F}_-=-q\vec{E};\end{array}\right\}\Rightarrow \left.\begin{array}{c} M=q(\vec{r}_+-\vec{r}_-)\times E=q\vec{l}\times E\\ \vec{r}_+-\vec{r}_-=\vec{l}\quad \text{si}\quad q\vec{l}=\vec{p}\end{array}\right\}\Rightarrow \boxed{\vec{M}=\vec{p}\times\vec{E}}$$

The torque on the dipole produces its rotation, bringing it parallel to the electric field lines.



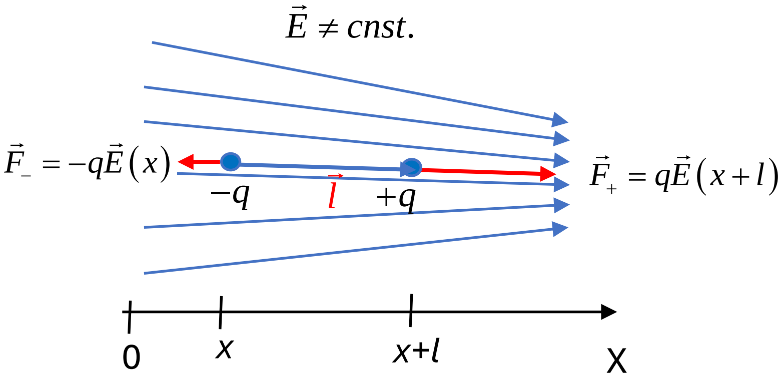
Application: Microwave oven

Food contains water and water molecules are natural dipoles. In the presence of an oscillating electric field, produced by the magnetron with a frequency of 2.45GHz, the water molecules will oscillate with a frequency of 2.45GHz. The collisions between water molecules and other molecules will transmit kinetic energy to them, heating them.

Magnetron: https://www.youtube.com/watch?v=bUsS5KUMLvw

1.3. Electric Field Force on a Dipole

- We consider a dipole placed in a variable electric field along the direction OX.
- Due to the variation of the field, the electric force acting on the charge +q is different from the one acting on the charge –q.
- The result is a **net force**, along OX, acting on the dipole, which is calculated as follows:



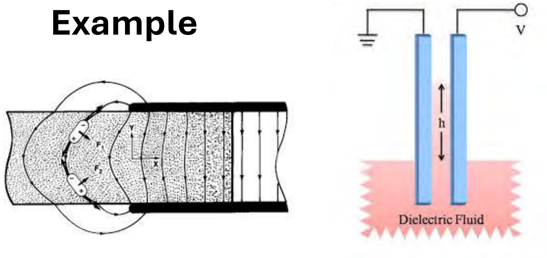
$$\left.\begin{aligned} F &= F_+ - F_- \\ F_+ &= qE(x+l); \\ F_- &= qE(x); \end{aligned}\right\} \Rightarrow F = q\left[E(x+l) - E(x)\right] = q\frac{\partial E}{\partial x} l \Leftrightarrow \boxed{F_x = p_x \frac{\partial E}{\partial x}}$$

The resultant force acting on the dipole

$$E(x+l) = E(x) + \frac{1}{1!}\frac{\partial E(x)}{\partial x} l + \frac{1}{2!}\frac{\partial^2 E(x)}{\partial x^2} l^2 + \dots$$

***Taylor series** can be limited to the first-order derivative if l is very small*

Example



https://www.youtube.com/watch?v=aBAdSE2jC8Y

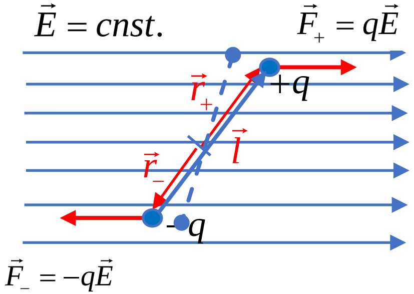
- This is where the OX component of the force was obtained because we assumed that the intensity only varies along OX. However, this expression can also be generalized for the OY and OZ directorates and thus it is obtained:

$$\boxed{\vec{F} = (\vec{p} \cdot \nabla)\vec{E}}$$

- It is observed that an electric force acts on a dipole only if the electric field is inhomogeneous.

1.4. The energy of a dipole in the electric field

- We consider a dipole placed in a constant electric field. The two forces acting on the dipole produce an elementary mechanical work at its rotation at a small angle.



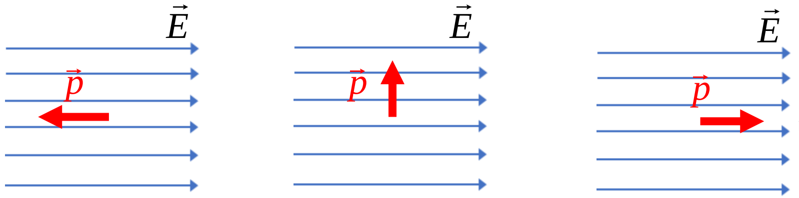
- We calculate elementary mechanical work as it follows:

$$\left.\begin{aligned} dL &= \vec{F}_+ \cdot d\vec{r}_+ + \vec{F}_- \cdot d\vec{r}_- \\ F_+ &= qE; \\ \vec{F}_- &= -q\vec{E}; \end{aligned}\right\} \Rightarrow dL = qE \cdot d\left(\vec{r}_+ - \vec{r}_-\right) = qE \cdot d\vec{l} = E \cdot d\left(q\vec{l}\right) \underset{\vec{E}=cnst}{=} d\left(\vec{p} \cdot E\right)$$

Elementary mechanical work is performed on account of the variation of potential energies: $dL = -dW_p$

$$\boxed{W_p = -\vec{p} \cdot \vec{E}}$$ ← Potential energy of a dipole placed in an electric field

- The energies for 3 cases:



$W_p = pE$ $\quad$ $W_p = 0$ $\quad$ $W_p = -pE$

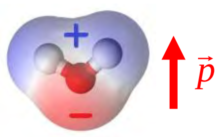
Analyzing the expression of the potential energy results that the dipole will pass into the minimum energy state $W_{pmin} = -pE$ when aligned parallel to the electric field lines

2. Dielectric materials in an electric field

- **Dielectric materials** consist of electric dipoles and are **isolators** in terms of charge transport. These dipoles can be permanent (as is the case with water molecules) or they can be induced (glass, mica, Teflon) by an external electric field.

Permanent dipole (Water, Acetone)

$\vec{E} = 0$



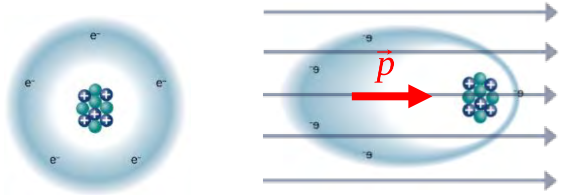
$\vec{p} \neq 0$

$p_{H_2O} = 1.84D$

$1D = Debye = 3.34 \cdot 10^{-30}\, Cm$

Induced dipole (cyclohexane, glass, mica, Teflon)

$\vec{E} = 0 \quad \vec{p} = 0 \qquad \vec{E} \neq 0 \quad \vec{p} \neq 0$

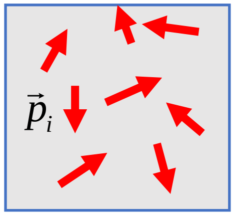


The induced dipole moment p depends on the characteristics of the molecule and the field strength

- By placing a dielectric material in an electric field, the dipoles of that material orient themselves parallel to the electric field (polarize)

The dielectric polarization *is defined as the dipole moment of the unit volume:*

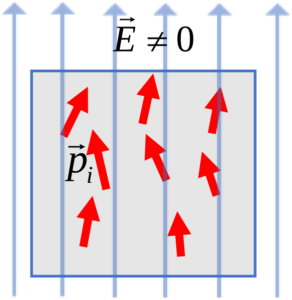
$\vec{E} = 0$



Randomly oriented dipoles

$\vec{P} = 0$

$\vec{E} \neq 0$



Parallel oriented dipoles

$\vec{P} \neq 0$

$$\boxed{\vec{P} = \frac{\sum_i \vec{p}_i}{V}}$$

$\vec{P}$ – polarization (C/m^2)

$\vec{p}_i$ – dipole moment of atom i (Cm);

V – volume of the material (m^3).

Observations:

- Polarization shows the degree of order of dipoles and their size.
- Polarization depends on the material, temperature and external electric field

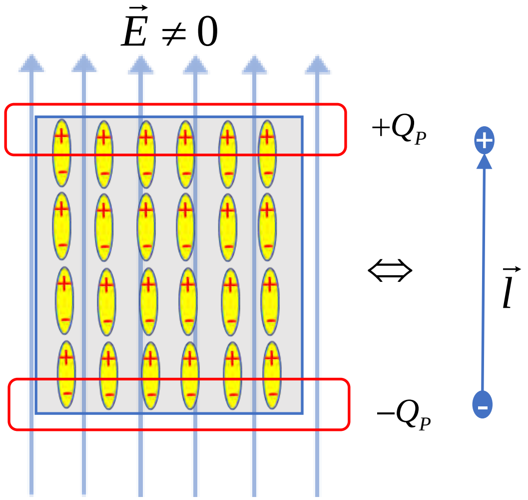
$$\boxed{\vec{P} = \varepsilon_0 \chi \vec{E}}$$

ε_0 – *electric permitivity of vacuum;*

χ – *electric susceptibility of the material.*

2.1 Bounded charge density (σ_p) and polarization P

- Consider a dielectric placed in an electric field. Its dipoles will be oriented as in the figure below



The material is equivalent to a total **dipole moment**

$$\Leftrightarrow \quad p_{tot} = Q_p l \quad \Rightarrow$$

The polarization of the material can be calculated using the relationship of definition:

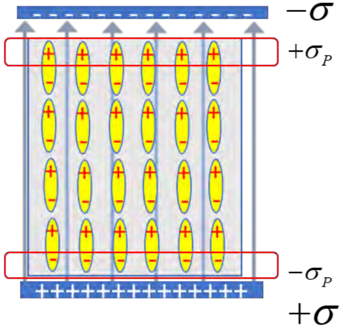
$$P = \frac{\sum_i p_i}{V} = \frac{p_{tot}}{V} = \frac{Q_p l}{V} = \frac{\sigma_p S l}{S l} \Leftrightarrow \boxed{P = \sigma_p}$$

$\sigma_P = \frac{Q_p}{S}$ – bounded charge density on the surface of the material

- **Observations**:
    1. The charges inside the material compensate each other. Only polarization charges Q_p remain on the top and bottom surface.
    2. Q_p charges are also called bounded charges because they cannot leave the surface of the material.

2.2. The intensity of the electric field inside the dielectric

- We consider a dielectric placed in the electric field produced between the plates of a charged capacitor with the surface density of free electric charge σ. We aim to calculate the effect of the presence of the dielectric on the intensity of the electric field inside the capacitor.



- The intensity of the electric field between the capacitor plates can be calculated based on Gauss's law if it is considered that the Gaussian surface also contains the bonded charge, σ_P which is the opposite sign of free charge σ

$$E=\frac{\sigma-\sigma_P}{\varepsilon_0}\text{ – intensity in the presence of the dielectric}$$

$$E_0=\frac{\sigma}{\varepsilon_0}\text{ – intensity in the absence of the dielectric (vacuum)}$$

$$\sigma_P=P=\varepsilon_0\chi E\text{ – bounded surface charge density}$$

$$\Rightarrow \boxed{E=E_0-\frac{P}{\varepsilon_0}},\quad P=\varepsilon_0\chi E \;\Rightarrow\; E=E_0-\chi E \;\Leftrightarrow\; \boxed{E=\frac{E_0}{1+\chi}=\frac{E_0}{\varepsilon_r}}$$

The intensity E inside the dielectric is smaller than E_0 in vacuum

σ – free surface charge density;

σ_P – bounded surface charge density;

$\boxed{\varepsilon_r=1+\chi}$ – relative electric permitivity of the material

$\boxed{\varepsilon=\varepsilon_0\varepsilon_r}$ – electric permitivity of the material

Electrical capacitance of the dielectric capacitor (see Chap.I.5)

$$C=\frac{Q}{U}\text{ – definition formula}$$

$$\left.\begin{aligned}&U=Ed\\&E=\frac{E_0}{\varepsilon_r}\end{aligned}\right\}\Rightarrow U=\frac{E_0}{\varepsilon_r}d$$

$$\Rightarrow C=\varepsilon_r\frac{Q}{E_0 d}$$

$$E_0=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 S}\text{ – intensity of the electric field inside the vacuum filled capacitor (see Chap.I.5)}$$

$$\Rightarrow \boxed{C=\varepsilon_0\varepsilon_r\frac{S}{d}=\varepsilon_r C_0}$$

The capacitance C of a capacitor with dielectric inside is bigger than that of the capacitor with air inside C_0

2.3. Electric Field Induction Vector

- As we have seen above, the strength of the electric field between the plates of a plane capacitor with dielectric is of the form:

$E = \frac{\sigma - \sigma_p}{\varepsilon_0}$ -intensity inside the dielectric material

$\boxed{\sigma_p = P}$ – dielectric polarisation =describes bounded charge per unit area

$\boxed{\sigma = D}$ – electric field induction=describes the free charge per unit area

$$\Rightarrow E = \frac{D - P}{\varepsilon_0} \Leftrightarrow \boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

Electric induction vector

(Unit in SI = C/m^2)

Note:

- The electric induction vector is used to describe the electric field inside dielectric materials;
- The electric induction vector D is related to the electric intensity E by the relation:

$$\left.\begin{array}{l} \vec{D} = \varepsilon_0 \vec{E} + \vec{P} \\ \vec{P} = \varepsilon_0 \chi \vec{E} \end{array}\right\} \Rightarrow \boxed{\vec{D} = \varepsilon_0 (1+\chi) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}}$$

- The magnetic induction **B** is related to the magnetic field **H** by the relation (see Chap. VII):

$$\boxed{\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}}$$

VII

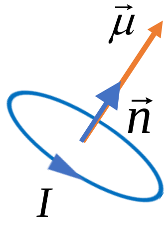
Materials in magnetic field

Content:

1. Magnetic moment and interactions with the field
2. Magnetic field inside materials
3. Diamagnetic materials
4. Paramagnetic Materials
5. Ferromagnetic Materials
6. Antiferromagnetic and ferrimagnetic materials

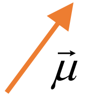
1. Magnetic moment and interactions with the field

- The **magnetic moment** plays in the case of magnetic materials the role played by the **dipole moment** in the case of dielectric materials. We can therefore say that magnetic materials are those materials that are made up of **atoms or molecules that possess a magnetic moment**.
- The magnetic moment of materials can exist **naturally** in them, or it can be **induced** by an external magnetic field
- We define the magnetic moment of a current loop by the relation:



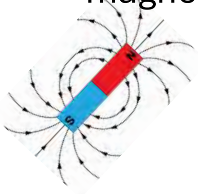
⇔

symbol



⇔

magnet



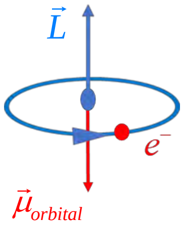
$$\vec{\mu} = IS\vec{n}$$

The magnetic moment is the equivalent of a small magnet

$\vec{\mu}$ – magnetic moment (Am^2);
I – intensity of the electric current in the loop (A);
S – area of the current loop (m^2);
$\vec{n}$ – unit vector oriented parallel to the direction of advancement of the corkscrew that rotates in the direction of the current.

- The magnetic moment, in the case of atoms/molecules that make up magnetic materials, comes from the **orbital motion of electrons** and/or the **magnetic spin moment of the electron** (an intrinsic property of the electron as an electric charge). The resulting magnetic moment is a contribution of the two moments, but the spin magnetic moment has a dominant role if there are unpaired electrons (paramagnetic, ferromagnetic, ferromagnetic, antiferromagnetic materials).

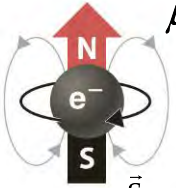
Orbital magnetic moment



$$\vec{\mu}_{orbital} = -\frac{e}{2m}\vec{L}$$

$\vec{L}$ – angular moment
e – charge of the electron
m – mass of the electron

Spin magnetic moment



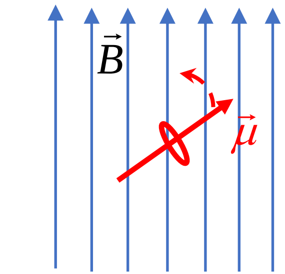
$$\vec{\mu}_{spin} = -g\frac{e}{2m}\vec{S}$$

$\vec{S}$ – spin kinetic moment
$g = 2.002319$ - g factor

Observations:

- According to **quantum mechanics**, an electron can have only two orientations of magnetic momentum in the magnetic field: one with the spin up (parallel to the field) and one with the spin down (antiparallel);
- Also, according to the Pauli exclusion principle, there can be only **2 electrons on an orbital** (1 spin up; 1 spin down).

1.1. The torque on a magnetic moment

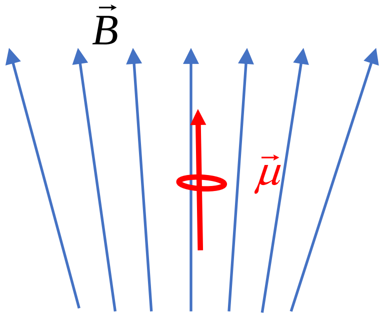


On a magnetic moment **μ** placed in a magnetic field **B** acts a torque:

$$\boxed{\vec{M} = \vec{\mu} \times \vec{B}}$$

This torque tends to orient the magnetic moment parallel to the field lines

1.2. The force exerted on a magnetic moment



On a magnetic moment **μ** placed in magnetic field **B** acts a force:

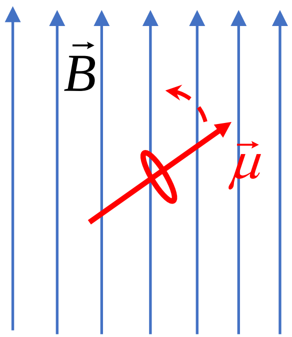
$$\boxed{\vec{F} = \left(\vec{\mu} \cdot \nabla\right) \vec{B}}$$

If the magnetic field acts only along the OZ axis, and the magnetic moment is oriented parallel to the field, then the force becomes:

$$F_z = \mu \frac{\partial B}{\partial z}$$

This magnetic force occurs only in inhomogeneous fields and is analogous to the electric force on a dipole.

1.3. The energy of a magnetic moment placed in the field



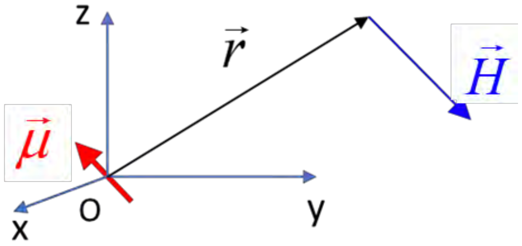
The magnetic moment **μ** placed in a magnetic field **B** will store the energy:

$$W = -\vec{\mu} \cdot \vec{B}$$

Analyzing this expression, it follows that the minimum energy state is the one in which the magnetic moment is parallel to the magnetic field lines (analogous to the electric dipole)

1.4. The intensity of the magnetic field created by the magnetic moment

A magnetic moment **μ** will create around it a magnetic field of intensity **H** that satisfies a relationship analogous to that of the electric dipole:



$$\vec{H} = \frac{1}{4\pi}\left[\frac{3(\vec{\mu}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{\mu}}{r^3}\right]$$

$H = \dfrac{B}{\mu_0}$ – intensity of the magnetic fiels (do not be cofused μ and μ_0)

$\vec{\mu}$ – magnetic moment

$\vec{r}$ – position vector

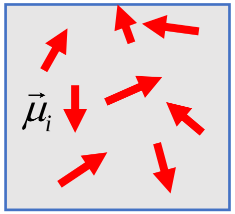
2. Magnetic field inside magnetic materials

- **Magnetic materials** are those materials that are made up of atoms or molecules that possess magnetic moments.
- **Magnetic moments** can be induced to atoms or molecules by an external magnetic field (diamagnetic materials) or intrinsically belong to them (paramagnetic, ferromagnetic, ferromagnetic, antiferromagnetic materials)

2.1. Magnetization vector (*M*)

- *We define the* ***magnetization*** *of a material as the* ***magnetic moment per unit volume:***

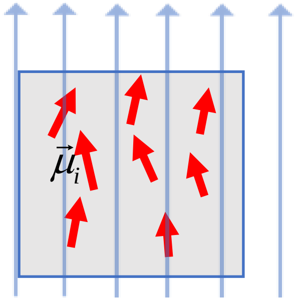
$\vec{B} = 0$



Randomly oriented magnetic moments

$\vec{M} = 0$

$\vec{B} \neq 0$



Magnetic moments oriented parallel to the field

$\vec{M} \neq 0$

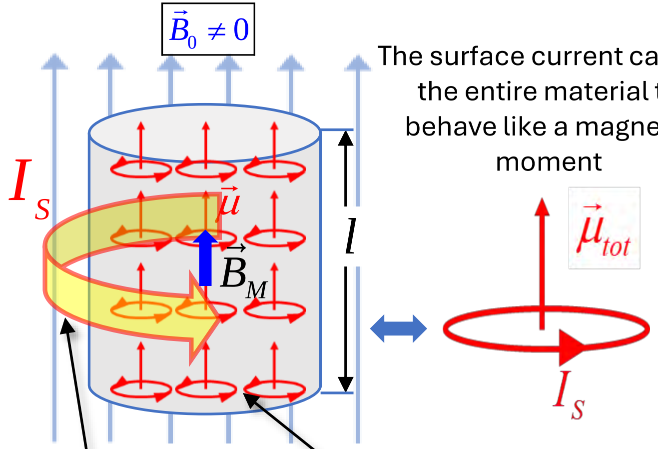
$$\vec{M} = \frac{\sum_i \vec{\mu}_i}{V}$$

$\vec{\mu}_i$ – magnetic moment of the atom/molecule i;
V – volume of the material

The unit of measurement for magnetization is: $[M]_{SI} = A/m$

2.2. Magnetic induction vector inside magnetic materials

- We consider a magnetic material made up of identical magnetic moments aligned parallel to the magnetic field, as in the figure below.



Inside the material the currents cancel each other out.

A surface current arises on the surface of the material.

Total magnetic moment of the material: $\mu_{tot} = I_S S$

The magnetization of the material can be calculated: $\vec{M} = \frac{\sum_i \vec{\mu}_i}{V} = \frac{\mu_{tot}}{V}\vec{k}$

$$\Rightarrow \boxed{\vec{M} = \frac{I_S}{l}\vec{k}}$$

$V = Sl$ – volume of the cylinder;

S-area of the basis; l-length of the cylinder.

- The surface current creates a magnetic field inside the material parallel to the outside field, similar to that produced by a solenoid:

$$B_M = \mu_0 \frac{I_S}{l} = \mu_0 M \Leftrightarrow \boxed{\vec{B}_M = \mu_0 \vec{M}} \leftarrow$$

The field produced by magnetization

- The total magnetic field inside the material is the sum of the external magnetic field **B_0** and the one produced by magnetization **B_M**:

$$\vec{B} = \vec{B}_0 + \vec{B}_M = \boxed{\vec{B} = \vec{B}_0 + \mu_0 \vec{M}} \leftarrow$$

The magnetic field inside a magnetic material

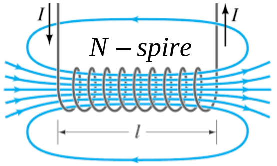
Note that the magnetization of materials depends on the intensity H of the magnetic field applied by the relationship:

$$\boxed{\vec{M} = \chi \vec{H}}$$

$\vec{H} = \frac{\vec{B}_0}{\mu_0}$ – intensity of the magnetic field

χ – magnetic susceptibility of the material

The magnetic field inside a solenoid



$$B = \mu_0 \frac{NI}{l} = \mu_0 \frac{I_S}{l}$$

since: $NI = I_S$

- Substituting in the expression of the total magnetic field the susceptibility dependence of the internal magnetic field, and considering the relationship between induction and magnetic intensity, we obtain:

$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M} \Longleftrightarrow \vec{B} = \mu_0 \vec{H} + \mu_0 \chi \vec{H} = \mu_0 (1+\chi) \vec{H} = \mu_0 \mu_r \vec{H} = \mu \vec{H} \Longleftrightarrow \boxed{\vec{B} = \mu \vec{H}}$$

$$\vec{M} = \chi \vec{H}$$

$$\vec{B}_0 = \mu_0 \vec{H}$$

$\mu = \mu_0 \mu_r$ – magnetic permeability

$\mu_r = (1+\chi)$ – relative permeability

The connection between magnetic induction and electric intensity in magnetic materials

Examples of magnetic materials

Types of Magnetic Materials	Material	Susceptibility	Limits
Diamagnetic	Superconductor	-1 (perfect diamagnet)	($-10^{-4}\div-10^{-9}$)
	Hg	$-2.9\cdot10^{-4}$	
	Cu	$-1.0\cdot10^{-5}$	
	H_2O	$-0.91\cdot10^{-5}$	$\chi < 0$
Paramagnetic	O_2 (lichid)	$3.6\cdot10^{-3}$	($10^{-6}\div-10^{-2}$)
	Mg	$1.2\cdot10^{-5}$	
	Al	$2.2\cdot10^{-5}$	
	Li	$1.4\cdot10^{-5}$	$\chi > 0$
Ferromagnetic	Fe	$6\cdot10^{3}$	($10^{-1}\div-10^{5}$)
	75Ni-Fe	$9\cdot10^{4}$	
	Co	-	
	Gd	-	$\chi >> 0$

It depends on the microstructure of the material in question

The magnetic susceptibility χ characterizes materials magnetically, classifying them into:

-diamagnetic $\left(-10^{-4} \le \chi \le -10^{-9}\right)(\chi < 0)$

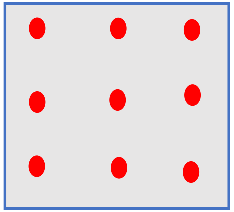
-paramagnetic $\left(10^{-6} \le \chi \le 10^{-2}\right)(\chi > 0)$

-feromagnetic $\left(10^{-1} \le \chi \le 10^{5}\right)(\chi >> 0)$

3. Diamagnetic materials

- **Diamagnetic materials** are made up of atoms/molecules that **do not have their own magnetic moment** (ex. Cu, C, H_2O, wood).
- When introduced into the magnetic field, these materials receive an **induced magnetic moment, oriented antiparallel to the field lines**: $\vec{\mu}_{ind} \uparrow\downarrow \vec{B}_0$

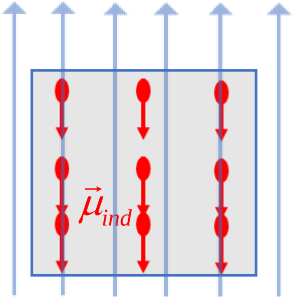
$\vec{B}_0 = 0$



Atoms do not possess magnetic moments

$$\vec{M} = 0$$

$\vec{B}_0 \neq 0$



Induced magnetic moments, oriented antiparallel to the field

$$\vec{M} = \chi \vec{H} = \chi \frac{\vec{B}_0}{\mu_0} \quad \boxed{\chi < 0}$$

Observations:

- The induced magnetic moment occurs as a result of the change that the external magnetic field produces in the orbits of the electrons;
- The susceptibility constant depends only on the nature of the material and is negative

$$\boxed{\chi < 0}$$

- The susceptibility constant does not depend on temperature;
- The susceptibility constant does not depend on the size of the external field $\mathbf{B_0}$.

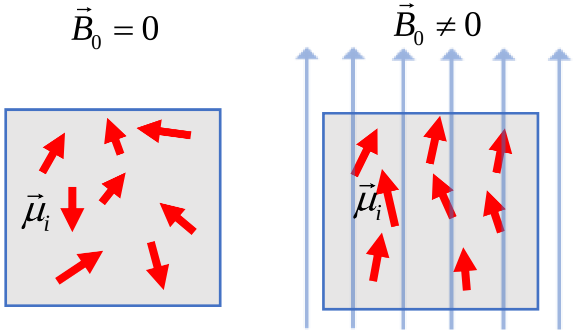
Levitation

If a diamagnetic material (even a frog) is placed in the magnetic field, it can levitate, under certain conditions. Below are some experiments.

https://www.youtube.com/watch?v=VC3r9-OaWes
https://www.youtube.com/watch?v=X5EoUD-BIss
https://www.youtube.com/watch?v=KUsVqc0ywM

4. Paramagnetic Materials

- **Paramagnetic materials** are made up of atoms/molecules that **have their own magnetic moment** (ex. Al, Li, O_2).
- The magnetic moment of the atoms of these materials is due to the **spin of unpaired electrons**, because the orbital magnetic moment can be neglected.
- When inserted into the magnetic field, the **magnetic moments align parallel to the field lines:** $\vec{\mu}_i \uparrow\uparrow \vec{B}_0$



Randomly oriented magnetic moments

$\vec{M} = 0$

Magnetic moments oriented parallel to the field

$\vec{M} = \chi\vec{H} = \chi\frac{\vec{B}_0}{\mu_0}$ $\boxed{\chi > 0}$

Observations:

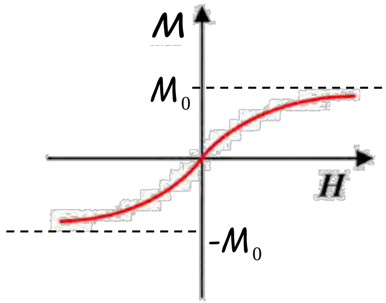
- The susceptibility constant depends on the nature of the material and the temperature

$$\boxed{\chi = \frac{C}{T}}$$

C – Curie's constant (depends on material) (K);
T – absolute temperature (K)

- The magnetization of the material disappears as soon as the magnetic field is removed, the magnetic moments settling disorderly

- Magnetization **M** depends on the applied external field intensity **H** as in the figure:

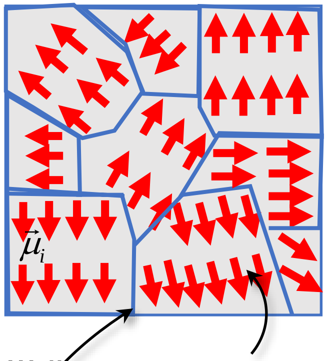


Saturation magnetization
=all magnetic moments aligned in parallel

5. Ferromagnetic materials

- **Ferromagnetic materials** are formed by atoms/molecules that **possess their own magnetic moment** and in addition **these moments are locally oriented**, forming magnetization domains also called **Weiss domains** (e.g. Fe, Co). These areas are separated by **Bloch walls**.
- The magnetic moment of the atoms of these materials is also due to the **spin of the unpaired electrons**, however, compared to paramagnetic materials, in the case of ferromagnetic materials there are interactions between these spins.
- When introduced into the magnetic field, the **magnetic domains parallel to the field increase** at the expense of those aligned antiparallel, until saturation is reached.

$\vec{B}_0 = 0$

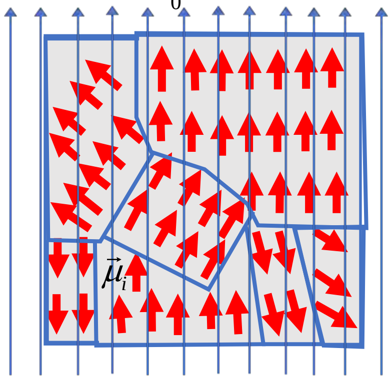


Bloch Wall

Weiss Domain
with all the spins aligned parallel (10nm÷10μm depending on the material)

Randomly oriented magnetic domains $\vec{M} = 0$

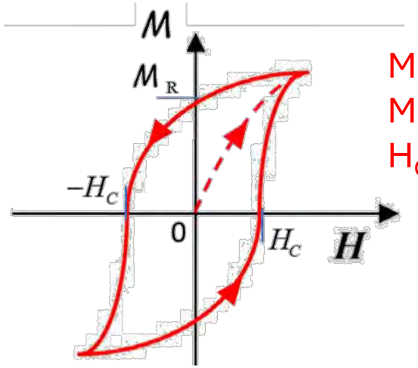
$\vec{B}_0 \neq 0$



Magnetic domains oriented predominantly parallel to the external field

$\vec{M} = \chi \vec{H}$ $\boxed{\chi \gg 0}$

The hysteresis curve of magnetization



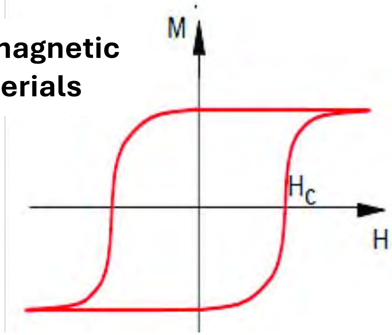
M-magnetization;
M_R-residual magnetization;
H_C-coercive field

The magnetization of ferromagnetic materials presents the phenomenon of hysteresis, i.e. when the intensity of the H field increases, the magnetization has different values than when it decreases. Thus, for H=0 we have a remanent magnetization M_R on the return and only by applying a coercive field H_C will $M=0$ be obtained.

Obs.: We cannot speak of a single susceptibility constant, since it depends on the external magnetic field

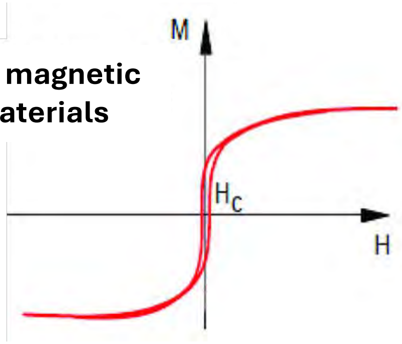
- Depending on the shape of the hysteresis curve, magnetic materials can be classified into **hard** magnetic materials and **soft** magnetic materials

Hard magnetic materials



- ❑ Materials with high remanence magnetization and high coercive field.
- ❑ Used as permanent magnets

Soft magnetic materials



- ❑ Materials with low residual magnetization and small coercive field. .
- ❑ Used for sensors, transformers and when fast magnetization manipulation is desired.

- Above a certain temperature, called the **Curie temperature** (T_C), ferromagnetic materials lose their properties (the Weiss domains are destroyed) and turn into paramagnetic materials. The Curie temperature depends on the nature of the material. For example, T_{CFe}=1043 K, and above this temperature the iron becomes paramagnetic.

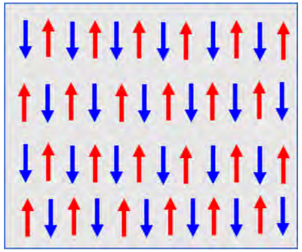
- In the **paramagnetic temperature** ($T>T_C$) region, the **Curie-Weiss law** describes the susceptibility:

$$\chi = \frac{C}{T - T_C}$$

C – Curie's constant (depends on material) (K);
T – absolute temperature (K);
T_C – Curie's temperature (K)

6. Antiferromagnetic and ferrimagnetic materials

- In some magnetic materials, the magnetic moments forming the Weiss domains can be found in other configurations than those described above.
- Materials in which the magnetic moments assemble as layers in which the magnetic moments cancel each other are called antiferromagnetic (e.g. NiO, MnO, FeO).

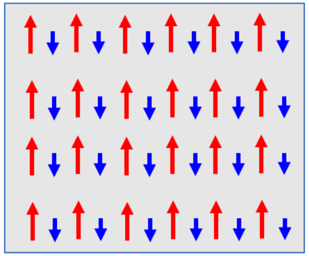


$\vec{M} = 0$

The materials remain antiferromagnetic up to a certain temperature called the **Neel temperature** (T_N). Above this, they become paramagnetic.

Antiferromagnetism plays an important role in the phenomenon of **giant magnetoresistance** (Nobel 2007) which consists of controlling the electrical resistance of a material by means of a magnetic field (Applications: field sensors, hard disks, micro mechanical systems).

- Materials in which the antiparallel magnetic moments assemble as layers in which the magnetic moments do not cancel are called ferrimagnetic (ex. ferrite: Fe_3O_4, $BaFe_{12}O_{19}$).



$\vec{M} \neq 0$

The materials remain ferromagnetic up to a certain temperature called the **Curie temperature (T_C)**. Above this, they become paramagnetic.

Ferrimagnetic materials have a **high electrical resistivity**, which means that they will generate small eddy currents and are therefore useful in transformers or high-frequency coils

VIII
Elements of quantum physics

Content:

1. The evolution of ideas in quantum physics
2. The photoelectric effect and the quantum theory
3. De Broglie's hypothesis and the wave-particle duality
4. Heisenberg's principle of uncertainty
5. Postulates of quantum physics
6. Schrödinger's equation
7. Applications to Schrödinger's equation
8. Energy levels of atoms. Atomic orbitals

1. The evolution of ideas in quantum physics

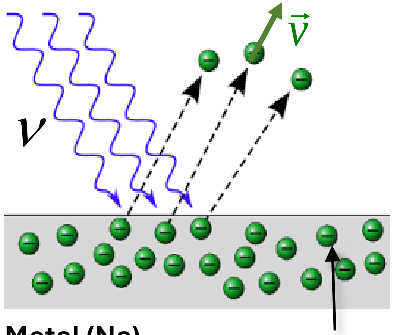
- **Quantum physics** describes the evolution over time of systems of microparticles (atoms, molecules, electrons, protons) as classical physics describes the evolution of macroscopic systems (objects in the world around us).
- Quantum physics has its origin in several experiments that classical physics could not explain, and therefore it was necessary to introduce new hypotheses and ideas, which sometimes seem absurd to someone accustomed to the experience gained in the classical (macroscopic) world.

• **The experiments that classical physics could not explain:** 1. Photoelectric effect; 2. Black body radiation; 3. Spectral lines and stability of the atom; 4. Electron diffraction.	Their interpretation required the introduction of **new concepts**: ❑ Quantization of physical quantities (energy, momentum, angular momentum); ❑ Wave-corpuscle dualism; ❑ The uncertainty principle; ❑ Operators corresponding to physical quantities; ❑ Quantum states and probabilities of occupying these states, etc.

- The foundations of quantum physics (also known as quantum mechanics) were laid in the early twentieth century by Max Planck, Niels Bohr, Werner Heisenberg, Louis de Broglie, Arthur Compton, Albert Einstein, Erwin Schrödinger, Max Born, John von Neumann, Paul Dirac, Enrico Fermi, Wolfgang Pauli, Max von Laue, Freeman Dyson, David Hilbert, Wilhelm Wien, Satyendra Nath Bose, Arnold Sommerfeld and others.
- The development of quantum mechanics continues to this day both in terms of formalism/ideas and applications (all the semiconductors used in electronics would not be possible without quantum physics).

2. The photoelectric effect and quantum theory

- ***The photoelectric effect*** *consists of the emission of electrons by metals (Na, Fe, Al) if they are radiated with radiation of a certain frequency*



Experimental observations on the photoelectric effect:

- Electrons are emitted instantaneously ($<10^{-9}$s);
- Increasing the intensity of radiation (light), preserving the frequency (color) produces an increase in the number of photoelectrons emitted;
- Increasing the intensity does not lead to an increase in the kinetic energy of photoelectrons;
- A frequency $\nu<\nu_{\_threshold}$ radiation does not produce photoelectrons independent of its intensity;
- The kinetic energy of photoelectrons increases by increasing the frequency of radiation.

Einstein's explanations (Nobel 1921):

- Radiation (and light) is made up of particles (quanta) called photons;
- The energy of a photon is:

$$\boxed{E_f = h\nu} - \text{Einstein's } 1^{st} \text{ law}$$

$$h = 6.626 \cdot 10^{-34}\, Js - \text{ Planck's constant}$$

- The energy of a quantum is transferred into extraction energy W_{ext} and kinetic energy E_C

$$\boxed{h\nu = W_{ext} + m\frac{v^2}{2}} - \text{Einstein's secon law}$$

$h\nu$ – energy of the incident photon;

W_{ext} – extraction enegy of the electron (material specific quantity);

m – mass of the electron;

v – velocity of the emitted electron.

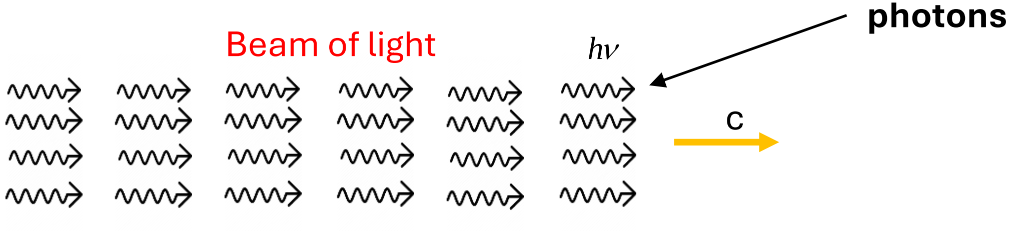
Extraction energies of several metals

Metal	W_{ext} (eV)
Al	4.08
Fe	4.5
Zn	4.3
Na	2.28

$$1eV = 1.6 \cdot 10^{-19}\, CV = 1.6 \cdot 10^{-19}\, J$$

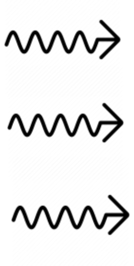
According to Einstein's theory:

Light (radiation) is made up of photons



The photon = a particle (quantum) made of light (or radiation) having energy, mass and momentum:

Theory of relativity: **E=mc²** ⇒



$$\boxed{E_f = h\nu} - \text{photon energy}$$

$$\boxed{m_f = \frac{h\nu}{c^2}} - \text{photon mass;}$$

$$\boxed{p_f = m_f c = \frac{h\nu}{c}} - \text{photon momentum;}$$

$h = 6.626 \cdot 10^{-34}\, Js$ – Planck's constant;

ν – photon frequency

$c = 3 \cdot 10^{-8}$ m/s – speed of light in vacuum;

(in fact $c = 299792458$ m/s)

Example: mass of red light photon (λ=700 nm):

$$m_{red} = \frac{h\nu}{c^2} = \frac{hc}{\lambda c^2} = \frac{h}{\lambda c} = \frac{6.626 \cdot 10^{-34}}{700 \cdot 10^{-9} \cdot 3 \cdot 10^{8}} = 3.15 \cdot 10^{-36}\, kg$$

$$m_{electron} = 9.1 \cdot 10^{-31}\, kg$$

Observation: This describes a moving photon. Because its rest mass is zero, a photon cannot be at rest.

3. De Broglie's hypothesis and wave-particle duality

- Experiments related to the photoelectric effect and blackbody radiation led to the conclusion that light (considered to be a wave) behaves in some experiments as a stream of particles, each carrying energy $h\nu$
- Starting from the wave-corpuscle **dualism** of light, de Broglie formulated in 1924 the following hypothesis: ***If light behaves like a particle in certain experiments, it means that there are experiments in which particles behave like waves, having properties specific to them (diffraction, interference)*** (Nobel, 1929).
- According to de Broglie, **each particle has an associated wavelength λ**:

$$\lambda = \frac{h}{p}$$

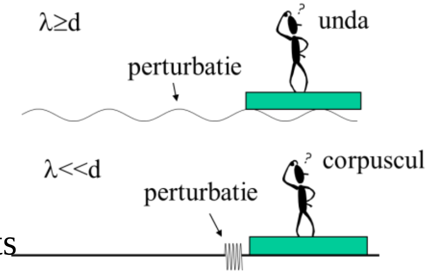
$h = 6.626 \cdot 10^{-34}\ Js$ – Planck's constant

$p = mv$ – momentum of the particle

- On the size of the associated wavelength depends whether a wave or particle (corpuscle) character will manifest in an experiment, as follows:

$\lambda \geq d$ - *wave* character manifests

$\lambda << d$ - *particle* character manifests



A man sitting on the raft feels the wave as a wave or as a corpuscle, depending on the size of the raft relative to the wavelength.

Example:

- A body with a mass of 1 g travelling at velocity v=1m/s will have an associated wavelength:

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \cdot 10^{-34}}{10^{-3} \cdot 1} = 6.626 \cdot 10^{-31} m$$

$\Rightarrow \lambda << d$ -dimmmension of any objects

- An electron accelerated to a potential difference of 54V will have a wavelength λ=0.165 nm

$\Rightarrow \lambda$ - the order of magnitude of the inter-atomic layer distance in a crystal (in the case of Ni = 0.091nm)

$\Rightarrow$ diffraction effects can be observed

- The wave character of electrons was first demonstrated by Davisson-Germer (1927) by diffraction experiments on a nickel crystal (Nobel, 1937). A description of the experiment can be found at: https://www.youtube.com/watch?v=Ho7K27B_Uu8

4. Heisenberg's principle of uncertainty

- In quantum physics, the result of measuring a physical quantity is not deterministic (single) but is characterized by a probability distribution. The value measured by an observer (instrument) is a weighted average of the possible values of that physical quantity.

- According to **Heisenberg's principle of indeterminacy** (1927) In a given measurement, no matter how large, both the position and momentum of a particle cannot be determined with precision, and the standard deviation satisfies the relationship:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Δp – standard deviation (error) in measuring momentum

Δx – standard deviation (error) in measuring position

$\hbar = \frac{h}{2\pi}$ – reduced Plank constant (Dirac's constant)

- This principle also applies to other conjugate pairs of physical quantities (whose operators do not commute), for example in the case of **energy and time**:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

ΔE – standard deviation (error) in measuring energy

Δt – standard deviation (error) in measuring time

Note: Heisenberg's principle can be explained on the basis of **wave–particle dualism**, being a fundamental effect in quantum physics and cannot be attributed to the influence that the observer introduces on the outcome of the experiment (as is sometimes misinterpreted). To understand this principle, it is recommended to view the video clip at the links: https://www.youtube.com/watch?v=TQKELOE9eY4

https://www.youtube.com/watch?v=qwt6wUUD2QI

Standard deviation

$$\Delta x = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}}$$ – standard deviation;

$\bar{x} = \sum_{i=1}^{n} x_i$ – aritmetic mean;

n – nr. of measurements;

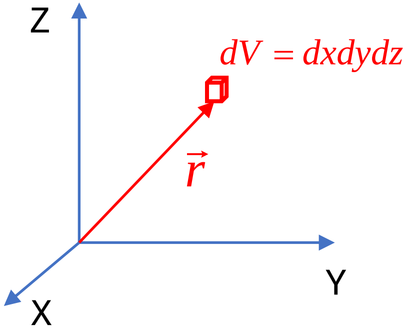
x_i – result of a measurement.

5. Postulates of quantum physics

- Quantum physics (QF) has already more than a century of development and in the present course we have allocated only 4 hours for this topic. It is therefore obvious that we cannot synthesize even its most important ideas. However, in order to get by further and to be able to read a book on solid state physics or microelectronics, we will try to synthesize some ideas here. Thus, it is enough to introduce some of the basic ideas of quantum physics in the form of **postulates**.
- By analogy with classical mechanics, which is based on **4 postulates** (law of inertia, law of force, law of action and reaction, law of superposition of forces) we will restrict ourselves here to 4 postulates, although thus we will not encompass all QF ideas

Postulate I:
The state of a physical system can be described by means of a state function (wave function), $\Psi(\vec{r}, t)$, which contains all the information about the system.

If the system consists of a single particle, the state function ψ allows calculation of the **probability dP to have the particle in a certain region of space** dV, indicated by the position vector **r**



$$\boxed{dP = \Psi^*(\vec{r},t)\Psi(\vec{r},t)dV} \text{ – probability to have the particle inside dV}$$

$$\Psi^*(\vec{r},t)\Psi(\vec{r},t) \text{ – probability density}$$

$$\Psi^*(r,t) \text{ – complex conjugate function of } \Psi(r,t)$$

Since the probability of finding the particle somewhere (anywhere) in the Universe is 1, we have:

$$\boxed{\iiint_{(\infty)} \Psi^*(\vec{r},t)\Psi(\vec{r},t)dV = 1} \text{ -nonmalization condition of the wave function}$$

Note: Here we referred to the system consisting of 1 particle. However, we have the same significance for the wave function for systems consisting of many particles.

Postulate II:

Every observable physical quantity A (e.g. position, energy, momentum, angular momentum) corresponds in quantum physics to a Hermitian operator $\hat{A}$, such that measurements (observables) of the physical quantity A result in values a equal to the operator's eigenvalues $\hat{A}$.

Eigen values *a* are those that satisfy the equation with eigenvalues:

$$\boxed{\hat{A}\Psi = a\Psi}$$

Ψ – wave function (eigen function)

$\hat{A}$ – operator of the physical quantity

a – eigen value

Fundamental operators

Physical quantity	Operator
Position: x	$\hat{x} = x$
Momentum: p_x	$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$
Total energy: E	$\hat{E} = i\hbar \frac{\partial}{\partial t}$

The operators for y and z components have similar forms

Postulate III:

The relationships between the operators associated with physical quantities are the same as between the physical quantities they represent

Example: the kinetic energy operator

physical quantity *operator*

$$E_c = m\frac{v^2}{2} = \frac{p^2}{2m} \Rightarrow \hat{E}_c = \frac{\hat{p}_x^{\,2}}{2m} = \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x}\right)^2 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

Postulate IV:

Value of an observable represented by the operator $\hat{A}$, of the Hermitian type, is given by the average value calculated as follows:

$$\boxed{\langle A \rangle = \iiint\limits_{(\infty)} \Psi^*(\vec{r},t)\hat{A}\Psi(\vec{r},t)\,dV}$$

6.Schrödinger'sequation

- As we remember from Newton's classical mechanics course, Newton's 2nd law ($\vec{F} = d\vec{p}/dt$) allows obtaining of position vectors of a system, if the initial conditions are known. This way we can predict the evolution of the system over time.
- In quantum mechanics, all information about the physical system is contained in the state (wave) function $\Psi(\vec{r},t)$, so by knowing the wave function we get information about the system.
- The state function in quantum mechanics is obtained as the solution of a differential equation called Schrödinger's equation (1926). This equation will be written below in various forms to be used in practical applications.

<u>According to postulate II:</u> Every observable physical quantity A (e.g. position, energy, momentum, angular momentum) corresponds in quantum physics to a Hermitian operator $\hat{A}$, such that measurements (observables) of the physical quantity A result in values a equal to the operator's eigenvalues $\hat{A}$.

Temporary Schrödinger equation—describes the time dependence of the state (wave) function

- Operator $\hat{A}$ satisfies the equation with eigenvalues: $\hat{A}\Psi = a\Psi$

- We consider the operator $\widehat{A}$ as the total energy operator $\widehat{E}$ of the system, having eigenvalues E: $\hat{E} = i\hbar\frac{\partial}{\partial t}$

$$\hat{E}\Psi = E\Psi \Leftrightarrow i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = E\Psi(\vec{r},t)$$

If the energy of the system is constant and consists of kinetic energy and potential energy, then the corresponding operator is called the **Hamiltonian operator**, $\widehat{H}$ and the eigenvalues E of this operator are the very values that energy can have to the system, that is, the operator satisfies the equation with eigenvalues:

$$\hat{H}\Psi(\vec{r},t) = E\Psi(\vec{r},t)$$

Schrödinger staționary equation (spatial)-describes the position dependence of the state function

$$\hat{H} = \hat{E}_c + U - \text{Hamiltonian operator of the system}$$

- From the combination of the two equations above gives **Schrödinger's general equation**:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \hat{H}\Psi(\vec{r},t)$$

General Schrödinger equation - describes the position and time dependence of the state function

The case of a single particle

- In the case of a particle moving along the OX axis in a potential energy field U(x), the Hamiltonian operator is:

$$\hat{H} = \hat{E}_c + U(x) = \frac{\hat{p}^2}{2m} + U(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$$

- Introducing this Hamiltonian into the **general Schrödinger** equation:

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

$$\Rightarrow \quad i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right]\Psi(x,t)$$

General Schrödinger's equation for a one-dimensional moving particle

- Introducing the Hamiltonian above into the **stationary Schrödinger** equation, we have:

$$\hat{H}\Psi = E\Psi \Leftrightarrow \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U\right)\Psi = E\Psi \Leftrightarrow \frac{\partial^2}{\partial x^2}\Psi(x,t) + \frac{2m}{\hbar^2}\left[E - U(x)\right]\Psi(x,t) = 0$$

Stationary (spatial) Schrödinger equation for a one-dimensional moving particle

- If the particle can perform a **three-dimensional** motion, the **stationary Schrödinger equation** becomes:

$$\nabla^2\Psi(\vec{r},t) + \frac{2m}{\hbar^2}\left[E - U(x,y,z)\right]\Psi(\vec{r},t) = 0$$

Obs.: This equation allows the calculation of the energy levels and orbitals of hydrogen atoms. In that case the Laplace operator is written in spherical coordinates.

$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ – Laplace operator

$\Psi(\vec{r},t)$ – wave function;

E-total energy of the particle;

$U(x,y,z)$ – potential energy of the particle;

m-particle's mass; $\hbar = h/2\pi$-reduced Planck's constant

6. Applications of Schrödinger's equation

6.1. The free particle

- In the following we will describe with the help of Schrödinger's equation the simplest quantum system: the free particle in one-dimensional motion, II OX, in the positive sense

free particle $\Longleftrightarrow$ $U(x)=0$

- The Schrödinger equations that provide the wave function for the free particle as well as their solutions are described below:

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=E\Psi(x,t)$$ $\Rightarrow$ $\Psi(x,t)=\Psi(x,0)e^{-i\frac{E}{\hbar}t}$ $\Leftrightarrow$ $\Psi(x,t)=\Psi(x,0)e^{-i\omega t}$ -Time dependence of the wave function

$\omega=\frac{E}{\hbar}$ – angular frequency

$$\frac{\partial^2\Psi(x,t)}{\partial x^2}+\frac{2mE}{\hbar^2}\Psi(x,t)=0$$

-if we substitute the time dependence of the wave function, found in the time equation

$\Rightarrow$

$$\frac{\partial^2\Psi(x,0)}{\partial x^2}+\frac{2mE}{\hbar^2}\Psi(x,0)=0$$

We note:

$k^2 =\Leftrightarrow k=\frac{1}{\hbar}\sqrt{2mE}$ – *wavenumber*

$\Rightarrow$

$$\frac{\partial^2\Psi(x,0)}{\partial x^2}+k^2\Psi(x,0)=0$$

Equation similar to that of the harmonic oscillator

$$\frac{d^2x}{dt^2}+\omega^2x=0$$

$\Psi(x,0)=Ae^{ikx}+Be^{-ikx}$

-Spatial dependence of the wave function

$\Rightarrow$

The total wave function:

$\Rightarrow$ $\Psi(x,t)=Ae^{-i(\omega t-kx)}+Be^{i(\omega t+kx)}$

progressive *regressive*

$\Longrightarrow$

Since the displacement is II OX, in a positive sense, only the progressive solution will be preserved

$$\Psi(x,t)=Ae^{-i(\omega t-kx)}$$ $\longleftarrow$ The wave function of the free particle

Constant A must be found from the condition of normalization of the wave function

Normalization condition for the wave function:

$$\int_{-\infty}^{+\infty} \Psi^*(x,t)\Psi(x,t)\,dx = 1$$

The normalization condition says that the probability of finding the particle anywhere on the OX axis between $-\infty$ and $+\infty$ is 1

$$\Psi(x,t) = Ae^{-i(\omega t - kx)}$$

The wave function for free particle

$$\Psi^*(x,t) = Ae^{i(\omega t - kx)}$$

The conjugate wave function for free particle

$$\Rightarrow \int_{-\infty}^{+\infty} A^2 dx = 1 \Rightarrow \boxed{A \to 0}$$

An absurd conclusion!

⬇

There are no completely free particles in the universe !!

⬇

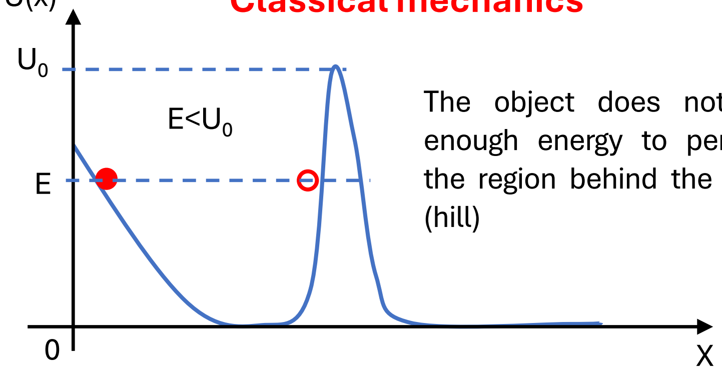
$$\boxed{U(x) \neq 0}$$

The potential energy of a particle will always be **nonzero**, so it will be under the action of some force fields from other particles.

6.2. The tunnel effect

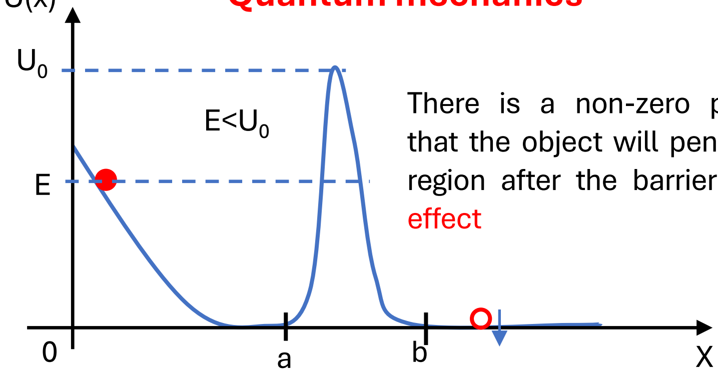
- **Tunnel effect** consists of passing a quantum object through a barrier of potential energy higher than the energy of that object.
- To illustrate the tunneling effect, consider the quantum object to be a ball rolling down a slope and meeting a hill. We will describe what happens from a classical and quantum point of view with that object.

Classical mechanics



The object does not have enough energy to penetrate the region behind the barrier (hill)

Quantum mechanics



There is a non-zero probability that the object will penetrate the region after the barrier = tunnel effect

Explanation

The tunneling effect can be described using the stationary Schrödinger equation:

$$\frac{\partial^2\Psi(x)}{\partial x^2}+\frac{2m}{\hbar^2}\left[E-U(x)\right]\Psi(x)=0$$

E – total energy; $U(x)$ – potential energy; U_0 – barrier height; m – particle mass.

This allows the calculation of the wave function in different regions of space and thus of the probability density in different regions. It is observed that the probability density is different from zero and in the region after the barrier

$$\Psi^*(x)\Psi(x)>0$$

Knowledge of the wave function allows calculating the **transparency coefficient T** of the barrier (**$T=I_{transmitted}/I_{incident}$**) in case of current intensity)

$$T=\exp\left\{-\frac{2}{\hbar}\int_a^b\sqrt{2m\left[U(x)-E\right]}dx\right\}$$

There is an exponential dependence on the thickness of the barrier ⇔ high sensitivity to thickness

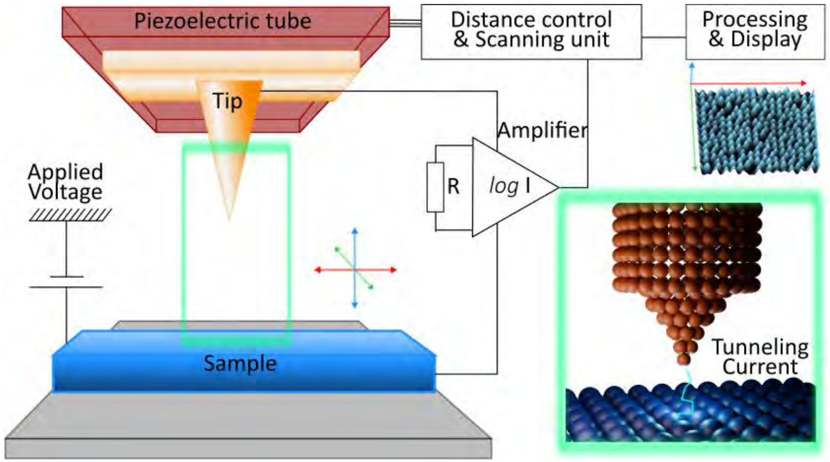
The tunneling microscope

- The tunneling effect explains several experimental observations such as α decay of radioactive nuclei, emission of free electrons from metals or kinetics of chemical reactions. The tunneling effect also has practical applications such as the tunneling diode or the **scanning tunneling microscope (STM)**.

STM operating principle

A very sharp tip (possibly a single atom) is passed on the surface of an invesrigated sample using piezoelectric transducers. The tunneling current is strongly dependent on the distance between the atoms and the tip (see rectangular barrier). Image resolutions of 0.1nm-lateral and 0.01nm-depth can be obtained. It can also manipulate atoms.

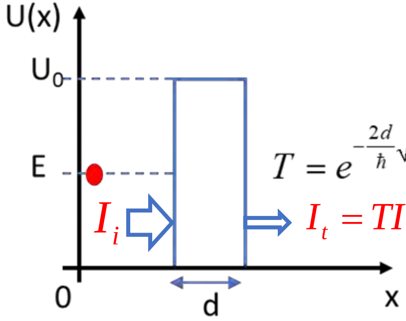
Diagram of a tunneling microscope



https://www.youtube.com/watch?v=wNEqRq6NyUw

Rectangular barrier

The transfer coefficient T of a current through a rectangular barrier depends on the width d of the barrier but also on the difference U_0-E between barrier height and particle energy. **The dependence on d is exponential** → **The transmitted** (tunneling) **current strongly depends on the thickness of the barrier (this effect is used under a tunneling microscope)**



$$T = e^{-\frac{2d}{\hbar}\sqrt{2m[U_0-E]}}$$

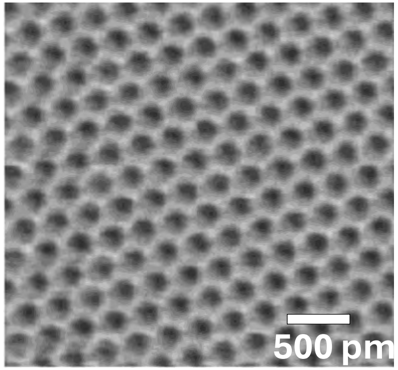
$$I_t = TI_i$$

T – transfer coefficient;

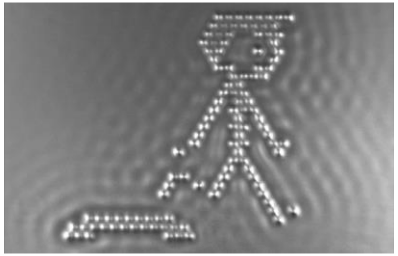
I_i – incident current;

I_t – transmitted currect (tunneling current).

Obtained images



Micrograph of graphene



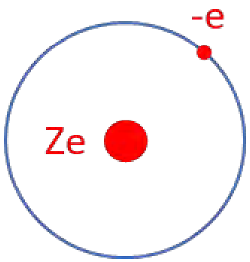
Atomic manipulation by **IBM**

https://www.youtube.com/watch?v=oSCX78-8-q0

6.3. Energy levels of atoms

- Schrödinger's stationary equation can also be applied to describe the distribution of electrons over energy levels in atoms.
- In the following we will restrict ourselves to the case of hydrogenoid atoms(H, He^{+}, Li^{2+}, Be^{3+} și B^{4+}), which are formed from the nucleus with charge Ze and an electron, since only this can be solved analytically (not by us). In the case of atoms with multiple electrons, because of their interactions, Schrödinger's equation can only be solved numerically.

- **Schrödinger's stationary** equation for such an atom is:



$$\nabla^2\Psi(\vec{r},t)+\frac{2m}{\hbar^2}\left[E-U(r)\right]\Psi(\vec{r},t)=0$$

E – *total energy of the electron*; m – *mass of the electron*;

$U(r)=-\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$ – *potential energy of the electron.*

- Due to the spherical symmetry of the atom, this equation is preferable to write in spherical coordinates:

$$\left\{-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+U(r)-\frac{\hbar^2}{2mr^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]\right\}\Psi(r,\theta,\varphi)=E\Psi(r,\theta,\varphi)$$

- After quite complicated calculations*, using a lot of tricks, and taking into account the normalizing condition, the solution becomes:

$$R_{n,l}(r)=\frac{2}{n^2a_0^{3/2}}\sqrt{\frac{(n-l-1)!}{(n+l)!}}\left(\frac{2r}{na_0}\right)^l e^{-\frac{r}{na_0}}L_{n-l-1}^{2l+1}\left(2r/na_0\right)$$

$$Y_{l,m}(\theta,\varphi)=\frac{(-1)^l}{2^l l!}\sqrt{\frac{(2l+1)(l+m)!}{4\pi(l-m)!}}\frac{e^{im\varphi}}{\sin^m\theta}\left(\frac{\partial}{\partial\cos\theta}\right)^{l-m}\sin^{2l}\theta.$$

$L_{n-l-1}^{2l+1}\left(2r/na_0\right)$ – *Laguere polynomials*

$$a_0=\frac{\hbar^2}{m_0e^2}=0.529177\cdot10^{-10}m – \textit{Bohr}\text{ radius}$$

$$\Psi_{n,l,m}(r,\theta,\varphi)=R_{n,l}(r)Y_{l,m}(\theta,\varphi)$$ -the wave function

$$E_n=-\frac{mZ^2e^4}{8h^2\varepsilon_0^2}\frac{1}{n^2}$$

The energy levels on which the electron is located are quantized (the electron can only have these energies)

Quantum numbers

$n=1,2,...,$ (principal quantuum number)

$l=0,1,2,...n-1$ (orbital quantuum number)

$m=-l,-l+1..-1,0,1,...l-1,l$ (magnetic quantuum number)

Pauli's principle: The quantum state characterized by the quantum numbers n, l, m can be occupied by 2 electrons: one with spin -1/2 and one with spin +1/2

*I. Ardelean, Introducere in mecanica cuantica, U.T. Pres, 2002, Cluj-Napoca

Degenerate energy states

For a main quantum number **n** we have **one energy**

$$E_n = -\frac{mZ^2e^4}{8h^2\varepsilon_0^2}\frac{1}{n^2}$$

but **more state functions**

$$\Psi_{n,l,m}(r,\theta,\varphi)$$

degenerate energy states

$n = 1,2,...,$ -principal quantum number
$l = 0,1,2,...n-1$ -orbital quantum number
$m = -l,-l+1..-1,0,1,...l-1,l$ -magnetic quantum number

$l = 0,\ 1,\ 2,\ 3,\ 4...$

↓ ↓ ↓ ↓ ↓

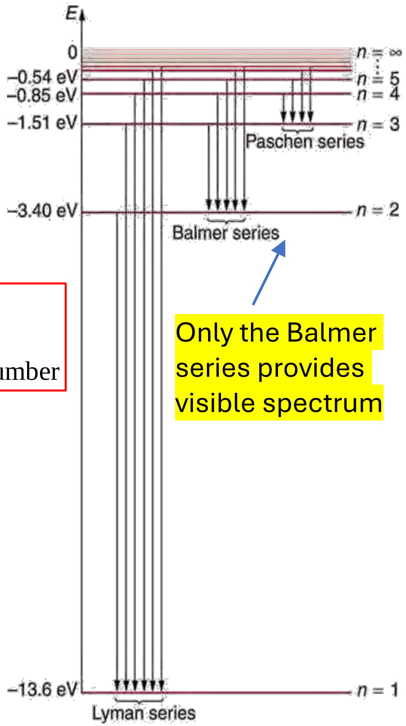
s p d e f ← **Atomic orbitals**

For each principal quantum number ***n*** there are *n^2* degenerate states, and each state can be occupied by 2 electrons. Consequently, an energy level *n* can be occupied by *$2n^2$* electrons

For a discussion of orbitals and their electron occupancy:

https://www.youtube.com/watch?v=Ewf7RlVNBSA

Energy levels and spectral lines



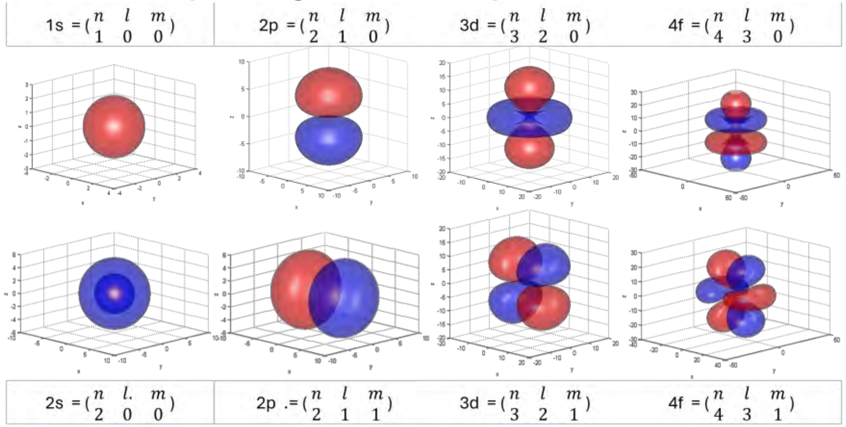
Only the Balmer series provides visible spectrum

Energy levels of the hydrogen atom and spectral series obtained by transitions between these levels.

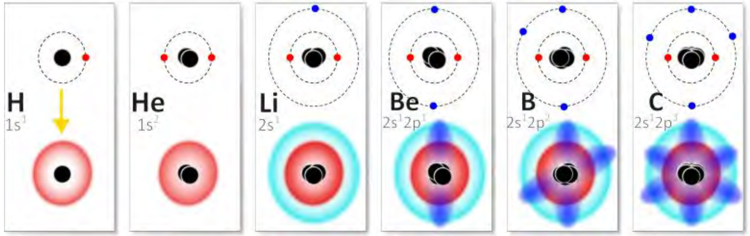
Atomic orbitals

- Atomic orbitals are the **spatial representation of probability density: $\psi^*_{n,l,m}\ \psi_{n,l,m}$.**
- Indicates where the electron will be with maximum probability(>90%)
- According to Pauli's principle, 2 electrons can be found on each orbital (same set of quantum numbers n, l, m)

Orbitals corresponding to different quantum numbers n,l,m



Comparison of semi-quantum and quantum representation



IX

Conductors, semi-conductors, insulators and superconductors

Content:

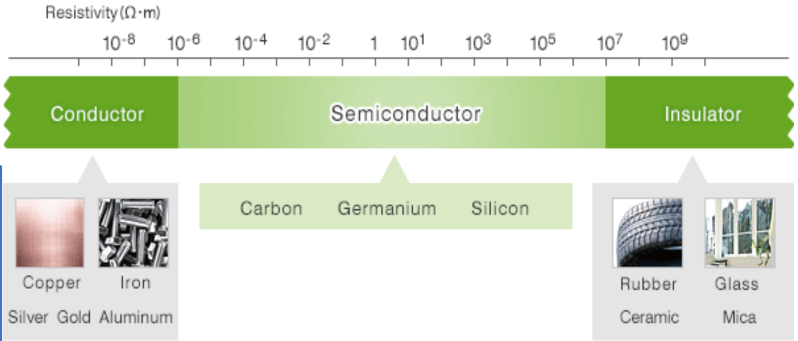
1. Introduction. Energy bands

From the point of view of electrical conduction, materials can be classified into **conductors**, **semiconductors**, **insulators** and **superconductors.** It is the electrical resistivity of materials that differentiates them essentially.

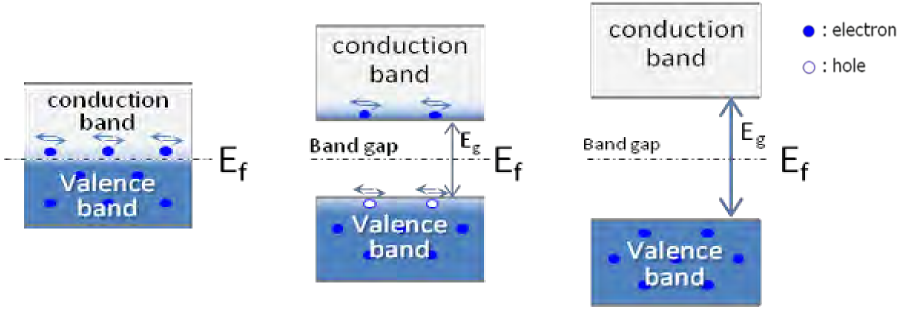
Superconductor
Resistivity=0

Superconductors are materials with zero electrical resistivity below a certain critical temperature(T_C).

The electrical resistance of such material is 0



The characteristic electrical properties are based on the energy difference separating the valence band from the conduction band



metal semiconductor insulator

https://www.hitachi-hightech.com/global/products/device/semiconductor/properties.html

2. Energy bands in solids

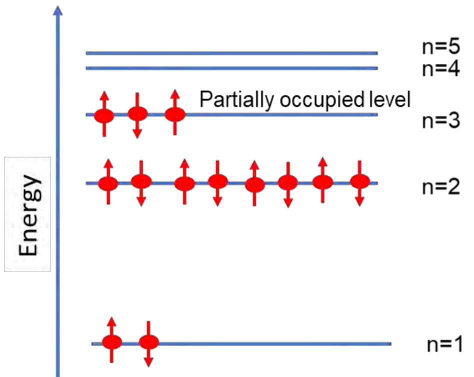
- As mentioned in the quantum physics section, the energy levels of individual atoms are quantized. This means that the electrons in the atom cannot have any energies but only those energies that are specified by the relationship:

$$E_n = -\frac{mZ^2e^4}{8h^2\varepsilon_0{}^2}\frac{1}{n^2} \quad n = 1, 2, 3, \ldots - \text{principal quantum number}$$

- On each energy level can sit a maximum of $2n^2$ electrons. This means that energy levels can be occupied totally or partially with electrons. For example, in the case of the Al atom, the level n=3 is only partially occupied by electrons (3) while it may be occupied by 18 electrons.

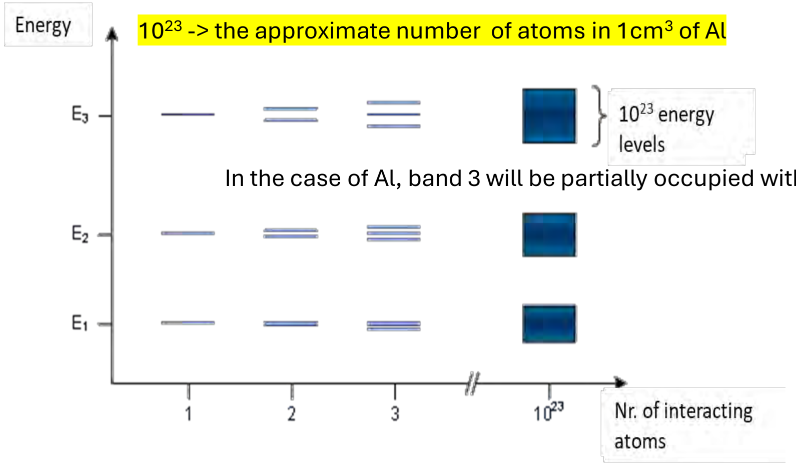
- In the case of atoms in solids (e.g. metallic Al) the energy levels split in the form of energy bands that will be in turn totally or partially occupied by electrons.

Discrete energy levels of the isolated aluminium atom (Al)



Aluminum has 13 electrons, with the configuration: $1s^22s^22p^63s^23p^1$

Energy levels split due to the interaction of atoms, forming energy bands

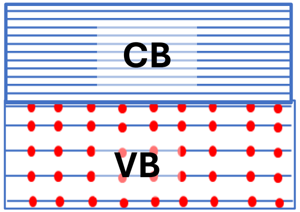


3. Conductors, semiconductors and insulators

- For electrical conduction, only the last two energy bands are important: the **valence band** and the **conduction band**.
- **Valence band (VB)** is the last energy band occupied or partially occupied by electrons.
- **Conduction band**(CB) is the first band free of electrons above the valence band, i.e. there are free energy levels.
- The valence band is sometimes separated from the conduction band by a band of forbidden energy ΔE (activation energy).
- Depending on the energy separation between the valence band and the conduction band, materials can be classified into: conductors, semiconductors and insulators (as shown in the figures below).

Note: *Electrons can flow (thus forming an electric current) only if there are available energy states within the band they occupy.*

Conductors



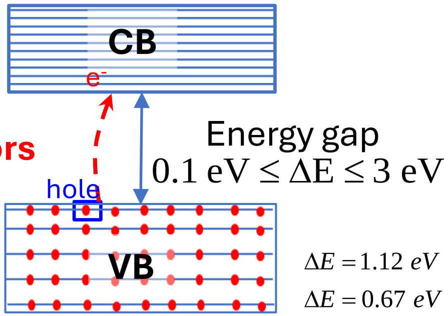
- Due to thermal motion, electrons from VB easily pass into CB where there are free energy levels, and thus, under the action of an external electric field, they can move.
- Because of electron collisions with the atomic lattice, it follows that electrical conductivity decreases with temperature:

$$\sigma = \frac{\sigma_0}{1+\alpha\left(T-T_0\right)}$$

σ_0 – conductivitaty at a temperature T_0;
T - absolute temperature (K)
α - constat of the material;

Valid over small temperature ranges

Semiconductors



$\Delta E = 1.12\ eV$ for *Si*

$\Delta E = 0.67\ eV$ for Ge

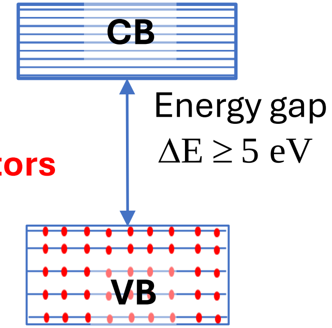
- By increasing the temperature, more and more electrons, electrons from VB pass over the forbidden band in CB, where there are free energy levels. The electron leaving VB leaves a gap (hole) in its place, which can be filled by another electron. Thus, under the action of an external electric field, a displacement of both electrons and holes occurs.
- The electrical conductivity of semiconductors increases by increasing the temperature according to the relationship:

$$\sigma = \sigma_0 e^{-\frac{\Delta E}{2kT}}$$

σ_0 – constant specific for the material
ΔE – activation energy of the semiconductor
k - Boltzmann's constant; T - absolute temperature (K)

$kT \cong 0.025eV$ at room temperature $\Rightarrow$ there are electrons transiting from VB to CB

Insulators



- By increasing the temperature there is little chance that electrons will pass from VB to CB and thus the material will conduct.
- Only at very high temperatures, or when the material is pierced and the electrons pass into CB, does the material become conductive.

4. Intrinsic and extrinsic semiconductors

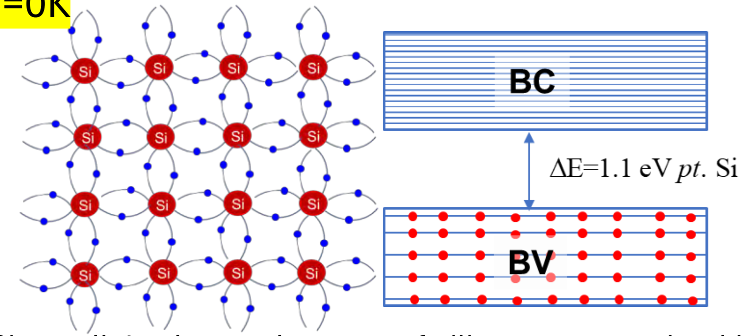
- Semiconductor materials are very numerous and can be chemically pure elements or combinations thereof. Typical representatives of semiconductor materials are silicon (Si) and germanium (Ge), which belong to group IV of the periodic table and have 4 electrons per valence layer. Semiconductors can be of two types: intrinsic and extrinsic.

4.1. Intrinsic semiconductors

They are pure materials, consisting of atoms found in group IV of the periodic system (e.g. Si, Ge) or combinations of atoms (e.g. GaAs, InSb, ZnS). To understand intrinsic semiconductors we will refer here to silicon (Si)

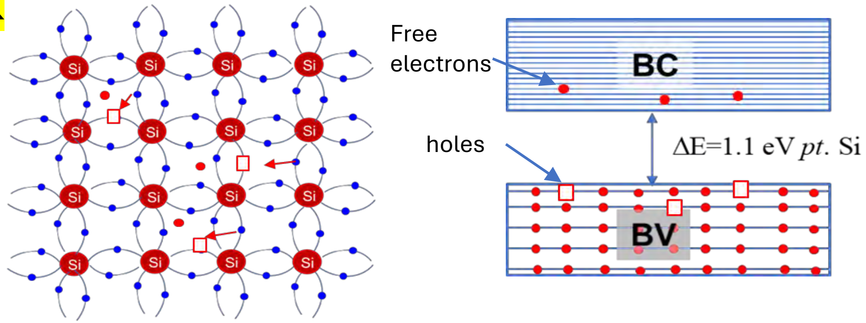
The atom of **Si** has 14 electrons, of which 4 are found on the last layer, forming covalent bonds with 4 other silicon atoms.

T=0K



Since all 4 valence electrons of silicon are contained in covalent chemical bonds, and therefore cannot flow through the network, we can say that at T=0K the Si electrons completely occupy the valence band. Consequently, at T=0K silicon is a perfect insulator

T>0K



At T>0K temperatures, some of the electrons **leave the valence band** and become **free**. This is equivalent to the passage of electrons from BV to BC. Instead of electrons, holes (surplus positive charge) remain in the BV that can be occupied by other electrons in the BV. Thus, under the action of an external electric field, a current consisting of electrons and holes can arise.

The electrical conductivity of intrinsic semiconductors depends on temperature:

The conductivity increases with temperature ←

$$\sigma = \sigma_0 e^{-\frac{\Delta E}{2kT}}$$

σ_0 – constant depending on material

ΔE – activation energy of the semiconductor

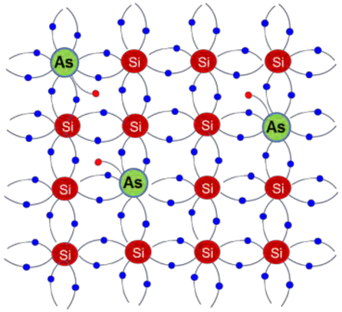
k - Boltzmann's constant; T - absolute temperature (K)

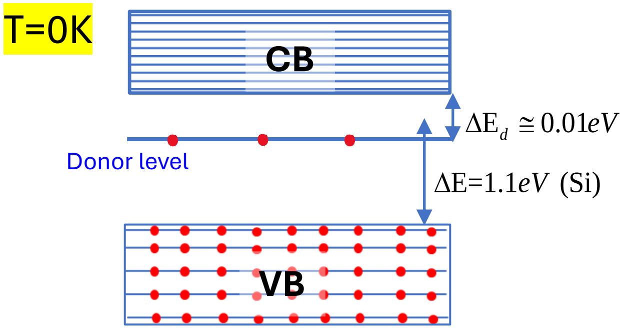
4.2. Extrinsic Semiconductors

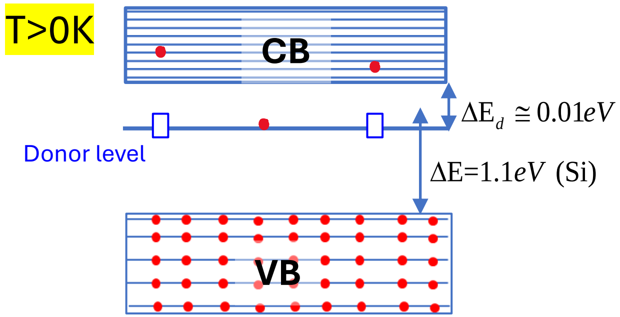
- They are those semiconductors that are obtained by introducing impurity atoms into pure silicon or germanium
- The degree of doping with impurity atoms is generally very small (1 atom per million Si atoms) so that the impurity atom is surrounded only by Si (or Ge) atoms.
- Extrinsic semiconductors can be n-type or p-type.

n-type semiconductors

If silicon (Si) doping is done with **pentavalent elements**, n-type semiconductors are obtained from group V-th of the periodic system (N, P, As, Sb)







- By introducing an arsenic (As) atom into the lattice, which possesses 5 valence electrons, results in a quasi-free electron (which is not trapped in the covalent bond). This electron can be very easily broken by the As atom, at T>0K temperatures, and thus circulate in the network in the form of a current of electrons, if an external electric field (as a result of a voltage) is applied.
- In the interpretation based on energy bands, the quasi-free electron is represented on an energy level very close to BC (donor level) and from where it can easily jump into BC. The passage from the donor level to the BC is favored by the increase in temperature and therefore the electrical conductivity of the material depends on the temperature:

$$\sigma = \sigma_0 e^{-\frac{\Delta E_d}{2kT}}$$

σ_0 – constant of material

ΔE_d – activation energy of the donor level $(\Delta E_d << \Delta E)$

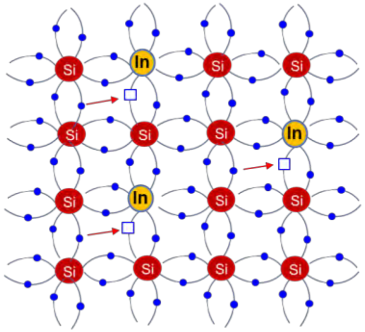
k - Boltzmann's constant;

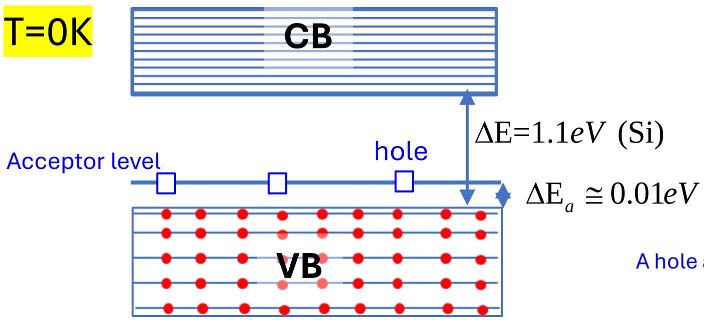
T - absolute temperature (K)

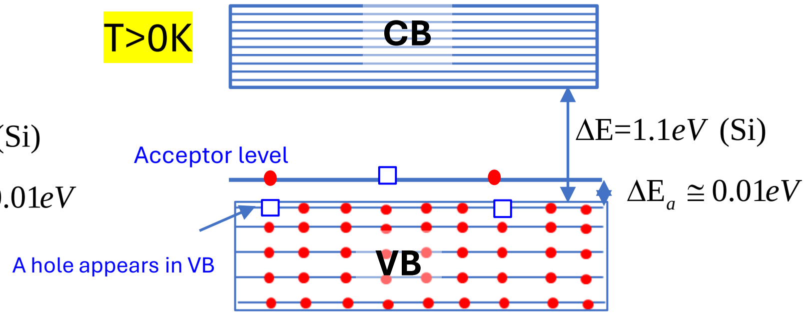
At low temperatures, electrons from the donor level, which have reached BC, dominate the current and that is why semiconductors are called n-type. There is no transport of holes here because the holes created at the donor level are fixed positive ions and do not move through the lattice, therefore, they do not directly contribute to the electrical current transport

p-type semiconductors

If silicon doping is done with **trivalent** elements, **p-type** semiconductors are obtained from group III of the periodic system (B, Al, Ga, In)







- By introducing an indium (In) atom into the lattice, which possesses 3 valence electrons, an incomplete covalent bond results. To complete the bond, at T>0K temperatures, an electron can be moved from a neighboring atom and in this case the hole is moved to the neighboring atom. So, there is a displacement of the hole through the network that is formally equivalent to the displacement of a positive charge. In the absence of an external electric field, this displacement of the holes is random, but in the presence of the field, a current of holes (positive charges) arises.
- In the interpretation based on energy bands, the possibility of a covalent bond to accept an electron from another atom is represented by an acceptor energy level near BV. The accepting energy level is occupied with as many holes as the atoms of impurity we have. At temperature T>0K, the electrons in the BV can easily pass to the acceptor level, occupying the holes. In their place, the electrons leave gaps in the BV that can be occupied by other electrons that in turn leave gaps, and thus a gap shift occurs. The passage of electrons from the BV to the acceptor level is favored by temperature, thus the conductivity increases with increasing temperature:

$$\sigma = \sigma_0 e^{-\frac{\Delta E_a}{2kT}}$$

σ_0 – constant of material

ΔE_d – activation energy of the acceptor level $(\Delta E_a << \Delta E)$

k - Boltzmann's constant;

T - absolute temperature (K)

At low temperatures, the holes in the BV, which arrived there as a result of the transition of electrons from the BV to the acceptor level, dominate the current and that is why semiconductors are called p-type. There is no electron transport here because the electrons on the acceptor level have the energy determined and cannot change it under an external electric field.

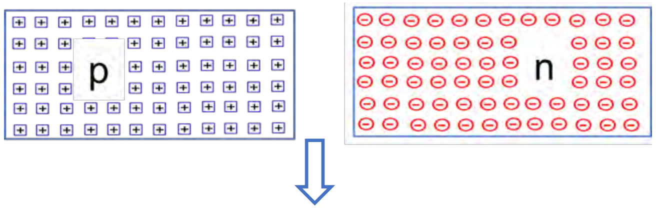
5. The p-n junction . The semiconductor diode

- By joining a p-type semiconductor (contains holes) with an n-type semiconductor (contains free electrons) a junction (area) is obtained in which the electrons from n will migrate to p (to fill in the holes). As a result of this migration process, in region n there is a surplus of positive charge and in region p a negative charge accumulates.
- At equilibrium, an **internal electric field arises** inside the junction of the semiconductor, oriented as in the figure, a field that prevents the electrons from displacing further. The junction that is formed has dimensions of the order of 0.1μm ÷ 10 μm.

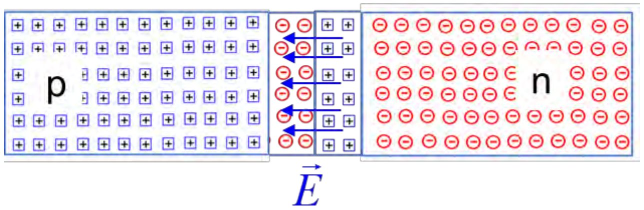
The p-n junction

The p-semiconductor is neutral but contains holes (the desire to attract electrons from n)

Semiconductor n is neutral, but contains free electrons (which can move)

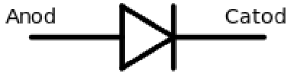


p-n junction



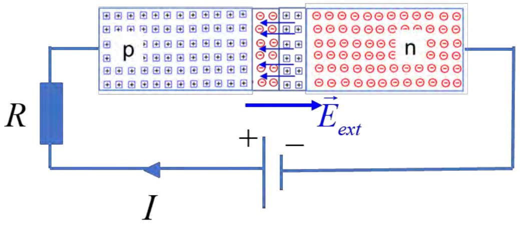
The internal electric field **E** prevents the transport of electrons from region n to p

The semiconductor diode



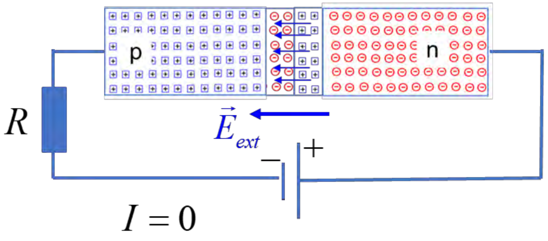
The arrow indicates the direction of current flow through the diode

Direct polarization



The external electric field $\mathbf{E}_{ext}$ opposes the internal one, thus reducing the width of the junction and allowing the transport of electrons from n to p. In this case the diode **conducts**.

Inverse polarization

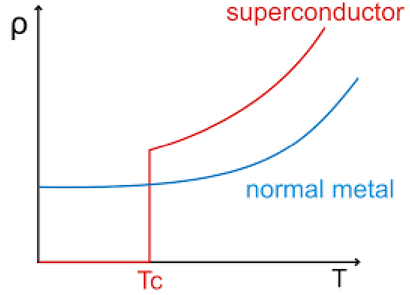


External electric field $\mathbf{E}_{ext}$ It is parallel to the internal one, thus increasing the width of the junction and blocking the transport of electrons from n to p. In this case the diode **does not conduct**.

6. Superconducting materials

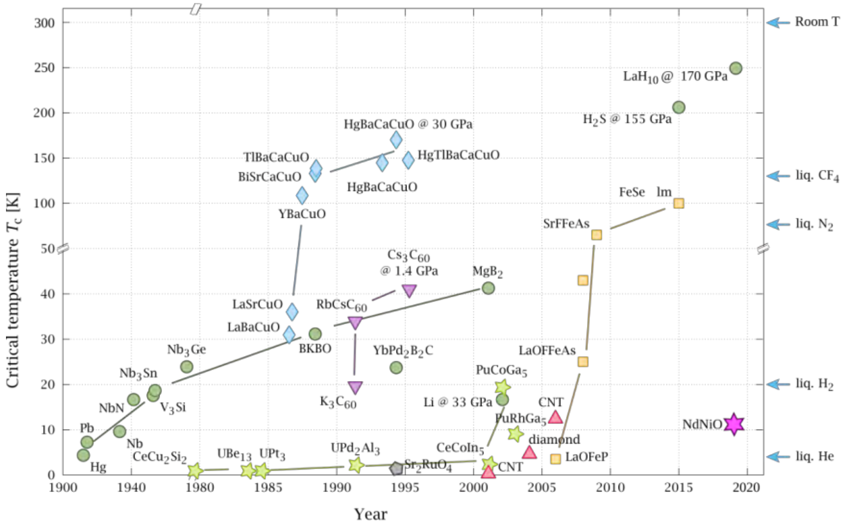
- Superconductors are materials with zero electrical resistivity, below a certain critical temperature (T_C)

The electrical resistance of such a material is 0



- If introduced into the magnetic field, superconductors deflect the field lines so that the magnetic field does not penetrate the superconductors. This effect of deflection of field lines is called the **Meissner effect** and allows the explanation of magnetic levitation on superconductors.
- We can say that inside the superconductor a magnetic field is created that compensates for the external field and thus $\chi = -1$, that is, we have a **perfect diamagnetic**.
- However, let us mention that the **superconducting state can be destroyed by the external magnetic field**, if it exceeds a certain critical value.

Critical temperature of certain materials



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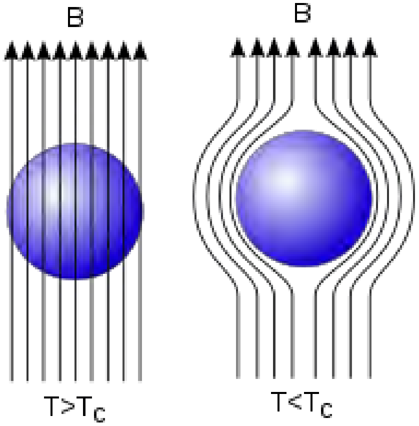
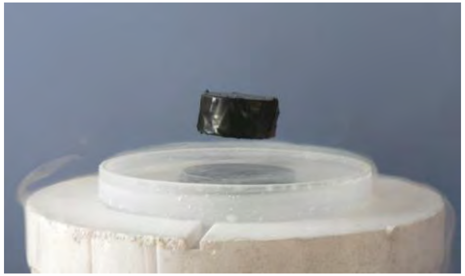
The Meissner effect



https://en.wikipedia.org/wiki/Meissner_effect

Levitation of a magnet on a superconductor



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